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Problems of Precision in Fuzzy Theories of Vagueness and Bayesian Epistemology

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Abstract

A common objection to theories of vagueness based on fuzzy logics centres on the idea that assigning a single numerical degree of truth—a real number between 0 and 1—to each vague statement is excessively precise. A common objection to Bayesian epistemology centres on the idea that assigning a single numerical degree of belief—a real number between 0 and 1—to each proposition is excessively precise. In this paper I explore possible parallels between these objections. In particular I argue that the only good objection along these lines to fuzzy theories of vagueness does not translate into a good objection to Bayesian epistemology. An important part of my argument consists in drawing a distinction between two different notions of degree of belief, which I call dispositional degree of belief and epistemic degree of belief.

1 Introduction

A common objection to theories of vagueness based on fuzzy logics centres on the idea that assigning a single numerical degree of truth—a real number between 0 and 1—to each proposition is excessively precise. A common objection to Bayesian epistemology centres on the idea that assigning a single numerical degree of belief—a real number between 0 and 1—to each proposition is excessively precise. There is a striking surface similarity between these objections. I have discussed the objection to fuzzy theories of vagueness elsewhere (Smith, 2008, 2011). I have argued that it is a powerful objection to a basic fuzzy theory of vagueness—but I have proposed a more sophisticated fuzzy theory, which avoids the objection. This paper discusses the argument against Bayesian epistemology. In particular it asks whether there is a good argument against Bayesian epistemology *of the same sort* as the good argument against the basic fuzzy theory of vagueness. I'll argue that there isn't: despite the surface similarity, a line of argument that is powerful in one context fails in the other. The main purpose of this investigation is to clarify the conceptual terrain—not to argue for (or against) Bayesianism. There is (as we'll see) another kind of argument against Bayesian views that works in a different way from the argument against fuzzy theories of vagueness—and although I'll *distinguish* this other kind of argument, I shall not *assess* it here.

The paper proceeds as follows. Section 2 introduces the basic fuzzy theory of vagueness and the objection that it involves artificial precision. Section 3 introduces Bayesian epistemology and the objection that *it* involves false precision. Section 4 introduces an important distinction between two kinds of degree of belief, which I call *dispositional* degree of belief and *epistemic* degree of belief. Section 5 explores whether there is a good argument, to the conclusion that Bayesian epistemology involves false precision, that runs along the same lines as the argument that fuzzy theories of vagueness involve artificial precision: I'll argue that there isn't. Section 6 turns to actual arguments in the literature (to the conclusion that Bayesian epistemology involves false precision). In light of the discussion in Section 5, we'll see that some of these arguments fail—while others are of a quite different nature from the artificial precision objection to fuzzy theories of vagueness.

2 Fuzzy Theories of Vagueness

A classical or crisp set is a collection of objects, where each object is either (definitely) in the set or (definitely) not in it. A fuzzy set is a collection of objects, where objects may be in the set to greater or lesser degrees. These degrees are modelled by real numbers from 0 (representing definite or total non-membership) to 1 (representing definite or total membership). A fuzzy set—or more precisely, a fuzzy subset S of some background universe U —is then a function from U to the real unit interval $[0, 1]$. For any object a in U , the number to which a is mapped by the function is a 's degree of membership in S .

The core idea of fuzzy theories of vagueness is that the extensions of vague predicates such as 'tall' and 'heavy' are fuzzy sets and (hence) that sentences may have intermediate degrees of truth. For example, the degree of truth of the sentence 'Bob is tall' will be the same as Bob's degree of membership in the fuzzy set of tall things. This degree is a real number in the interval $[0, 1]$. So in the fuzzy framework, reals in $[0, 1]$ do double duty as degrees of membership (of objects in sets) and degrees of truth (of sentences).

A classic objection to fuzzy theories of vagueness is that they involve excessive or artificial precision:

[Fuzzy logic] imposes artificial precision. . . [T]hough one is not obliged to require that a predicate either definitely applies or definitely does not apply, one *is* obliged to require that a predicate definitely applies to such-and-such, rather than to such-and-such other, degree (e.g. that a man 5 ft 10 in tall belongs to *tall* to degree 0.6 rather than 0.5) (Haack, 1979, 443)

One immediate objection which presents itself to [the fuzzy] line of

approach is the extremely artificial nature of the attaching of precise numerical values to sentences like ‘73 is a large number’ or ‘Picasso’s *Guernica* is beautiful’. In fact, it seems plausible to say that the nature of vague predicates precludes attaching precise numerical values just as much as it precludes attaching precise classical truth values. (Urquhart, 1986, 108)

[T]he degree theorist’s assignments impose precision in a form that is just as unacceptable as a classical true/false assignment. In so far as a degree theory avoids determinacy over whether a is F , the objection here is that it does so by enforcing determinacy over the *degree* to which a is F . All predications of ‘is red’ will receive a unique, exact value, but it seems inappropriate to associate our vague predicate ‘red’ with any particular exact function from objects to degrees of truth. For a start, what could determine which is the correct function, settling that my coat is red to degree 0.322 rather than 0.321? (Keefe, 1998, 571)

The objection to the fuzzy approach is that it is artificial or implausible or somehow inappropriate to associate each vague *predicate* in natural language with a particular function that assigns a real number to each object (the object’s degree of possession of the property picked out by that predicate) and to associate each vague *sentence* in natural language with one particular real number (the sentence’s degree of truth).

I have addressed this problem in detail in Smith (2008) and Smith (2011). I have argued that the problem is a genuine one and that at its core are considerations about how meanings are determined. It is generally accepted that language is a human artefact. The sounds we make mean what they do because of the kinds of situations in which we (and earlier speakers) have made (and would make) those sounds (e.g. had the word ‘dog’ always been used where ‘cat’ was used, and vice versa, then ‘dog’ would have meant what ‘cat’ in fact means, and vice versa). So there is an essential connection between *meaning* and *use*. More precisely, consider the following kinds of facts:

- All the facts as to what speakers actually say and write, including the circumstances in which these things are said and written, and any causal relations obtaining between speakers and their environments.
- All the facts as to what speakers are disposed to say and write in all kinds of possible circumstances.

There is widespread agreement in the literature that semantic facts are never primitive or brute—they are always determined by some other, meaning-determining

facts. There is also widespread agreement that the meaning-determining facts are the ones just itemised: facts about actual usage, counterfactual usage and usage dispositions.¹ Thus, if these facts are insufficient to determine (unique) meanings for some utterances, then those utterances have no (unique) meanings.

This generates a constraint on any theory of vagueness: if the theory says that vague predicates have meanings of such-and-such a kind (e.g. fuzzy sets), then we must be able to satisfy ourselves that the meaning-determining facts could indeed determine such meanings for actual vague predicates. To the extent that the meaning-determining facts do *not* appear sufficient to determine meanings for vague predicates of the kind posited by some theory of vagueness, that theory is undermined.

This is precisely where the basic fuzzy theory of vagueness—according to which each vague discourse is associated with a unique intended fuzzy model—runs into trouble. For it certainly seems that usage and usage dispositions do not suffice to pick out a particular function from objects to fuzzy truth values representing the extension of ‘is tall’ (and similarly for other vague predicates). Our tendencies to classify certain people as (clearly) tall, certain others as (clearly) not tall, and to hedge over other cases (the borderline cases)—including any tendencies we might have to be more or less confident or reticent about different borderline cases—just do not, it seems, form a rich enough base to determine a *unique* function that assigns a degree of tallness (a real number between 0 and 1) to each object. Keefe asks (in the passage quoted above) “what could determine which is the correct function, settling that my coat is red to degree 0.322 rather than 0.321”? The answer, I have argued, is that what could determine this is our usage (actual and counterfactual/dispositional). But now the problem: our usage *does not* (it would seem) determine a unique function. Hence the basic fuzzy theory of vagueness is the wrong model of vague discourse.

The story does not end there: I have also presented and argued for a solution to the problem. The solution is to move from a basic fuzzy theory of vagueness (on which each vague discourse is associated with a unique intended fuzzy model) to a theory that I call *fuzzy plurivaluationism* (on which each vague discourse is associated with multiple acceptable fuzzy models). But these further developments are not relevant for our present purposes. We shall turn now to a problem for Bayesian epistemology that is, on the surface, very similar to the artificial precision problem for the basic fuzzy theory of vagueness.

¹Some authors would also like to include other kinds of facts, such as facts concerning the eligibility as referents of objects and sets, or facts concerning the simplicity/complexity of interpretations. Even on these views, however, usage plays a central role.

3 Bayesian Epistemology

‘Bayesian epistemology’ denotes a family of views sharing three core tenets:

1. An agent’s ‘belief state’ can be represented by an assignment to propositions of real numbers between 0 and 1 inclusive.²
2. In so far as the agent is rational, this assignment will obey the laws of probability.³
3. In so far as the agent is rational, she will update her assignment in the face of new evidence by conditionalisation.^{4,5}

Tenet 1 is a descriptive claim and tenets 2 and 3 are normative claims. Bayesian epistemology is thus partially descriptive and partially normative. Tenet 1 says that agents can in fact be represented as assigning numbers to propositions (a descriptive claim), and tenet 2 says that these assignments should (although in fact they may not always) obey the laws of probability (a normative claim; and similarly for tenet 3).

In this paper my main focus is on tenet 1—and in particular, on a certain kind of objection to it.⁶ The objection is that—in supposing that agents can be represented by a unique assignment of real numbers to propositions—it involves excessive or artificial precision:

I find it wildly implausible to suppose (as some orthodox Bayesians have done) that actual investigators in general harbor precise, real-valued degrees of confidence for hypotheses. Even when construed as a regulative ideal, the requirement that investigators harbor such

²Different Bayesians use different terms where I have used ‘belief state’—e.g. ‘credal state’, ‘credences’, ‘degrees of belief’, ‘degrees of confidence’, and ‘subjective probabilities’. At this point I use the term ‘belief state’ with scare quotes and without an explanation of what the term means, because a key issue below—beginning in section 4—will be what, exactly, the agent’s ‘belief state’ is supposed to be.

³Taking propositions to be sets of possible worlds, the second tenet becomes the requirement that the assignment of real numbers to propositions constitutes a probability measure over the space of possible worlds.

⁴Given tenet 2, the agent’s assignment of real numbers to propositions can be referred to as a probability assignment. Tenet 3 then requires that the agent’s posterior probability assignment to a proposition P (after the evidence E comes in) is equal to her prior (before the evidence comes in) conditional probability of P given E .

⁵Some characterisations of Bayesianism (e.g. Joyce 2010, 282) add a fourth tenet: rational agents make decisions by maximising expected utility.

⁶Objections have also been made to tenets 2 and 3—but they are not my concern in this paper.

precise degrees of confidence looks as if it owes more to an unfortunate worship of false precision than it does to reason. (Kaplan, 1996, x)

the orthodox Bayesian account of belief is inaccurate. The nature of the inaccuracy is that it requires point-valued credences, which carries a commitment to the existence of sharp lines where, intuitively, there aren't any (Rinard, 2017, 262)

In general, our opinion is certainly not expressible in precise numerical probabilities. (Van Fraassen, 1990, 345)

The objection is that it is artificial or implausible or somehow inappropriate to represent an agent's belief state by an assignment of unique numerical values to each proposition.

Why? Is there an argument here? There are indeed arguments in the literature. Before we look at these existing arguments in section 6, however, I want (in section 5) to explore whether there is a good argument to this conclusion that runs along similar lines to the dispositional argument against precision in the case of fuzzy theories of vagueness—and before undertaking this exploration, I shall (in section 4) introduce an important distinction.

4 Two Aspects of Thinking Agents

There is an influential view, going back to Ramsey, according to which degrees of belief are strengths of behavioural dispositions:

the degree of a belief is a causal property of it, which we can express vaguely as the extent to which we are prepared to act on it. . . . it is not asserted that a belief is an idea which does actually lead to action, but one which would lead to action in suitable circumstances. . . . The difference [between believing more firmly and believing less firmly] seems to me to lie in how far we should act on these beliefs. (Ramsey, 1926, 65–6)

On this view, one's degree of belief that *S* is a measure of the strength of one's tendency to act as if *S*.

There is another widespread view of degrees of belief, according to which an agent's degrees of belief constitute her view about how the world is, as justified by the evidence available to her. For example, Walley (1991, 14) writes that “Epistemic probabilities. . . depend on the available evidence” and Sturgeon (2008, 159) writes of “evidence and attitude aptly based on it.”

On the first conception—which I’ll call the *dispositional* conception—degrees of belief are essentially related to (possible) action: they are (strengths of) behavioural dispositions. On the second conception—which I’ll call the *epistemic* conception—degrees of belief are essentially related to evidence and rational judgement: they are evidence-based opinions about the way the world is.

Most authors seem simply to adopt one conception without mentioning or acknowledging the other. Some authors do note the distinction—but even then there is a tendency to write as if what we have here are two rival conceptions of one thing (‘degrees of belief’). I think it is much more fruitful to view this distinction—at least initially—as a distinction between two aspects of thinking agents. Both aspects are present—and we need accounts of both.

On the one hand, thinking agents need to keep a running view of the way the world is, and this view needs to be appropriately determined or justified by the evidence available to them. There should not be anything in this picture of the world that isn’t warranted by the available evidence. Now sometimes the evidence tells us clearly that something is or is not the case, but at other times the evidence is less decisive—and so we should suppose that the agent’s running view of how the world is will be a matter of degree rather than all-or-nothing: some ways the world might be are definitely ruled out as not being the case; some are definitely ruled in; but others are only more or less ruled out or in.

On the other hand, thinking agents act in the world—they make choices and decisions—and at any time, they have dispositions to make these choices in certain ways. Knowing how an agent will choose can be crucial to interacting with him. These dispositions are also a matter of degree, rather than all-or-nothing. An agent need not have a full-on tendency to make just one choice in a certain kind of situation—and no tendency at all to make any of the other choices that would also be possible in that situation. Rather, he may have stronger or weaker tendencies to make various choices.

Clearly both aspects are, and should be, present in typical thinking agents. There is no reason to think we can get a full understanding if we focus on one aspect alone. Nor should we assume, in advance, that these two aspects of thinking agents must really, at bottom, be the same aspect. There’s certainly reason to think there should be some relation between them—any view will need to allow that the choices of *rational* agents, at least, should be sensitive to their view of how the world is—but just how close the relationship should be is a big question. I cannot explore this question in detail here—but I can at least note some possible positions. Certain dispositionalist or functionalist views of belief would seem to suggest that ultimately there can be little or even no real distinction here: that having (say) a mid-strength tendency to act as if *P* is necessary and sufficient for the attribution of a mid-strength epistemic degree of belief that *P*. Other views of belief—for example certain representationalist views—would seem to allow room

for a real distinction between an evidence-based and rationally inferential realm of epistemic beliefs and a realm of behavioural dispositions. Yet other views would seem to demand a significant distinction here: see for example Gendler (2008a,b) on alief and belief. In the end, then, it *may* be that there is a good theoretical argument leading to one or more of the following conclusions: that the epistemic and dispositional aspects of thinking agents are ultimately one and the same aspect; that an agent's epistemic degree of belief that *S*—the degree to which *S*'s being the case forms part of her evidence-based picture of how the world is—should always equal her dispositional degree of belief that *S*—the strength of her tendency to act as if *S*; or that there is some tool—for example probability theory—that models both aspects of thinking agents in one go. But given the deep theoretical issues involved, we should not *begin* by assuming that any of these conclusions are true. We should start by acknowledging two aspects of thinking agents—epistemic and dispositional—and the need for theories of both of them, even if they might turn out in the end to be the same theory. Certainly we should not start with a theory—for example probability theory—and assume that it is the only possible tool available, or that we can use it to model everything.

While it is not common, the viewpoint that I have just outlined—according to which we have a distinction between two coexisting aspects of thinking agents (dispositional degrees of belief and epistemic degrees of belief), rather than two rival conceptions of one aspect ('degrees of belief')—is shared by some other authors. Kyburg Jr. (1983) is a possible example—and the Transferable Belief Model (Smets and Kennes, 1994) is a clear example of a well worked-out framework in which this distinction plays a crucial role.⁷

5 A Dispositional Argument Against Bayesianism?

The artificial precision problem for the basic fuzzy theory of vagueness ultimately boils down to usage dispositions: those dispositions are not rich or detailed enough to determine unique fuzzy sets as meanings for vague predicates. The question now is whether there is a genuine problem *of the same sort* for Bayesian epistemology: that is, a problem to do with our dispositions not being rich or detailed enough to determine unique numerical assignments to propositions. (In the Bayesian case, these numerical assignments represent degrees of belief, not degrees of truth.) Note that if there is such a problem, it is primarily a problem for tenet 1 of Bayesianism.

In the previous section I distinguished two notions of degree of belief: degree

⁷The epistemic/dispositional distinction that I have just drawn is not the same as the traditional distinction between occurrent and dispositional belief: the latter is most naturally viewed as a distinction amongst epistemic beliefs.

of belief that S as strength of tendency to act as if S (dispositional degree of belief); and degree of belief that S as firmness of evidence-based opinion as to whether S is true (epistemic degree of belief). When it comes to dispositional arguments, the action must centre on dispositional degrees of belief. Degrees of belief of this kind just are (strengths of) behavioural dispositions—so to the extent that an agent lacks behavioural dispositions, it *ipso facto* lacks (determinate) degrees of belief of this kind. Epistemic degrees of belief, on the other hand, either are not subject to a dispositional argument at all, or are subject to such an argument only via a dispositional argument concerning dispositional degrees of belief. Which of these possibilities one takes to be the case will depend on one's substantive view of the relationship between dispositional and epistemic degrees of belief. On a view on which epistemic degrees of belief are to a large extent independent of behavioural dispositions, there can be no dispositional argument against epistemic degrees of belief. On this sort of view, the notion of epistemic degree of belief—strength of opinion as justified by the available evidence—is not a dispositional notion. A purely contemplative being with no behavioural dispositions—for example a computer programme—might have degrees of belief in this sense. Lack of dispositions to do things is no threat to the existence or determinacy of degrees of belief of this kind. On a view on which epistemic degrees of belief are more closely tied to behavioural dispositions—for example a view on which having (say) a mid-strength tendency to act as if P is necessary and sufficient for the attribution of a mid-strength epistemic degree of belief that P —there will be room for a dispositional argument against determinate epistemic degrees of belief, but it will go via a dispositional argument against determinate dispositional degrees of belief. On this sort of view, a dispositional argument can get a grip on epistemic degrees of belief only by first getting a direct grip on dispositional degrees of belief and then transferring that grip via theoretical claims linking (or even identifying) dispositional and epistemic degrees of belief. Thus, either way, when it comes to dispositional arguments, the action must centre on dispositional degrees of belief.

So let's work with the conception of degree of belief as strength of tendency to act, and investigate whether a dispositional argument shows that our degrees of belief (of this kind) cannot be represented by unique numerical assignments. Let A be a typical human agent. The argument would be that for many or all propositions S , A 's behavioural dispositions are insufficiently rich to enable the attachment of a unique numerical degree to A 's tendency to act as if S . Now considered in the abstract, this may seem plausible. But thanks to de Finetti and others, we can see that it is incorrect. Consider an agent A and an amount of money that is neither so small that A would not care about winning or losing such an amount nor so big that A would not want to bet such an amount if there were any risk at all of losing the bet. For the sake of argument, let the amount be \$1. Now take a proposition S . There is to be a bet on S . There are two parties to the bet. One party will

take on the Pro-role: will win the bet if S turns out to be true (and lose if S turns out to be false). The other party will take on the Con-role: will win the bet if S turns out to be false (and lose if S turns out to be true). The *betting ratio* x of the bet determines how much is won and lost: x is a real number between 0 and 1 (inclusive) such that Pro pays $\$x$ to bet, Con pays $\$(1 - x)$ to bet, and the winner takes all (i.e. $\$x + \$(1 - x) = \$1$). Now A is to set the betting ratio x in such a way that she would be happy to play the Pro role or the Con role. (Compare cutting a cake into two pieces in such a way that you would be happy to get either of them.) The idea is that x can then be taken as a numerical representation of the strength of A 's tendency to act as if S .

Now it seems to me that for most agents A and most propositions S , if A were in a situation where a bet on S is to take place and A must set the betting ratio x , A would set some x . Of course there might be some cases in which A freezes up completely and is simply unable to set x —but I think that would not be the norm. If one really were required to set x , one would. Hence one *does* have dispositions to set x . So the dispositional argument fails: our behavioural dispositions *are* rich enough to underpin unique numerical assignments to degrees of belief (conceived as strengths of tendency to act).

Several issues need to be clarified at this point.

(1) Do not confuse sitting in an armchair thinking about an imaginary betting situation with really being in a situation where you must set the betting ratio. Maybe you have no idea what x you'd set when you imagine the situation. Maybe you can't imagine setting x at all. This is beside the point. My claim is that you *would* set x and hence you *do* have a disposition to set x .

(2) I have taken from de Finetti the idea of measuring degrees of belief by betting ratios—but I am not adopting the operationalist perspective that he expresses in passages such as the following:

In order to give an effective meaning to a notion—and not merely an appearance of such in a metaphysical-verbalistic sense—an operational definition is required. By this we mean a definition based on a criterion which allows us to measure it. We will therefore be concerned with giving an operational definition to the prevision of a random quantity, and hence to the probability of an event. (de Finetti, 1974, 76)

The picture I'm working with is this. An agent, in this actual world, has a determinate strength of tendency to act as if S . This is a dispositional property of the agent, grounded in facts about his brain and body. Betting behaviour is one, but *not* the only kind of behaviour that falls under the heading 'acting as if S ': you can act as if it will rain by betting that it will but also by taking your umbrella. The agent's strength of tendency to act as if S certainly isn't *defined* as the number he

would give in another, nearby world if he had to set a betting ratio (in that world). However, thinking about such a world enables us to see that the agent does indeed have rich and detailed dispositions: rich and detailed enough to warrant modelling his strength of tendency to act as if S as a particular real number. The situation is analogous to the case of length measurement. I suppose that the table before me has some determinate length. I do not *define* this length as the number I would get if I measured the table with a tape measure. But if someone were to doubt that there are any facts about the table that warrant modelling its length as a particular real number, we could point out that were we to lay a tape measure on top of the table, we would see a particular mark on the tape measure lined up with the edge of the table, and there would be a particular number associated with that mark. Even if we never measure the table, there are facts about it that ensure we would get a particular answer if we did—and this justifies modelling its length as a particular number (as opposed, say, to a coarse-grained model in which we model lengths as ‘short’, ‘medium’, and ‘long’).

Note that this line of thought is supposed to function only as a ‘proof of concept’—not as a specification of a standardised measurement system. If we want the *official* length of the table—say for some legal purpose, such as determining whether a contract has been breached—it has to be measured by a trained professional using certified measuring equipment. Similarly, if we wanted an official measurement of Bob’s degree of belief that S , we would need to settle on a particular formulation of the bet to be offered, we would need to train and certify people to offer such bets in a particular way, and so on. Presumably a body such as the Australian National Measurement Institute could settle such matters—no doubt with some measure of arbitrariness and stipulation—if there were a need or reason to do so. But my purpose here is not to specify a particular procedure by means of which we can actually obtain a unique number as the measure of Bob’s degree of belief that S : it is simply to make it plausible—by thinking about how agents would set betting ratios—that their dispositions are sufficiently rich and detailed to warrant modelling strengths of tendency to act as particular numbers.

With these points in mind, we can respond to several potential worries.

(2a) Do not object that de Finetti’s procedure fails to get us sufficient precision: that it could never justify assigning, say, 0.3 rather than 0.300001 as the measure of some tendency to act. The fact that we can only measure lengths to a certain level of precision—say, down to the nearest millimetre if we are using an ordinary tape measure—does not threaten the idea that objects have unique lengths which can be represented by real numbers. Limited precision is a given in all cases of numerical measurement: it is not a special problem for degrees of belief.

(2b) Consider an agent S in a particular context C , in which S is not in fact asked to set a betting ratio for a bet on a proposition P . I have said that if S were asked to set a betting ratio, she would. The worry now is that if we think about

this in the way advocated by Lewis (1973), there might not be a single determinate ratio that *S* would set: it may be that in some of the worlds most similar to the actual world, *S* says 0.29, while in others—in which the question is asked in a slightly different tone of voice, or by a different questioner—*S* says 0.3. One response to this worry is to say that we should not think about the situation in Lewis’s way, but rather in Stalnaker’s way, according to which there is a unique most similar world in which the agent is asked to set a betting ratio (Stalnaker, 1968). A second response is to say that even if we stick with Lewis’s conception, the resulting indeterminacy is not a special problem for degrees of belief, but is of a piece with indeterminacy in other cases of measurement. For example, if the in-fact-unmeasured table were measured, the resulting length might be slightly different depending on who performed the measurement and what kind of measuring device was used. If we wanted an official length we would need to have a standardised measuring system and accredited measurers—and similarly for betting ratios. There is nothing here to cast doubt on the thought that the table has a determinate length: it is precisely because the table is the determinate way it is that we would get *this* answer if we measured it in this way and *that* answer if we measured it in that way. Similarly, there is nothing here to cast doubt on the thought that the agent has a determinate strength of tendency to act as if *S*: it is precisely because the agent is the determinate way she is that she would set *this* ratio if asked in this way and *that* ratio if asked in that way.

(3) Someone might accept my core point that we *would* set certain betting ratios and hence *do* have dispositions to set them, but question the relevance of this to degrees of belief. That is, why should we take these betting ratios as degrees of belief? Well, because we are, at the moment, taking degree of belief to be strength of tendency to act—and it just seems undeniable that setting a higher betting ratio in a bet on *S* *just is* (one way of) acting more strongly as if *S* is true. To see the point more clearly, consider again the case of fuzzy theories of vagueness. Suppose I were to give you a corkboard and a pin. I then present you with pairs of objects and predicates. For each pair, you must stick the pin in the corkboard. (These are the only instructions I give you.) You would do so—and hence you have dispositions to do so. Now suppose I propose to measure the distance from the left side of the corkboard to the pin (in a system of units where the width of the corkboard is 1 unit) and take that to be a measure of the degree to which the object falls under the predicate. In this case, it seems clear that although the dispositions are present, they are irrelevant: there seems to be no good reason why we should take these dispositions to determine *degrees of truth* (or degrees of application of predicates, or degrees of membership in fuzzy sets). If we now return to the case of degrees of belief—conceived as strengths of tendency to act—I think it should be clear that there is no such problem: evidently, setting a higher betting ratio in a bet on *S* *just is* (one way of) acting more strongly as if *S* is true.

(4) It seems unlikely that the betting ratios that a typical actual agent would set will satisfy the probability axioms. But note that—if correct—this observation does not speak against tenet 1 of Bayesianism. Nor, moreover, does it speak against tenet 2: for tenet 2 of the Bayesian approach is normative. It holds, not that an agent’s degrees of belief *do* satisfy the laws of probability, but that they *should*—that in so far as they do not, the agent is irrational. And even if actual agents do not satisfy this norm, there seems to be a strong reason why they should—that is, a strong reason to accept it as a norm. The reason is that if an agent’s betting ratios do not satisfy the probability axioms, then the agent is susceptible to Dutch book—to a sure loss. Now some have objected to Dutch book arguments on the grounds that they are pragmatic and cannot justify claims about epistemic rationality. These objections may or may not be well founded when directed against Dutch book arguments to the conclusion that *epistemic* degrees of belief should obey the laws of probability. However for now we are talking about *dispositional* degrees of belief—strengths of tendency to act—and it seems quite clearly irrational to have tendencies that will leave you worse off *no matter what*. When we are thinking of degrees of belief in this practical sense—as tendencies to action—a pragmatic argument is just the sort of argument we need.

(5) It seems quite possible that the betting ratios that a typical actual agent would set will be variable, unstable, and sensitive to context. That is, given the very same proposition *S*, an agent might set a different ratio for a bet on *S* if asked to do so in context *C*₁ from that she would set in context *C*₂.^{8,9} Note again, however, that—if correct—this observation does not speak against tenet 1 of Bayesianism. It means that an agent’s behavioural dispositions might change with time and with context. This does not undermine my key claim that—at any given time and in any given context—the agent’s behavioural dispositions are sufficiently determinate to enable the strength of her tendency to act as if *S* to be measured by a

⁸Note that this is a different kind of variation from the one we considered under (2b) above. There we were talking about a certain agent *S* in a certain context *C*, and imagining how *S* would set a betting ratio if asked to do so in *C*. Given that *S* is not actually asked in *C*, this means considering nearby worlds in which *S* is asked—and the thought was that *S* might not set the very same ratio in all of these worlds. Now the situation is different. We are imagining *S* first in actual context *C*₁ and then in actual context *C*₂, and we are imagining how *S* would set a betting ratio if asked to do so in each of these contexts. So now we are considering the ratio *S* sets in a counterfactual situation similar to *C*₁ and the ratio *S* sets in a counterfactual situation similar to *C*₂, where *C*₁ and *C*₂ are different contexts in the actual world (as opposed to the ratios *S* sets in two counterfactual situations, both of which are similar to the same actual context *C*).

⁹There is psychological evidence that agents’ explicit judgements or estimates of probability are variable and situation-dependent. This does not automatically mean, however, that agents’ behavioural dispositions are variable and context-dependent—that would follow only if degrees of belief (in the dispositional sense) are straightforwardly mirrored in estimates of probability. The *possibility* that I am considering now is that agents’ behavioural dispositions *are* variable and context-dependent.

particular real number. Of course, if an agent's strength of tendency to act as if S changes between t_1 and t_2 , even though the agent receives no new evidence during that period that bears one way or the other on whether S is true, then further issues do arise. One issue is that the agent will have violated tenet 3 of Bayesianism: she will not have updated by conditionalisation. However, as in the case of tenet 2, tenet 3 is normative: it holds not that agents *do* always update by conditionalisation but that they *should*. The observation that ordinary folk do not conform to tenet 3 does not automatically mean that we should abandon it as a normative principle. (Should we abandon it? I think the answer is less clear here than in the case of tenet 2. There is a diachronic Dutch book argument for taking tenet 3 as a normative principle—but this argument seems less secure than the synchronic Dutch book argument for tenet 2. For example, in the Transferable Belief Model, updates need not respect conditionalisation—yet Smets (1993) argues that this model is *not* refuted by the diachronic Dutch book argument.) A second issue is that, even setting aside the particular question of conditionalisation, it might seem that the agent has done something *irrational* in changing her degree of belief that S when no relevant evidence has come in: for shouldn't degrees of belief be determined by the available evidence? Well, *epistemic* degrees of belief should be—but with *dispositional* degrees of belief (which are our current concern), change over time is surely to be expected, with or without change in evidence: degrees of belief, on this conception, are strengths of tendencies to act—and of course your dispositions may change over time, for all sorts of reasons.

(6) Someone might object that while it is true that an agent *would* set a particular betting ratio, this does not mean that she has any *disposition* to do so. Degree of belief, on the current conception, is supposed to be strength of *tendency* to act. The objection now is that the agent *has no* tendency to act. She would act, but this act would not be determined by any underlying tendency or disposition: it would be an undetermined act *ex nihilo* that she would come up with entirely on the fly. In response, I don't really have anything to say except that I find this completely implausible, as a general picture. Our actions may not always be rational or justified, but in general it is natural to suppose—at least in the absence of some compelling argument or evidence to the contrary—that they stem from underlying states of our brains and bodies that cause us to act in the ways we do. The objection under consideration is that when asked in some situation to specify a betting ratio on S , I might specify 0.3 (say)—but I could *just as well* have specified 0.4 or some other number (in the same situation): the number I specify pops out of nowhere, rather than being a manifestation of an underlying disposition or tendency to specify that number. Now here we must be careful to distinguish the two kinds of degree of belief. On the *epistemic* conception it may be true to say that 0.3 'comes from nowhere' in the sense that this number is not uniquely *justified* by the evidence available to me. But on the *dispositional* conception (which is our

current concern), the claim that my answer of 0.3 ‘comes from nowhere’ in the sense of popping out randomly—in such a way that other numbers could just as easily have popped out, and it simply happened for no underlying cause to be this number—seems to me utterly far-fetched.

6 Arguments in the Literature

In the previous section I argued that there is no good argument against tenet 1 of Bayesianism along the lines of the dispositional argument against the basic fuzzy theory of vagueness. In this section I turn to actual arguments in the literature that object to Bayesian epistemology on the grounds that it involves false precision. Some of these arguments can be seen as (more or less detailed gestures in the direction of) dispositional arguments—and can be seen to fail for reasons already discussed. Some of them can be seen as being (to a greater or lesser extent) in the same ballpark as the dispositional argument—and can also be seen to fail. Finally, some of them can be seen as entirely different kinds of argument. With respect to this last group, my aim here is to point out the differences (from dispositional arguments)—not to go on to assess whether these other arguments are sound.

Joyce (2010, 283) writes:

numerically sharp degrees of belief are psychologically unrealistic. It is rare, outside casinos, to find opinions that are anywhere near definite or univocal enough to admit of quantification. An agent with a precise credence for, say, the proposition that it will rain in Detroit next July 4th should be able to assign an exact ‘fair price’ to a wager that pays \$100 if the proposition is true and costs \$50 if it is false. The best most people can do, however, is to specify some vague range.

Joyce seems to be claiming that in general people could not and would not set a betting ratio if required to do so. This just strikes me as false. As discussed in section 5, for most agents A and most propositions S , if A were in a situation where a bet on S is to take place and A must set the betting ratio x , A would set some x . Of course there might be some cases in which A freezes up completely and is simply unable to set x —and of course if we simply *asked* A what x she would set if she were required to (as opposed to requiring her to set x), A might not be able to give an answer (recall point (1) in section 5)—but in general I think that if one were required to do so, one would set some x .

Walley (1991) writes:

We know, through introspection, that our beliefs about many matters are indeterminate. . . Imprecise probabilities are needed to model the indeterminacy (4)

It seems clear that indeterminacy exists. A little introspection should suffice to convince You that Your beliefs about many matters are presently indeterminate. (210)

There are different ways of interpreting this. Recall the distinction in section 4 between the dispositional conception of degrees of belief as strengths of tendency to act and the epistemic conception of degrees of belief as evidence-justified view of how the world is. Walley might be thinking of beliefs in the epistemic sense. Now perhaps it is plausible that our epistemic degrees of belief are accessible to introspection: they are, after all, supposed to constitute our considered, evidence-justified opinions about the way the world is. Interpreted this way, the argument is very different from a dispositional argument. Unlike a dispositional argument, it is directly concerned with epistemic degrees of belief, not dispositional degrees of belief. It forms a descriptive counterpart (introspection tells us that epistemic degrees of belief *are not* determinate) to the normative argument that we shall see below (epistemic degrees of belief *should not* be determinate). For our present purposes it is not relevant to assess these arguments: pointing out that they are of a completely different nature from the dispositional argument against the basic fuzzy theory of vagueness is enough.

So suppose we interpret Walley as talking about dispositional degrees of belief. Now if we are concerned with degrees of belief as strengths of tendency to act then there is no reason to think that introspection should have the final word. That is, our dispositions might be far more determinate than we can ascertain by introspection. For, in general, there is no reason to think that we can always imagine a situation in sufficient detail to trigger (in imagination) our dispositions and hence allow us to know how we would react in that situation. You might have no idea how you would react in a certain dangerous situation—or you might think that you would run away or freeze up completely. Nevertheless it might still be the case that you would act courageously in that situation, were you to face it. Similarly, as we have already discussed, if you imagine a betting situation, you might not have much idea what ratio you'd set—or you might think that you would not set one at all. Nevertheless, were you in such a situation and required to set a betting ratio, you would (in general, I maintain) do so.

Walley (1991) writes:

If You are required to choose between X and Y then You will, of course, choose one way or the other, but in cases of indeterminacy Your choice is simply not determined by Your current state of mind. Your mind is not 'made up'. (209–10)

It is essential to distinguish between choice and preference. A **choice** is a decision about how to act, that is made in a specific context. . . A

preference is an underlying disposition to choose in a particular way... You can choose a_1 over a_2 without having any preference between them. A choice can be **arbitrary**, in the sense that it is not determined by Your preferences, beliefs and values. (236–7)¹⁰

Again, there are different ways of interpreting this. We could interpret Walley as expressing the idea discussed under point (6) in section 5: even if there is determinate counterfactual behaviour, that does not mean that there are determinate behavioural *dispositions*. On the contrary, the behaviour ‘just happens’—rather than being a manifestation of an underlying disposition to behave in that way. I have already responded (in section 5) to this line of thought. A different interpretation is that sometimes we act in ways that we cannot justify on the basis of our underlying *epistemic* beliefs (and preferences and values). It’s not that our actions ‘just happen’ in the sense just mentioned: on the contrary, at any time, our brains and bodies are in such a state that we have definite behavioural dispositions. Rather, the point is that sometimes (or often) our actions are arbitrary in the sense that we have no more reason or justification for acting in the way we do than in some other possible way. Now this may be true—but it is a very different kind of point from the dispositional argument against the view that our dispositional degrees of belief can be represented by particular numbers.

Following the passage quoted near the beginning of this section, Joyce (2010) continues:

While psychological implausibility is one worry, a more decisive problem is that precise degrees of belief are the wrong response to the sorts of evidence that we typically receive. As argued in Joyce (2005), since the data we receive is often incomplete, imprecise or equivocal, the epistemically *right* response is often to have opinions that are similarly incomplete, imprecise or equivocal. (283)

Precise credences... always commit a believer to extremely definite beliefs about repeated events... even when the evidence comes nowhere close to warranting such beliefs (285)

A similar line of thought has been expressed by many other authors—for example Kaplan (1996):

The moral would seem to be that if we want to give evidence its due—if we want to maintain that a rational investigator ought to adopt only those states of opinion she has good reason to adopt—we had better conclude that Immodest Connectedness is not a principle you should want to satisfy. (27–8)

¹⁰Cf. also Walley (1991, 106, 245, 247, 533) and Eriksson and Hájek (2007, 189–90).

It is important to notice that my reason for rejecting as falsely precise Immodest Connectedness's demand that you place a monetary value on each well-mannered state of affairs is not what one might have expected. It is not that this demand is not humanly satisfiable. For if *that* were all that was wrong, the demand might still play a useful role as a regulative ideal—an ideal which might then be legitimately invoked to get you to 'solve' your decision problem as the orthodox Bayesian would have you do. My complaint about the orthodox Bayesian demand is rather that it imposes the wrong regulative ideal. For if you place a monetary value on each well-mannered state of affairs, you have a determinate assignment of *con*(-) to every hypothesis—and then you are not giving evidence its due. (29)¹¹

Walley (1991):

When there is little relevant evidence, even the 'ideal' probabilities are imprecise. (7)

When there is little or no relevant evidence, the probability model should be highly imprecise or vacuous. More generally, the precision of probability models should match the amount of information on which they are based (34)

'Vagueness' or lack of information should be reflected in imprecise probabilities (246)

and Levi (1985, 396):

it is sometimes rational to make no determinate probability judgment and, indeed, to make maximally indeterminate judgments. Here I am supposing... that refusal to make a determinate probability judgment does not derive from a lack of clarity about one's credal state. To the contrary, it may derive from a very clear and cool judgment that on the basis of the available evidence, making a numerically determinate judgment would be unwarranted and arbitrary.¹²

The argument is that degrees of belief should be determined and justified by the available evidence; but often the available evidence is sparse or vague; and in such cases, degrees of belief should not be precise. This may, or may not, be a powerful line of argument—but it suffices for present purposes to point out that it is clearly an argument that degrees of belief in the epistemic (rather than dispositional) sense

¹¹Cf. also Kaplan (1996, 24).

¹²Levi here refers to Peirce, Fisher, Neyman, Pearson and Kyburg as examples of earlier authors who held similar anti-Bayesian views. Cf. also Levi (1974, 394–5).

should be imprecise—and it is clearly a very different kind of argument from the dispositional argument.

7 Conclusion

The artificial precision argument against fuzzy theories of vagueness and the false precision objection to Bayesian epistemology have the same surface form. A more detailed investigation, however, has revealed deep differences. The kind of dispositional argument that carries weight in the fuzzy case does not, I have argued, go through in the Bayesian case. The main purpose of this investigation has been to shed light on the conceptual landscape. But it may be that there is a more substantive conclusion in the offing. Suppose that the Bayesian account is correct for dispositional degrees of belief. (Note that I have not shown this: I have defended the Bayesian view—and in particular, its first tenet—against a certain objection; I have not given a complete defence of the full Bayesian account.) Suppose also that the final kind of objection considered in section 6 is correct—in which case the Bayesian theory is inadequate as an account of epistemic degrees of belief. (Note that this goes far beyond anything argued in this paper—but just suppose for the moment that it is true, in order to see what follows.) In that case, we need different accounts of dispositional degrees of belief and epistemic degrees of belief. That sort of two-level picture is not unheard of—it is one of the core ideas of the Transferable Belief Model (Smets and Kennes, 1994)—but it is not common: most of the debate in this area is concerned with finding the right model of ‘degrees of belief’ (or ‘credences’, or whatever other term is chosen). The more substantive conclusion that may be in the offing is that what we really need are two different models (and an account of their interactions): one for dispositional degrees of belief and one for epistemic degrees of belief.¹³

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