Measuring and Modeling Truth

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Abstract

Philosophers, linguists and others interested in problems concerning natural language frequently employ tools from logic and model theory. The question arises as to the proper interpretation of the formal methods employed—of the relationship between, on the one hand, the formal languages and their set-theoretic models and, on the other hand, the objects of ultimate interest: natural language and the meanings and truth conditions of its constituent words, phrases and sentences. Two familiar answers to this question are descriptivism and instrumentalism. More recently, a third answer has been proposed: the logic-as-modeling view. This paper seeks to clarify and assess this view of logic. The conclusion is that we can successfully adopt the modeling perspective on a given piece of logical machinery only if we have to hand some other machinery to which we take the descriptive attitude. Thus, logic-as-modeling is not a full-fledged alternative to the descriptive view—for it cannot stand alone: it can at best be an addition to the descriptive perspective. The paper first presents the argument in a general, abstract form, before working through a detailed case study. The case examined is the one with respect to which the logic-as-modeling view has been developed in the greatest detail in the literature: the case of fuzzy model theory as an account of vagueness in natural language.

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1 The Argument in the Abstract

Philosophers, linguists and others interested in problems concerning natural language frequently employ tools from logic and model theory. The question arises as to the proper interpretation of the formal methods employed—of the relationship between, on the one hand, the formal languages and their set-theoretic models and, on the other hand, the objects of ultimate interest: natural language and the meanings and truth conditions of its constituent words, phrases and sentences. Two familiar answers to this question are *descriptivism* and *instrumentalism.* The descriptivist regards model theory as giving a literal (although not necessarily complete) description of the relationship between language and the world: a system of model theory as a whole tells us about the kinds of relationships that a language may have to a world; what is going on in the intended model of a particular discourse tells us (something about) the actual relationship between that discourse and the world. The instrumentalist denies this: model theory can be useful in various ways—for example, it might provide a useful calculus for predicting speakers' assertions—but it does not provide a literal description (not even a partial one) of the meanings or truth conditions of natural language expressions.[1]

More recently, a third answer has been proposed: the *logic-as-modeling* view, according to which formal languages together with systems of model theory provide mock-ups of natural languages and their semantic properties.[2] The key thing about such mock-ups, which distinguishes them from descriptions, is that while descriptions may simplify and approximate, some aspects of mock-ups are not even *intended* to represent—not even in an approximate or simplified way—an aspect of the thing modeled. Such features of a mock-up are called *artifacts*. Cook [2002, p.236] gives an example:

> a model ship might have, deep in its interior, supports situated where the engine room is located in the actual ship. Although the supports do not represent anything real on the actual ship, they are not necessarily useless or eliminable as a result, since they might be crucial to the structural integrity of the model.

Other parts of a mock-up—those which are intended to represent aspects of the thing modeled—are called *representors* [Shapiro, 2006, p.50]. The logic-as-modeling view thus combines aspects of descriptivism and of instrumentalism: some parts of the formal machinery are viewed in the way the descriptivist views them—as representing aspects of natural language and its semantics—while other parts are viewed in the way the instrumentalist views them. As Cook [2002, p.236] puts is: "parts of a logical model, including objects and relations intimately involved in the semantics, might be there just to facilitate the mathematics or to simplify our manipulations of the model."

This paper seeks to clarify and assess the logic-as-modeling view. The conclusion will be that there could be situations in which one might wish to use some formal theory while regarding it as a mock-up—*however*, we can successfully adopt the modeling perspective on a given piece of logical machinery only if we have to hand some *other* machinery to which we take the *descriptive* attitude. Thus, logic-as-modeling is not a full-fledged alternative to the descriptive view—for it cannot stand alone: it can at best be an addition to the descriptive perspective.

The argument to this conclusion will be presented first in a general, abstract form. The argument has two steps. Step one: A requirement on a mock-up being useful is that we know which parts of it are artifacts and which are representors. For—to consider Cook's example of the model ship—although it is of course not a problem that the model contains supports which correspond to nothing on the real ship, we would soon get into serious trouble in trying to use the model to draw conclusions about the real ship if we did not *know* that the supports were artifacts. The proponents of the modeling view stress this point themselves—for example:

> Of course, saying that the account is meant to be a model, and thus that certain unattractive parts of the semantics are artifactual, is not enough. We have yet to determine in general which aspects of the model are artifacts and which are representors ... Without knowing in more detail what is representor and what is artifact we cannot draw any useful insights from the model, since we do not know what parts of it are intended to provide such information. [Cook, 2002, pp.240–1]

> it must be determined which features of formal languages correspond to relevant features of correct reasoning in natural language, and which features do not. Otherwise, there is a danger of inferring something about the target ... on the basis of an artifact of an otherwise good model. [Shapiro, 2006, p.50]

Step two: Once we know which parts of a mock-up are artifacts and which are representors, we will have available a distinct theory to which we take the descriptive attitude: namely, the mock-up minus the artifacts. Consider the model ship again. Once we know which parts of it are representors—say, the outer surface of the hull, the number and dimensions of the masts (and so on)—and which parts are artifacts—say, the supports, the thickness of the hull (and so on)—then we have a mental picture of the ship to which we take the *descriptive* attitude.

Of course, this picture is not *complete*: for example, it tells us nothing about the part of the ship corresponding to the part of the model where the supports are. But it was never part of the descriptive view that formal theories must provide *complete* descriptions of their subject matter. The contrast between the descriptive view and the modeling view was that the latter allows for artifacts—parts of the mock-up which represent nothing about the thing modeled—while the descriptive view does not countenance artifacts. So when we take the descriptive attitude to a formal system, we regard every aspect of the *system* as representing something about the subject matter of ultimate interest. That is quite different from thinking that every aspect of the *subject matter* is represented in the system—that is, that the system provides a *complete* description of the subject matter.

The upshot so far—as applied to the case of the model ship—is this: for the model ship to be useful for purposes of drawing conclusions about the real ship modeled, we must know which parts of it are artifacts; but once we know this, we have another representation of the ship, to which we take the descriptive attitude. So the modeling attitude to formal theories is not on a par with the descriptive and instrumental attitudes: for the modeling perspective cannot operate alone—it always requires the descriptive perspective as chaperone.[3]

This is *not* to say that the modeling perspective cannot be useful. In the case of the ship, it obviously is useful in some situations. For the representation to which we take the descriptive attitude, which must be available if we are successfully to take the modeling attitude towards the model ship, is a *mental* picture (formed by subtracting from the model ship the parts which are artifacts). Suppose we wish to do tank testing in order to develop a design for a new rudder for the real ship. Then the mental picture is no good: we need the physical model to go into the tank. So the fact that a mock-up must—if it is to be of any use—be accompanied by a description does *not* automatically mean that mock-ups can always be discarded in favor of their accompanying descriptions.

In philosophy, however, the modeling perspective is vulnerable. For philosophers do not do tank-testing. Typically, their goal is conceptual clarity. In such a context, could a mock-up ever have any advantage over a description? Only one such kind of situation comes to mind. Suppose that two systems of model theory give rise to the same consequence relation, but one system is much simpler than the other. If we take the descriptive attitude to the more complex system, we would still have good reason to retain the simpler system.[4] However, in this case we would naturally take the *instrumental* attitude towards it: it is useful for determining consequences, and that is *all* we care about—the fact that some aspects of it may be seen as representors is simply irrelevant in this context. This type of case, then, offers no comfort to the logic-as-modeling approach.

In any case, the main argument of this paper is that the modeling perspective can be adopted only as an *addition* to the descriptive perspective—not as a full-fledged alternative. We can adopt the modeling perspective on some formalism only if we are prepared to adopt the descriptive perspective on some other formalism. (The subsidiary point is then that, in theory, this leaves it open whether it might be useful to continue to employ the first formalism—i.e. the one to which we take the modeling attitude—as opposed to abandoning it in favor of the second formalism—i.e. the one to which we take the descriptive attitude. In practical contexts, we might have good reason to continue to use the first formalism; in philosophical contexts, this seems less likely.) This section has presented the argument in a general, abstract form. The remainder of the paper works through a detailed case study. The case to be examined is the one with respect to which the logic-as-modeling view has been developed in the greatest detail [Cook, 2002]: the case of fuzzy model theory as an account of vagueness in natural language.

2 The Fuzzy Account of Vagueness

Consider the account of vagueness in natural language based on fuzzy model theory. In order to understand what this account is, we need to distinguish pure model theory and model-theoretic semantics (MTS). MTS requires an additional notion that does not figure in pure model theory: the notion of the *intended model*—or some other notion which plays a similar role. That role is to distinguish one (or perhaps some) of the infinity of models of a given formal language countenanced in pure model theory as the one(s) relevant to questions of the (actual) meaning and truth (simpliciter) of utterances in some discourse. Questions of (actual) meaning and truth (simpliciter) are of central interest in natural language semantics—but pure model theory cannot (fully) answer them: for it tells us only that a well-formed formula (wff) is true on this model and false on that one (etc.). If we want to know whether a given statement is true (simpliciter), then we need to single out a particular model (or perhaps a class of models): truth simpliciter will then be truth relative to this model.[5] Weiner [2004, p.165] sums up the MTS perspective very nicely:

> Natural language (or at least a cleaned up version of a fragment of natural language) is to be understood as a formal language along with an intended interpretation. Truth, for sentences of natural language, is to be understood as truth under the intended interpretation.

We get a system of MTS by combining a system of pure model theory with some notion that plays the role of distinguishing some model(s) as the ones relevant to questions of (actual) meaning and truth (simpliciter) of utterances in some discourse. The simplest choice of model theory is classical model theory. The simplest choice of auxiliary notion is the idea that for each discourse, there is a *unique* relevant model: the 'intended model'. Combining these two choices yields the 'classical semantic picture' [Smith, 2008, §1.2]. It is the version of MTS which underlies epistemic theories of vagueness such as those advocated by Sorensen [1988, ch.6; 2001] and Williamson [1992; 1994, ch's 7–8].

The 'basic fuzzy theory of vagueness' (as it will be called here) differs from the classical semantic picture (only) by replacing classical model theory with fuzzy model theory. So it retains the idea that each discourse is associated with a unique intended model—only this time, that model is fuzzy (it assigns *fuzzy* subsets of the domain as extensions of unary predicates, and so on), not classical.

One of the biggest problems faced by the basic fuzzy theory of vagueness is the problem of 'artificial precision'.[6] Each of the following passages gives a nice statement of the problem:[7]

> [Fuzzy logic] imposes artificial precision ... [T]hough one is not obliged to require that a predicate either definitely applies or definitely does not apply, one *is* obliged to require that a predicate definitely applies to such-and-such, rather than to such-and-such other, degree (e.g. that a man 5 ft 10 in tall belongs to *tall* to degree 0.6 rather than 0.5) [Haack, 1979, p.443]

> One immediate objection which presents itself to [the fuzzy] line of approach is the extremely artificial nature of the attaching of precise numerical values to sentences like '73 is a large number' or 'Picasso's *Guernica* is beautiful'. In fact, it seems plausible to say that the nature of vague predicates precludes attaching precise numerical values just as much as it precludes attaching precise classical truth values. [Urquhart, 1986, p.108]

> [T]he degree theorist's assignments impose precision in a form that is just as unacceptable as a classical true/false assignment.

> In so far as a degree theory avoids determinacy over whether a is F, the objection here is that it does so by enforcing determinacy over the *degree* to which a is F. All predications of "is red" will receive a unique, exact value, but it seems inappropriate to associate our vague predicate "red" with any particular exact function from objects to degrees of truth. For a start, what could determine which is the correct function, settling that my coat is red to degree 0.322 rather than 0.321? [Keefe, 1998, p.571]

In a nutshell, the problem for the basic fuzzy view is this: it is artificial/implausible/inappropriate to associate each vague *predicate* in natural language with a particular function which assigns one particular fuzzy truth value (i.e. real number between 0 and 1) to each object (the object's degree of possession of the property picked out by that predicate); likewise, it is artificial/implausible/inappropriate to associate each vague *sentence* in natural language with a particular fuzzy truth value (the sentence's degree of truth). But this is exactly what the basic fuzzy view does: it associates each vague predicate (as used in some discourse) with its extension on the unique intended model (of that discourse), and each vague sentence with its truth value on that model.

The following three sections discuss three possible responses to this problem. The third is the logic-as-modeling approach. The other two are needed in order to illustrate the claim of the present paper that the modeling approach can be employed successfully only when we have to hand theories to which we take the descriptive attitude.

3 Measuring Truth on an Ordinal Scale

The first response to be considered holds that when we assign fuzzy truth values to sentences, the only thing about the assignments that is meaningful is the relative *ordering* of the values assigned. Views of this sort have been advocated by—amongst others—Goguen, Machina and Hyde:[8]

> We certainly do not want to claim there is some *absolute* [fuzzy] set representing 'short'. ... It appears that many arguments about fuzzy sets do not depend on particular values of functions ... This raises the problem of *measuring* fuzzy sets ... Probably we should not expect particular numerical values of shortness to be meaningful (except 0 and 1), but rather their *ordering* ... degree of membership may be measured by an *ordinal scale*. [Goguen, 1968–69, pp.331–2]

> the assignment of exact values usually doesn't matter much ... what is of importance instead is the ordering relation between the values of various propositions. [Machina, 1976, p.188]

> The foregoing account ... requires only a totally-ordered dense set of values. The choice of a specific value from among the infinitely many possible ... is arbitrary except in so far as it preserves ordering requirements imposed by the structure of higher-order vagueness. No significance attaches to the choice of value apart from these ordering requirements. [Hyde, 2008, p.207]

However, this view has never been fully articulated in the literature. It seems that there would be two different ways of spelling it out—corresponding to two different ways of thinking about what is going on when we assign numbers to objects to measure their lengths, weights, temperatures and so on:[9]

(i) *Realism.* On the first way of thinking about measurement, there are certain entities—*lengths*—and each object has a *unique* length. However, we do not have special *names* for these entities, so we refer to them by assigning real numbers to them—that is, we use real numbers as names for the lengths. A way of assigning numbers to lengths is acceptable if the structure of the lengths is mirrored in relations between the numbers assigned: if a is longer than b, then a's name (which is a number) is greater than b's—etc. Now it turns out that there is more than one acceptable way of naming lengths by real numbers. For example, under one system of assigning names to lengths (the system which we call 'measuring in feet'), a certain length gets the name 3; under a different—but equally acceptable—system of naming the lengths (the system which we call 'measuring in centimeters'), the *very same length* gets the name 91.44. A statement about lengths made in terms of real numbers—that is, using real numbers as names for the lengths—is *meaningful* only if it holds (or fails to hold) across *all* acceptable ways of naming the lengths. So, for example, it is meaningful to say that my boat is half as long as yours, but it is not meaningful to say that the length of my boat is prime.

(ii) *Nominalism.* On the second way of thinking about measurement, there are no such entities as lengths: there are only the objects which (on the first view) have lengths (i.e. boats, roads, pieces of string, etc.) and the real numbers. Again, we represent the facts about 'the lengths' of objects by assigning numbers to them—and there are many admissible ways of doing so.

On the realist view, the complete set of length facts about some objects is encapsulated in the assignment to each of them of a *unique* length. There is, however, no unique *description* of these facts in terms of real numbers: the complete description comprises those statements which hold across *all* acceptable ways of assigning real number names to the lengths. On the nominalist view, the complete set of length facts about some objects is encapsulated in a whole *set* of assignments of numbers to them: all the acceptable assignments. In *practice*, then, the two views come to the same thing: we measure lengths by associating real numbers with objects, and the associations are not unique. When we look at the underlying details, however, the two views are quite different: on the realist view, the multiplicity of associations between objects and numbers represents a lack of one way of *describing* the length facts, which consist in the assignment of a *unique* length (where lengths are *not* numbers: they are distinct entities) to each object; on the nominalist view, the multiplicity of associations *is* the complete set of facts.

Returning now to the fuzzy view, the idea that truth is measured on an ordinal scale is similarly subject to two different developments: realist and nominalist. On the realist way of looking at things, the truth values of the system are *not* real numbers in the interval $[0, 1]$ (as they are in fuzzy model theory). The real interval $[0, 1]$ comprises some entities, together with some structure—an order structure, a metric structure—and some operations—addition, subtraction, and so on. Now suppose we retain the entities and the order structure, but discard the metric structure, and hence also any operations defined in terms of it (e.g. subtraction). This gives us a new structure—and its elements are the truth values of the new sort of model theory now under consideration. Figuratively, one can think of the new structure as a rubbery unit interval, fixed at each end: its end-points have fixed positions, but between them, none of the other elements has a fixed position. They can be squeezed or stretched left or right at will—but they can never leapfrog one another: their *order* is fixed. Let us fix on some terminology: fuzzy truth values (ftv's) are reals in the interval $[0, 1]$; rubbery truth values (rtv's) are elements in the structure just described—the rubbery unit interval. Now the idea behind the realist way of spelling out the view that truth is measured on an ordinal scale is to replace fuzzy models with rubbery models (i.e. models which assign rubbery sets to predicates—where a rubbery set is a function from the domain to the rtv's—and rubbery truth values to wffs), while retaining the idea that each discourse has a unique intended model (a rubbery model this time, rather than a fuzzy one).

Note that developing rubbery model theory will involve (amongst other things) specifying truth conditions for conjunctions, conditionals, negations and so on. In fuzzy model theory, we have many options; three of the most important sets of options are shown in Figure 1. In rubbery model theory we have fewer options. We have an ordering of the rubbery truth values which allows us to make sense of the operations max and min, and the endpoints of the rubbery interval are fixed, so we can make sense of picking out the values 0 and 1. However there is no metric structure—there are no (fixed) distances between rubbery truth values—and so we cannot make sense of an

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\begin{array}{llcl}
\text{Łukasiewicz:} & x \wedge y & = & \max(0, x+y-1) \\
& x \rightarrow y & = & \begin{cases} 1 & \text{if } x \leq y \\ 1-x+y & \text{if } x > y \end{cases} \\
& \neg x & = & 1-x \\
\text{Gödel:} & x \wedge y & = & \min(x, y) \\
& x \rightarrow y & = & \begin{cases} 1 & \text{if } x \leq y \\ y & \text{if } x > y \end{cases} \\
& \neg x & = & \begin{cases} 1 & \text{if } x = 0 \\ 0 & \text{otherwise} \end{cases} \\
\text{Product:} & x \wedge y & = & x \cdot y \\
& x \rightarrow y & = & \begin{cases} 1 & \text{if } x \leq y \\ y/x & \text{if } x > y \end{cases} \\
& \neg x & = & \begin{cases} 1 & \text{if } x = 0 \\ 0 & \text{otherwise} \end{cases}
\end{array}
$$

Figure 1: Conjunctions, conditionals and negations in three fuzzy logics

expression such as '$1 - x$', which speaks of the distance between the truth values 1 and x. Nor can we make sense of multiplying or dividing rubbery truth values. Hence, the Łukasiewicz operations and the Product conjunction and conditional are not available in rubbery model theory—but the Gödel operations are available.

Although we have introduced the rubbery unit interval, we have not introduced any new *names* for the rubbery truth values. Rather, we use reals in $[0, 1]$—that is, fuzzy truth values—as names for the rubbery truth values. A way of assigning ftv's to rtv's is acceptable if the structure of the rtv's is mirrored in relations between the ftv's assigned: if a is truer than b, then a's name (which is an ftv, which is a number) is greater than b's. Now of course there is more than one acceptable way of naming rtv's by ftv's: for any acceptable way of mapping sentences to fuzzy truth values, any mapping obtained by composing it with an order-preserving and endpoint-fixing transformation of $[0,1]$ is equally acceptable. A statement about rtv's made in terms of ftv's is *meaningful* only if it holds (or fails to hold) across *all* acceptable ways of naming the rtv's. So, for example, it is meaningful to say that one sentence is truer than another, but not that it is twice as true.

On the nominalist way of developing the idea that truth is measured on an ordinal scale, there are no rubbery truth values, in addition to the fuzzy truth values: there are only the fuzzy truth values. We represent the facts about the truth of statements by assigning fuzzy truth values to them—and there are many acceptable ways of doing so: for any acceptable way of mapping sentences to fuzzy truth values, any mapping obtained by composing it with an order-preserving and endpoint-fixing transformation of $[0,1]$ is equally acceptable.

On the nominalist approach, the space of possible truth conditions for conjunctions, conditionals, negations and so on is reduced in the same way as on the realist view—but for a different reason. We cannot, for example, say that the truth value of $\neg\alpha$ is 1 minus the truth value of α, for although this relationship is well-defined for each assignment of fuzzy truth values, it cannot hold across *all* acceptable assignments.

On the realist view, the complete set of facts about the truth of some statements is encapsulated in the assignment to each statement of a *unique* rubbery truth value. There is, however, no unique *description* of these facts in terms of ftv's—and we have no special names for the rtv's: so the complete description comprises those statements made in terms of ftv's which hold across *all* acceptable ways of assigning ftv's as names to the rtv's. On the nominalist view, the complete set of facts about the truth of some statements is encapsulated in a whole *set* of assignments of ftv's to them: all the acceptable assignments. In *practice*, then, the two views come to the same thing: we measure truth by associating real numbers with statements, and the associations are not unique. When we look at the underlying details, however, the two views are quite different: on the realist view, the multiplicity of associations between statements and numbers represents a lack of a unique way of *describing* the semantic facts, which consist in the assignment of a *unique* truth value (where truth values are *not* numbers: they are distinct entities) to each statement; on the nominalist view, the multiplicity of associations *is* the complete set of facts.

In the basic fuzzy theory of vagueness, the semantics of vague discourse is modeled by the assignment of a single fuzzy model to the formal language. In the realist version of the ordinal view, the semantics of vague discourse is modeled by the assignment of a single model to the formal language—but it is a rubbery model, not a fuzzy one. In the nominalist version of the ordinal view, the semantics of vague discourse is modeled by the assignment of fuzzy models to the formal language—but many such models are assigned, not just one. Either way, the problem of artificial precision is sidestepped: vague statements are *not* assigned unique fuzzy truth values; they are either assigned unique non-fuzzy truth values, or they are assigned non-unique fuzzy truth values.

4 Interval-Valued Fuzzy Sets

A second response to the problem of artificial precision consists in moving from fuzzy sets to interval-valued fuzzy sets.[10] Where a fuzzy subset of a background universal set U is a function $F : U \to [0, 1]$, an interval-valued fuzzy set is a function $I : U \to \mathcal{E}([0, 1])$, where $\mathcal{E}([0, 1])$ is the family of all closed intervals of reals in $[0, 1]$. For example, where Bob might be assigned 0.3 by the fuzzy set of bald men, indicating that he is bald to degree 0.3, he might be assigned the interval $[0.2, 0.4]$ by the interval-valued fuzzy set of bald men, indicating that he is bald to a degree between 0.2 and 0.4 (inclusive).

It is natural to extend intersection, union and complement operations on fuzzy sets defined thus [Zadeh, 1965, pp.340–1]:

$$\begin{aligned} F \cap G(u) &= \min(F(u), G(u)) \\ F \cup G(u) &= \max(F(u), G(u)) \\ F^c(u) &= 1 - F(u) \end{aligned}$$

to operations on interval-valued fuzzy sets as follows, where $I(u) = [I_*(u), I^*(u)]$ [Dubois and Prade, 2005, p.2]:

$$\begin{aligned} I \cap J(u) &= [\min(I_*(u), J_*(u)), \min(I^*(u), J^*(u))] \\ I \cup J(u) &= [\max(I_*(u), J_*(u)), \max(I^*(u), J^*(u))] \\ I^c(u) &= [1 - I^*(u), 1 - I_*(u)] \end{aligned}$$

The most straightforward way of implementing MTS based on the idea of interval-valued fuzzy sets is the realist way: fuzzy models are replaced by interval-valued fuzzy models, which assign intervals of reals—not single reals—to wffs; each vague discourse is associated with a unique intended model; hence each statement in the discourse has a unique truth value—but this truth value is an interval, not a particular number. Alternatively, we could proceed in the nominalist way. Instead of associating each discourse with a unique non-fuzzy model, we associate it with multiple fuzzy models: those models which have the same domain as the unique intended interval-valued model $\mathfrak{M}$ countenanced on the realist approach, and which are such that the extension of each predicate sends each object in the domain to a real which is in the interval to which that object is sent by the extension of that predicate on $\mathfrak{M}$. Note however that there is an important difference between the nominalist versions of the ordinal view and the interval-valued view: in the ordinal case, the acceptable fuzzy models can all be generated by taking a single fuzzy model and applying to it a certain sort of transformation of the fuzzy truth values; in the interval-valued case, the acceptable models cannot (in general) be generated in this sort of way.

5 The Basic Fuzzy View as a Mock-Up

As mentioned earlier, the most detailed development of the logic-as-modeling view in the literature is Cook's [2002] discussion of fuzzy model theory as an account of vagueness in natural language. Cook argues that viewing the basic fuzzy account as providing a mock-up, rather than a description, of the semantics of vague language allows one to sidestep the problem of artificial precision:

> In essence, the idea is to treat the problematic parts of the degree-theoretic picture, namely the assignment of particular real numbers to sentences, as mere artifacts. ... If the problematic parts of the account are not intended actually to describe anything occurring in the phenomenon in the first place, then they certainly cannot be *misdescribing*. [p.237]

As already noted, Cook recognizes that in order to make good on this line of thought, he needs to specify which aspects of the mock-up are artifacts and which are not. His *general* approach to this question is as follows:

> there are real verities in the world. We use the real numbers to model these verities, however, as a matter of convenience, and many (but not all) of the properties holding of them are artifactual ... although sentences do have real verities, these verities are not real numbers but are only *modeled* by real numbers [p.239]

That is, there are real degrees of truth ('verities'): it is useful to use the fuzzy truth values (reals in $[0, 1]$) to model them—but in reality the verities and the fuzzy truth values are distinct entities. So far so good—but this (as Cook is fully aware) still leaves the *specific* details wide open. We still want to know *which* properties of the fuzzy truth values represent properties of the verities, and which properties of the ftv's are mere artifacts. First, Cook holds that the *ordering* of the ftv's is representative [p.241]. Note that if that was *all* that was representative about the ftv's, we would be straight back to the realist version of the ordinal view. (Realist because verities are regarded as distinct entities from ftv's. In this case, the verities would just be the rubbery truth values introduced in §3 above.) However, Cook thinks that there is more about the ftv's that is representative than just their ordering:

> small changes in the real numbers assigned to sentences are often artifactual, and will not affect the relations, logical or otherwise, between the sentences. Clearly, however, large changes in these assignments will change these relations ... In other words, if we are given two sentences such that the real number assigned to the first is significantly smaller than the one assigned to the second, then we can conclude that there is a real difference in degree of truth between the two sentences. A small difference, however, is not necessarily indicative of any actual difference in verity [p.241]

This is *suggestive* of the interval-valued view: the real verities are intervals; two ftv's represent the *same* verity (interval) if they are both inside it. However Cook does not spell out the view in this way—indeed, he does not spell out the view in full detail at all. This, however, is not the criticism being made here. There are clearly ways of (fully) spelling out a view which involves a combination of the ordinal and interval-valued approaches. Rather, the present point is as follows. If the view is *not* spelled out, then the modeling approach is not useful: as noted in this paper, and by the proponents of the logic-as-modeling view themselves, in order for a model to be useful, we need to know which aspects of it are representors and which are artifacts. On the other hand, if the view *is* spelled out, then we have to hand a distinct theory to which we take the *descriptive* approach. In the present case, this distinct theory will be one which countenances truth values distinct from ftv's—they might be intervals, or rubbery truth values, or elements in some structure which combines aspects of both the ordinal and interval-valued views—and which regards each statement in a vague discourse as being assigned a unique one of these truth values (i.e. its value on the unique intended model).

6 Conclusion

Examination of the specific case—the case of the basic fuzzy theory of vagueness, the problem of artificial precision for this view, and the logic-as-modeling solution to this problem—reinforces the general conclusion drawn in §1: the modeling perspective does not provide a full-fledged alternative to the descriptive perspective, because we can adopt the modeling perspective on some formalism only if we are prepared to adopt the descriptive perspective on some other formalism.[11]

Notes

[1]For further discussion of these two positions, see Cook [2002, p.234] and Smith [2008, §2.1.3.1].

[2]For discussions of this view see Corcoran [1973], Sánchez-Miguel [1993], Edgington [1997], Shapiro [1998], Cook [2000], Shapiro [2001], Martínez [2001], Cook [2002] and Shapiro [2006, ch.2]. (Note that in some of these discussions, the focus is on viewing formal languages together with systems of *deduction* as mock-ups of chains of *reasoning* in natural language.) Proponents of the view usually speak of formal theories as *models*, where 'model' is used in the sense it has in, for example, 'model aeroplane' or 'Bohr's model of the atom'. In order to avoid confusion between models in this sense, and models in the sense of 'model theory', this paper follows Sánchez-Miguel [1993, p.123, p.127 n.3] in using the term 'mock-up' in place of 'model' in the former sense.

[3]This is different from an objection to the modeling perspective that has been made in the literature. Keefe [2000, p.55] notes—as was noted earlier in the present paper—that when it comes to the logic-as-modeling approach, "What is needed is an explicit, systematic account of how the model corresponds to or applies to natural language, stating which aspects of the model are representational, and justifying the treatment of others as mere artifacts." She then continues: "It is far from clear how this could be done." So Keefe's objection is that proponents of the logic-as-modeling approach have not, and perhaps cannot, make good on the requirement that they say which parts of their models are artifacts. The present point is different: here there

is no general pessimism about the possibility of specifying the artifacts; the point is that when the artifacts are specified, we then have to hand another representation of the modeled phenomena, to which we take the descriptive attitude.

[4]For example, the propositional calculus for three-valued Łukasiewicz logic with disjunction, conjunction and negation is the same as the propositional calculus for fuzzy logic with disjunction, conjunction and negation defined in terms of max, min and subtraction from 1 (for the details, see Nguyen and Walker [2000, 68–70]). For certain purposes, the three-valued system is easier to work with, because we can do *truth tables* with three truth values, but not with infinitely many truth values.

[5]Cf. Lepore [1983, p.181]: "A theory of meaning ... is concerned only with a single interpretation of a language, the correct or intended one: so its fundamental notion is that of meaning or truth—simpliciter."

[6]The term 'higher-order vagueness' is used more widely in the literature in reference to this problem, but this term is also applied to problems which are rather different in character from the problem for the fuzzy view under discussion here; the term 'artificial precision' is therefore used in this paper. Cook [2011] uses the term 'problem of inappropriate precision' to denote a general kind of problem, particular versions of which confront a number of different theories of vagueness; the particular version of this general problem which confronts the fuzzy theory of vagueness is what is here called the problem of artificial precision.

[7]For further statements of the problem see Copeland [1997, pp.521–2], Goguen [1968–69, p.332; 1979, p.54], Lakoff [1973, p.462, p.481], Machina

[1976, p.187], Rolf [1984, pp.223–4], Schwartz [1990, p.46], Tye [1995, p.11], Williamson [1994, pp.127–8] and Keefe [2000, pp.113–4].

[8]See also Sanford [1975, p.29], Goguen [1979, p.59], Hájek [1999, pp.162–3] and Weatherson [2005].

[9]In what follows, for the sake of simplicity, the focus is on the case of length—but the discussion applies, mutatis mutandis, to the measurement of other attributes.

[10]See Zadeh [1975], Grattan-Guinness [1976], Klir and Yuan [1995, p.16] and Dubois and Prade [2005].

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