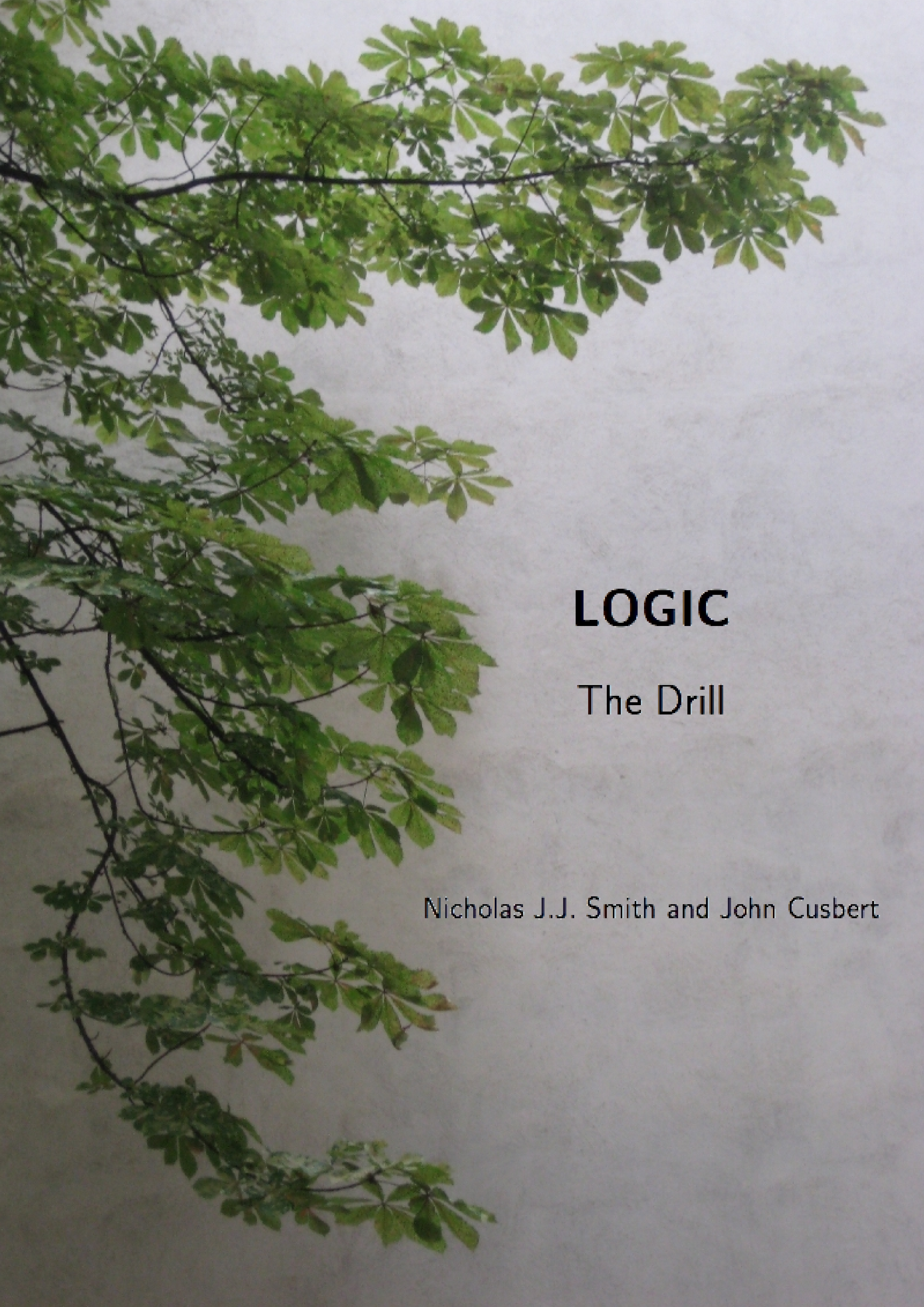

LOGIC

The Drill

Nicholas J.J. Smith and John Cusbert

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Preface

The first part of this volume contains all the exercise questions that appear in *Logic: The Laws of Truth* by Nicholas J.J. Smith (Princeton University Press, 2012). The second part contains answers to almost all of these exercises. Both the questions and the answers are a collaborative effort between Nicholas J.J. Smith and John Cusbert.

One obvious use of this work is as a solutions manual for readers of *Logic: The Laws of Truth*—but it should also be of use to readers of other logic books. Students of logic need a large number of worked examples and exercise problems *with solutions*: the more the better. This volume should help to meet that need.

After each question, a cross-reference of the form '[A p.x]' appears. This indicates the page on which the answer to that question can be found. You can click on the cross-reference to be taken directly to the answer. Each answer then contains a cross-reference of the form '[Q p.x]' which leads back to the corresponding question. Other blue items are also links: for example, clicking on an entry in the Contents pages takes you directly to the relevant section, and at the end of each exercise set and each answer set there is a link back to the Contents.

If you find any errors—or have any other comments or suggestions—please email us at:

`logicthedrill@gmail.com`

The latest version of this work can be found at:

`http://www.personal.usyd.edu.au/~njjsmith/lawsoftruth/`

Any significant revisions (e.g. corrections or additions to the exercises or answers) will be documented on the copyright page.

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Questions

Chapter 1

Propositions and Arguments

Exercises 1.2.1

Classify the following as propositions or nonpropositions.

1. Los Angeles is a long way from New York. [A p.78]
2. Let's go to Los Angeles! [A p.78]
3. Los Angeles, whoopee! [A p.78]
4. Would that Los Angeles were not so far away. [A p.78]
5. I really wish Los Angeles were nearer to New York. [A p.78]
6. I think we should go to Los Angeles. [A p.78]
7. I hate Los Angeles. [A p.78]
8. Los Angeles is great! [A p.78]
9. If only Los Angeles were closer. [A p.78]
10. Go to Los Angeles! [A p.78]

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Exercises 1.3.1

Represent the following lines of reasoning as arguments.

1. If the stock market crashes, thousands of experienced investors will lose a lot of money. So the stock market won't crash. [A p.78]

2. Diamond is harder than topaz, topaz is harder than quartz, quartz is harder than calcite, and calcite is harder than talc, therefore diamond is harder than talc. [A p.79]

3. Any friend of yours is a friend of mine; and you're friends with everyone on the volleyball team. Hence, if Sally's on the volleyball team, she's a friend of mine. [A p.79]

4. When a politician engages in shady business dealings, it ends up on page one of the newspapers. No South Australian senator has ever appeared on page one of a newspaper. Thus, no South Australian senator engages in shady business dealings. [A p.79]

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Exercises 1.4.1

State whether each of the following arguments is valid or invalid.

1. All dogs are mammals.
   All mammals are animals.

   All dogs are animals. [A p.79]

2. All dogs are mammals.
   All dogs are animals.

   All mammals are animals. [A p.79]

3. All dogs are mammals.
   No fish are mammals.

   No fish are dogs. [A p.79]

4. All fish are mammals.
All mammals are robots.

All fish are robots. [A p.79]

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Exercises 1.5.1

1. Which of the arguments in Exercise 1.4.1 are sound? [A p.79]
2. Find an argument in Exercise 1.4.1 that has all true premises and a true conclusion but is not valid and hence not sound. [A p.79]
3. Find an argument in Exercise 1.4.1 that has false premises and a false conclusion but is valid. [A p.79]

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Exercises 1.6.1.1

1. What is the negand of:
   (i) Bob is not a good student [A p.80]
   (ii) I haven't decided not to go to the party. [A p.80]
   (iii) Mars isn't the closest planet to the sun. [A p.80]
   (iv) It is not the case that Alice is late. [A p.80]
   (v) I don't like scrambled eggs. [A p.80]
   (vi) Scrambled eggs aren't good for you. [A p.80]
2. If a proposition is true, its double negation is. . . ? [A p.80]
3. If a proposition's double negation is false, the proposition is. . . ? [A p.80]

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Exercises 1.6.2.1

What are the conjuncts of the following propositions?

1. The sun is shining, and I am happy. [A p.80]
2. Maisie and Rosie are my friends. [A p.80]
3. Sailing is fun, and snowboarding is too. [A p.80]
4. We watched the movie and ate popcorn. [A p.80]
5. Sue does not want the red bicycle, and she does not like the blue one. [A p.80]
6. The road to the campsite is long and uneven. [A p.80]

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Exercises 1.6.4.1

What are the (a) antecedents and (b) consequents of the following propositions?

1. If that's pistachio ice cream, it doesn't taste the way it should. [A p.80]
2. That tastes the way it should only if it isn't pistachio ice cream. [A p.80]
3. If that is supposed to taste that way, then it isn't pistachio ice cream. [A p.81]
4. If you pressed the red button, then your cup contains coffee. [A p.81]
5. Your cup does not contain coffee if you pressed the green button. [A p.81]
6. Your cup contains hot chocolate only if you pressed the green button. [A p.81]

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Exercises 1.6.6

State what sort of compound proposition each of the following is, and identify its components. Do the same for the components.

1. If it is sunny and windy tomorrow, we shall go sailing or kite flying. [A p.81]
2. If it rains or snows tomorrow, we shall not go sailing or kite flying. [A p.81]
3. Either he'll stay here and we'll come back and collect him later, or he'll come with us and we'll all come back together. [A p.81]
4. Jane is a talented painter and a wonderful sculptor, and if she remains interested in art, her work will one day be of the highest quality. [A p.81]
5. It's not the case that the unemployment rate will both increase and decrease in the next quarter. [A p.82]
6. Your sunburn will get worse and become painful if you don't stop swimming during the daytime. [A p.82]
7. Either Steven won't get the job, or I'll leave and all my clients will leave. [A p.82]
8. The Tigers will not lose if and only if both Thompson and Thomson get injured. [A p.82]
9. Fido will wag his tail if you give him dinner at 6 this evening, and if you don't, then he will bark. [A p.82]
10. It will rain or snow today—or else it won't. [A p.83]

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Chapter 2

The Language of Propositional Logic

Exercises 2.3.3

Using the glossary:

A:	Aristotle was a philosopher
B:	Paper burns
F:	Fire is hot

translate the following from PL into English.

1. $\neg A$ [A p.84]
2. $(A \wedge B)$ [A p.84]
3. $(A \wedge \neg B)$ [A p.84]
4. $(\neg F \wedge \neg B)$ [A p.84]
5. $\neg(F \wedge B)$ [A p.84]

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Exercises 2.3.5

Using the glossary of Exercise 2.3.3, translate the following from PL into English.

1. $((A \wedge B) \vee F)$ [A p.84]
2. $(\neg A \vee \neg B)$ [A p.84]
3. $((A \vee B) \wedge \neg(A \wedge B))$ [A p.84]
4. $\neg(A \vee F)$ [A p.84]
5. $(A \wedge (B \vee F))$ [A p.85]

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Exercises 2.3.8

1. Using the glossary:

   B: The sky is blue
   G: Grass is green
   R: Roses are red
   W: Snow is white
   Y: Bananas are yellow

   translate the following from PL into English.

   (i) $(W \rightarrow B)$ [A p.85]
   (ii) $(W \leftrightarrow (W \wedge \neg R))$ [A p.85]
   (iii) $\neg(R \rightarrow \neg W)$ [A p.85]
   (iv) $((R \vee W) \rightarrow (R \wedge \neg W))$ [A p.85]
   (v) $((W \wedge W) \vee (R \wedge \neg B))$ [A p.85]
   (vi) $(G \vee (W \rightarrow R))$ [A p.85]
   (vii) $((Y \leftrightarrow Y) \wedge (\neg Y \leftrightarrow \neg Y))$ [A p.85]
   (viii) $((B \rightarrow W) \rightarrow (\neg W \rightarrow \neg B))$ [A p.85]
   (ix) $(((R \wedge W) \wedge B) \rightarrow (Y \vee G))$ [A p.85]

(x) $\neg(\neg R \wedge (\neg W \vee G))$ [A p.85]

2. Translate the following from English into PL.

   (i) Only if the sky is blue is snow white. [A p.86]

   (ii) The sky is blue if, and only if, snow is white and roses are not red. [A p.86]

   (iii) It's not true that if roses are red, then snow is not white. [A p.86]

   (iv) If snow and roses are red, then roses are red and/or snow isn't. [A p.86]

   (v) Jim is tall if and only if Maisy is, and Maisy is tall only if Nora is not. [A p.86]

   (vi) Jim is tall only if Nora or Maisy is. [A p.86]

   (vii) If Jim is tall, then either Maisy is tall or Nora isn't. [A p.86]

   (viii) Either snow is white and Maisy is tall, or snow is white and she isn't. [A p.86]

   (ix) If Jim is tall and Jim is not tall, then the sky both is and is not blue. [A p.86]

   (x) If Maisy is tall and the sky is blue, then Jim is tall and the sky is not blue. [A p.86]

3. Translate the following from English into PL.

   (i) If it is snowing, we are not kite flying. [A p.87]

   (ii) If it is sunny and it is windy, then we are sailing or kite flying. [A p.87]

   (iii) Only if it is windy are we kite flying, and only if it is windy are we sailing. [A p.87]

   (iv) We are sailing or kite flying—or skiing. [A p.87]

   (v) If—and only if—it is windy, we are sailing. [A p.87]

   (vi) We are skiing only if it is windy or snowing. [A p.87]

   (vii) We are skiing only if it is both windy and snowing. [A p.87]

   (viii) If it is sunny, then if it is windy, we are kite flying. [A p.87]

   (ix) We are sailing only if it is sunny, windy, and not snowing. [A p.87]

(x) If it is sunny and windy, we're sailing, and if it is snowing and not windy, we're skiing. [A p.87]

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Exercises 2.5.1

1. State whether each of the following is a wff of PL.

   (i) $((A \to B))$ [A p.87]

   (ii) $(A \to\to B)$ [A p.87]

   (iii) $(A \to (A \to A))$ [A p.87]

   (iv) $A \to ((A \to A))$ [A p.87]

   (v) $((A \wedge B)\wedge)A$ [A p.87]

   (vi) $(A \vee (A \vee (A \vee (A \vee (A \vee (A \vee (A \vee A)))))))$ [A p.87]

   (vii) $((AA \vee \wedge BC))$ [A p.87]

   (viii) $((A \vee A) \wedge BC))$ [A p.87]

   (ix) ABC [A p.87]

   (x) $((A \vee A) \wedge ((A \vee A) \wedge ((A \vee A) \wedge A)))$ [A p.87]

2. Give recursive definitions of the following.

   (i) The set of all odd numbers. [A p.88]

   (ii) The set of all numbers divisible by five. [A p.88]

   (iii) The set of all "words" (finite strings of letters) that use only (but not necessarily both of) the letters *a* and *b*. [A p.88]

   (iv) The set containing all of Bob's ancestors. [A p.88]

   (v) The set of all cackles: hah hah hah, hah hah hah hah, hah hah hah hah hah, and so on. [A p.88]

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Exercises 2.5.3.1

Write out a construction for each of the following wffs, and state the main connective.

1. $(\neg P \vee (Q \wedge R))$ [A p.89]
2. $\neg(P \wedge (Q \vee R))$ [A p.89]
3. $((\neg P \wedge \neg Q) \vee \neg R)$ [A p.89]
4. $((P \to Q) \vee (R \to S))$ [A p.89]
5. $(((P \leftrightarrow Q) \leftrightarrow R) \leftrightarrow S)$ [A p.89]
6. $((\neg P \wedge \neg\neg P) \to (P \wedge \neg P))$ [A p.90]

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Exercises 2.5.4.1

1. For each of the remaining orderings (2–6) of the connectives $\to, \wedge$, and $\vee$ given in §2.5.4, state which disambiguation (1–5) results from restoring parentheses to our original expression in this order.
[A p.90]

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Exercises 2.5.5.1

1. Write the following in the notation of this book:
   (i) $\vee \neg P \wedge QR$ [A p.90]
   (ii) $\neg \wedge \vee PQR$ [A p.90]
   (iii) $\wedge \neg \vee PQR$ [A p.90]
   (iv) $\vee \wedge \neg P \neg Q \neg R$ [A p.90]
   (v) $\leftrightarrow\leftrightarrow\leftrightarrow PQRS$ [A p.90]
2. Write the following in Polish notation:
   (i) $\neg(P \wedge (Q \vee R))$ [A p.91]

(ii) $([P \to (Q \vee R)] \to S)$ [A p.91]

(iii) $[(P \to Q) \vee (R \to S)]$ [A p.91]

(iv) $(P \to [(Q \vee R) \to S])$ [A p.91]

(v) $[(\neg P \wedge \neg\neg P) \to (P \wedge \neg P)]$ [A p.91]

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Chapter 3

Semantics of Propositional Logic

Exercises 3.2.1

Determine the truth values of the following wffs, given the truth values for their basic components, which are written under those components.

1. $$\begin{array}{ccccc} (\neg P & \wedge & (Q & \vee & R)) \\ \text{T} & & \text{T} & & \text{F} \end{array}$$ [A p.92]

2. $$\begin{array}{ccccc} \neg(P & \vee & (Q & \rightarrow & R)) \\ \text{T} & & \text{T} & & \text{F} \end{array}$$ [A p.92]

3. $$\begin{array}{ccccccc} (\neg\neg P & \wedge & (Q & \rightarrow & (R & \vee & P))) \\ \text{F} & & \text{T} & & \text{T} & & \text{F} \end{array}$$ [A p.92]

4. $$\begin{array}{ccccccc} (\neg\neg P & \wedge & (Q & \rightarrow & (R & \vee & P))) \\ \text{T} & & \text{F} & & \text{F} & & \text{T} \end{array}$$ [A p.92]

5. $$\begin{array}{ccccccc} ((P & \vee & Q) & \rightarrow & (P & \vee & P)) \\ \text{F} & & \text{T} & & \text{F} & & \text{F} \end{array}$$ [A p.92]

6. $$\begin{array}{ccccccc} ((P & \vee & Q) & \rightarrow & (P & \vee & P)) \\ \text{T} & & \text{F} & & \text{T} & & \text{T} \end{array}$$ [A p.93]

7. $$\begin{array}{ccccccc} (P & \rightarrow & (Q & \rightarrow & (R & \rightarrow & S))) \\ \text{T} & & \text{T} & & \text{T} & & \text{F} \end{array}$$ [A p.93]

8. $$\begin{array}{ccccccc} (P & \rightarrow & (Q & \rightarrow & (R & \rightarrow & S))) \\ \text{F} & & \text{T} & & \text{F} & & \text{T} \end{array}$$ [A p.93]

9. $$\begin{array}{ccccccc} \neg(((\neg P & \leftrightarrow & P) & \leftrightarrow & Q) & \rightarrow & R) \\ \text{F} & & \text{F} & & \text{F} & & \text{F} \end{array}$$ [A p.93]

10. $\begin{array}{ccccccc} \neg(((\neg P & \leftrightarrow & P) & \leftrightarrow & Q) & \to & R) \\ \text{T} & & \text{T} & & \text{T} & & \text{T} \end{array}$ [A p.93]

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Exercises 3.3.1

Draw up truth tables for the following propositions.

1. $((P \wedge Q) \vee P)$ [A p.93]
2. $(P \wedge (P \vee P))$ [A p.94]
3. $\neg(\neg P \wedge \neg Q)$ [A p.94]
4. $(Q \to (Q \wedge \neg Q))$ [A p.94]
5. $(P \to (Q \to R))$ [A p.94]
6. $((P \vee Q) \leftrightarrow (P \wedge Q))$ [A p.94]
7. $\neg((P \wedge Q) \leftrightarrow Q)$ [A p.94]
8. $(((P \to \neg P) \to \neg P) \to \neg P)$ [A p.94]
9. $\neg(P \wedge (Q \wedge R))$ [A p.95]
10. $((\neg R \vee S) \wedge (S \vee \neg T))$ [A p.95]

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Exercises 3.4.1

Draw up a joint truth table for each of the following groups of propositions.

1. $(P \to Q)$ and $(Q \to P)$ [A p.95]
2. $\neg(P \leftrightarrow Q)$ and $((P \vee Q) \wedge \neg(P \wedge Q))$ [A p.95]
3. $\neg(P \wedge \neg Q)$ and $\neg Q$ [A p.95]
4. $((P \to Q) \wedge R)$ and $(P \vee (Q \vee R))$ [A p.96]
5. $((P \wedge Q) \wedge (\neg R \wedge \neg S))$ and $((P \vee (R \to Q)) \wedge S)$ [A p.96]

6. $(P \wedge \neg P)$ and $(Q \wedge \neg Q)$ [A p.96]
7. $(P \vee (Q \leftrightarrow R))$ and $((Q \rightarrow P) \wedge Q)$ [A p.96]
8. $\neg((P \wedge Q) \wedge R)$ and $((P \rightarrow Q) \leftrightarrow (P \rightarrow R))$ [A p.97]
9. $(P \vee Q)$, $\neg P$ and $(Q \vee Q)$ [A p.97]
10. $(P \rightarrow (Q \rightarrow (R \rightarrow S)))$ and $\neg S$ [A p.97]

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Exercises 3.5.1

1. Can the meaning of any of our two-place connectives $(\wedge, \vee, \rightarrow, \leftrightarrow)$ be specified as the truth function f_2^2 defined in Figure 3.2? [A p.98]
2. Define truth functions f_4^2 and f_5^2 such that the meanings of $\wedge$ and $\rightarrow$ (respectively) can be specified as these truth functions. [A p.98]
3. Suppose we introduce a new one-place connective $\star$ and specify its meaning as the truth function f_1^1 defined in Figure 3.2. What is the truth value of $\star A$ when A is T? [A p.98]
4. What truth values do you need to know to determine the truth value of $\star(A \rightarrow B)$?
   (i) The truth values of A and B.
   (ii) The truth value of A but not of B.
   (iii) The truth value of B but not of A.
   (iv) None. [A p.98]
5. Which of our connectives could have its meaning specified as the two-place function $g(x,y)$ defined as follows?

$$g(x,y) = f_3^2(f_2^1(x), y)$$

[A p.98]

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Chapter 4

Uses of Truth Tables

Exercises 4.1.2

Use truth tables to determine whether each of the following arguments is valid. For any argument that is not valid, give a counterexample.

1. $A \vee B$
   $A \to C$
   $\therefore\ (B \to C) \to C$ [A p.99]

2. $\neg A$
   $\therefore\ \neg((A \to B) \wedge (B \to C)) \vee C$ [A p.99]

3. $(A \wedge \neg B) \to C$
   $\neg C$
   B
   $\therefore\ \neg A$ [A p.100]

4. $(A \wedge B) \leftrightarrow C$
   A
   $\therefore\ C \to B$ [A p.100]

5. $(\neg A \wedge \neg B) \leftrightarrow \neg C$
   $\neg(A \vee B)$
   $\therefore\ C \to \neg C$ [A p.100]

6. $A \vee B$
   $\neg A \vee C$
   $B \to C$
   $\therefore\ C$ [A p.101]

7. $\neg(A \vee B) \leftrightarrow \neg C$
   $\neg A \wedge \neg B$
   $\therefore\ C \wedge \neg C$ [A p.101]

8. $\neg(A \wedge B) \rightarrow (C \vee A)$
   $\neg A \vee \neg B$
   A
   $\therefore\ \neg(C \vee \neg C)$ [A p.101]

9. $A \rightarrow (B \wedge C)$
   $B \leftrightarrow \neg C$
   $\therefore\ \neg A$ [A p.102]

10. $A \rightarrow B$
    $B \rightarrow C$
    $\neg C$
    $\therefore\ \neg A$ [A p.102]

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Exercises 4.2.1

Write out truth tables for the following propositions, and state whether each is a tautology, a contradiction, or neither.

1. $((P \vee Q) \rightarrow P)$ [A p.102]
2. $(\neg P \wedge (Q \vee R))$ [A p.103]
3. $((\neg P \vee Q) \leftrightarrow (P \wedge \neg Q))$ [A p.103]
4. $(P \rightarrow (Q \rightarrow (R \rightarrow P)))$ [A p.103]
5. $(P \rightarrow ((P \rightarrow Q) \rightarrow Q))$ [A p.103]
6. $(P \rightarrow ((Q \rightarrow P) \rightarrow Q))$ [A p.104]
7. $((P \rightarrow Q) \vee \neg(Q \wedge \neg Q))$ [A p.104]
8. $((P \rightarrow Q) \vee \neg(Q \wedge \neg P))$ [A p.104]
9. $((P \wedge Q) \leftrightarrow (Q \leftrightarrow P))$ [A p.104]
10. $\neg((P \wedge Q) \rightarrow (Q \leftrightarrow P))$ [A p.104]

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Exercises 4.3.1

Write out joint truth tables for the following pairs of propositions, and state in each case whether the two propositions are (a) jointly satisfiable, (b) equivalent, (c) contradictory, (d) contraries.

1. $(P \rightarrow Q)$ and $\neg(P \wedge \neg Q)$ [A p.105]
2. $(P \wedge Q)$ and $(P \wedge \neg Q)$ [A p.105]
3. $\neg(P \leftrightarrow Q)$ and $\neg(P \rightarrow Q) \vee \neg(P \vee \neg Q)$ [A p.105]
4. $(P \rightarrow (Q \rightarrow R))$ and $((P \rightarrow Q) \rightarrow R)$ [A p.106]
5. $(P \wedge (Q \wedge \neg Q))$ and $\neg(Q \rightarrow \neg(R \wedge \neg Q))$ [A p.106]
6. $(P \wedge \neg P)$ and $(R \vee \neg R)$ [A p.107]
7. $(P \wedge \neg P)$ and $\neg(Q \rightarrow Q)$ [A p.107]
8. $((P \rightarrow Q) \rightarrow R)$ and $\neg(P \vee \neg(Q \wedge \neg R))$ [A p.108]
9. $(P \leftrightarrow Q)$ and $((P \wedge Q) \vee (\neg P \wedge \neg Q))$ [A p.108]
10. $(P \leftrightarrow Q)$ and $((P \wedge Q) \vee (\neg P \wedge \neg Q))$ [A p.109]

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Exercises 4.4.1

Write out a joint truth table for the propositions in each of the following sets, and state whether each set is satisfiable.

1. $\{(P \vee Q), \neg(P \wedge Q), P\}$ [A p.109]
2. $\{\neg(P \rightarrow Q), (P \leftrightarrow Q), (\neg P \vee Q)\}$ [A p.109]
3. $\{(P \rightarrow \neg P), (P \vee \neg P), (\neg P \rightarrow P)\}$ [A p.109]
4. $\{((P \vee Q) \vee R), (\neg P \rightarrow \neg Q), (\neg Q \rightarrow \neg R), \neg P\}$ [A p.110]
5. $\{(P \leftrightarrow Q), (Q \vee R), (R \rightarrow P)\}$ [A p.110]
6. $\{(\neg P \rightarrow \neg Q), (P \leftrightarrow Q), P\}$ [A p.110]
7. $\{\neg P, (P \rightarrow (P \rightarrow P)), (\neg P \leftrightarrow P)\}$ [A p.110]

8. $\{(P \vee \neg Q), (P \to R), \neg R, (\neg R \to Q)\}$ [A p.111]

9. $\{\neg R, \neg P, ((Q \to \neg Q) \to R)\}$ [A p.111]

10. $\{(\neg P \vee \neg Q), \neg(P \wedge \neg Q), (P \vee \neg Q), \neg(\neg P \wedge \neg Q)\}$ [A p.111]

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Chapter 5

Logical Form

Exercises 5.1.1

For each of the following propositions, give three correct answers to the question "what is the form of this proposition?"

1. $\neg(R \to (R \to Q))$ [A p.112]
2. $(R \vee P) \to (R \vee P)$ [A p.112]
3. $P \wedge (\neg P \to Q)$ [A p.112]
4. $((\neg P \vee Q) \wedge P) \leftrightarrow R$ [A p.112]

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Exercises 5.2.1

1. The following propositions all have three logical forms in common. State what the three forms are, and in each case, show what replacements of variables by propositions are required to obtain the three propositions from the form.

   (i) $\neg\neg C$

   (ii) $\neg\neg(A \wedge B)$

   (iii) $\neg\neg(C \wedge \neg D)$ [A p.113]

2. State whether the given propositions are instances of the given form. If so, show what replacements of variables by propositions are required to obtain the proposition from the form.

   (i) Form: $\neg(\alpha \to \beta)$
   Propositions:

   (a) $\neg(P \to Q)$ [A p.113]
   (b) $\neg(R \to Q)$ [A p.113]
   (c) $\neg(R \to (R \to Q))$ [A p.113]

   (ii) Form: $\neg(\alpha \to (\alpha \to \beta))$
   Propositions:

   (a) $\neg(P \to (P \to Q))$ [A p.113]
   (b) $\neg(P \to (P \to P))$ [A p.113]
   (c) $\neg(P \to (Q \to P))$ [A p.113]

   (iii) Form: $(\alpha \vee \beta) \to (\alpha \wedge \beta)$
   Propositions:

   (a) $(\neg P \vee Q) \to (\neg P \wedge Q)$ [A p.113]
   (b) $(P \vee \neg P) \to (P \wedge \neg P)$ [A p.113]
   (c) $\neg(R \vee S) \to \neg(R \wedge S)$ [A p.113]

   (iv) Form: $\alpha \vee (\neg\beta \vee \alpha)$
   Propositions:

   (a) $(P \vee Q) \vee (Q \vee (P \vee Q))$ [A p.113]
   (b) $Q \vee (\neg Q \vee (Q \wedge Q))$ [A p.113]
   (c) $\neg P \vee (\neg\neg P \vee \neg P)$ [A p.113]

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Exercises 5.3.1

For each of the following arguments, give four correct answers to the question "what is the form of this argument?" For each form, show what replacements of variables by propositions are required to obtain the argument from the form.

1. $\neg(R \to (R \to Q))$
   $\therefore\ R \vee (R \to Q)$ [A p.114]

2. $(P \wedge Q) \to Q$
   $\neg Q$
   $\therefore\ \neg(P \wedge Q)$ [A p.114]

3. $\neg Q \to (R \to S)$
   $\neg Q$
   $\therefore\ R \to S$ [A p.115]

4. $(P \to \neg Q) \vee (\neg Q \to P)$
   $\neg(\neg Q \to P)$
   $\therefore\ P \to \neg Q$ [A p.115]

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Exercises 5.4.1

For each of the following arguments, (i) show that it is an instance of the form:

α
$\alpha \to \beta$
$\therefore\ \beta$

by stating what substitutions of propositions for variables have to be made to otbain the argument from the form, and (ii) show by producing a truth table for the argument that it is valid.

1. P
   $P \to Q$
   $\therefore\ Q$ [A p.116]

2. $(A \wedge B)$
   $(A \wedge B) \to (B \vee C)$
   $\therefore\ (B \vee C)$ [A p.116]

3. $(A \vee \neg A)$
   $(A \vee \neg A) \to (A \wedge \neg A)$
   $\therefore\ (A \wedge \neg A)$ [A p.116]

4. $(P \to \neg P)$
   $(P \to \neg P) \to (P \to (Q \wedge \neg R))$
   $\therefore\ (P \to (Q \wedge \neg R))$ [A p.116]

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Exercises 5.5.1

1. (i) Show by producing a truth table for the following argument form that it is invalid:

$$\begin{array}{ll} & \alpha \\ \therefore & \beta \end{array}$$

[A p.117]

   (ii) Give an instance of the above argument form that is valid; show that it is valid by producing a truth table for the argument.

[A p.117]

2. While it is not true in general that every instance of an invalid argument form is an invalid argument, there are some invalid argument forms whose instances are always invalid arguments. Give an example of such an argument form. [A p.117]

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Chapter 6

Connectives: Translation and Adequacy

Exercises 6.5.1

Translate the following arguments into PL and then assess them for validity (you may use shortcuts in your truth tables).

1. Bob is happy if and only if it is raining. Either it is raining or the sun is shining. So Bob is happy only if the sun is not shining. [A p.118]

2. If I have neither money nor a card, I shall walk. If I walk, I shall get tired or have a rest. So if I have a rest, I have money. [A p.119]

3. Maisy is upset only if there is thunder. If there is thunder, then there is lightning. Therefore, either Maisy is not upset, or there is lightning. [A p.121]

4. The car started only if you turned the key and pressed the accelerator. If you turned the key but did not press the accelerator, then the car did not start. The car did not start—so either you pressed the accelerator but did not turn the key, or you neither turned the key nor pressed the accelerator. [A p.122]

5. Either Maisy isn't barking, or there is a robber outside. If there is a robber outside and Maisy is not barking, then she is either asleep or depressed. Maisy is neither asleep nor depressed. Hence Maisy is barking if and only if there is a robber outside. [A p.123]

6. If it isn't sunny, then either it is too windy or we are sailing. We are having fun if we are sailing. It is not sunny and it isn't too windy either—hence we are having fun. [A p.124]

7. Either you came through Singleton and Maitland, or you came through Newcastle. You didn't come through either Singleton or Maitland—you came through Cessnock. Therefore, you came through both Newcastle and Cessnock. [A p.125]

8. We shall have lobster for lunch, provided that the shop is open. Either the shop will be open, or it is Sunday. If it is Sunday, we shall go to a restaurant and have lobster for lunch. So we shall have lobster for lunch. [A p.126]

9. Catch Billy a fish, and you will feed him for a day. Teach him to fish, and you'll feed him for life. So either you won't feed Billy for life, or you will teach him to fish. [A p.127]

10. I'll be happy if the Tigers win. Moreover, they will win—or else they won't. However, assuming they don't, it will be a draw. Therefore, if it's not a draw, and they don't win, I'll be happy. [A p.128]

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Exercises 6.6.3

1. State whether each of the following is a functionally complete set of connectives. Justify your answers.

   (i) $\{\rightarrow, \neg\}$ [A p.128]

   (ii) $\{\leftrightarrow, \veebar\}$ [A p.129]

   (iii) $\{②_{15}\}$ (The connective $②_{15}$ is often symbolized by $\downarrow$; another common symbol for this connective is NOR.) [A p.130]

   (iv) $\{\rightarrow, \wedge\}$ [A p.130]

   (v) $\{\neg, ②_{12}\}$ [A p.131]

   (vi) $\{\vee, ②_{4}\}$ [A p.131]

2. Give the truth table for each of the following propositions.

   (i) $B\, ②_{14}\, A$ [A p.131]

(ii) $(A \text{②}_{11} B) \text{②}_{15} B$ [A p.131]

(iii) $\neg(A \vee (A \text{②}_6 B))$ [A p.131]

(iv) $A \leftrightarrow (A \text{②}_3 \neg B)$ [A p.131]

(v) $(A \text{②}_{12} B) \veebar (B \text{②}_{12} A)$ [A p.132]

(vi) $(A \text{②}_{12} B) \veebar (B \text{②}_{16} A)$ [A p.132]

3. Consider the three-place connectives $\sharp$ and $\natural$, whose truth tables are as follows:

α	β	γ	$\sharp(\alpha, \beta, \gamma)$	$\natural(\alpha, \beta, \gamma)$
T	T	T	T	F
T	T	F	F	F
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	F	T
F	F	T	T	F
F	F	F	T	F

(i) Define $\sharp$ using only (but not necessarily all of) the connectives $\vee$, $\wedge$, and $\neg$. [A p.132]

(ii) Do the same for $\natural$. [A p.132]

4. State a proposition involving only the connectives $\neg$ and $\wedge$ that is equivalent to the given proposition.

(i) $\neg(A \to B)$ [A p.132]

(ii) $\neg(A \vee B)$ [A p.132]

(iii) $\neg A \vee \neg B$ [A p.132]

(iv) $\neg(\neg A \vee B)$ [A p.132]

(v) $A \leftrightarrow B$ [A p.132]

(vi) $(A \to B) \vee (B \to A)$ [A p.132]

5. (i) What is the dual of ①_1? [A p.132]

(ii) What is the dual of $\to$? [A p.133]

(iii) Which one-place connectives are their own duals? [A p.133]

(iv) Which two-place connectives are their own duals? [A p.133]

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Chapter 7

Trees for Propositional Logic

Exercises 7.2.1.1

Apply the appropriate tree rule to each of the following propositions.

1. $(\neg A \vee \neg B)$ [A p.134]
2. $(\neg A \rightarrow B)$ [A p.134]
3. $((A \rightarrow B) \wedge B)$ [A p.134]
4. $((A \leftrightarrow B) \leftrightarrow B)$ [A p.134]
5. $\neg(A \leftrightarrow \neg\neg A)$ [A p.134]
6. $\neg(\neg A \vee B)$ [A p.135]

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Exercises 7.2.2.1

Construct finished trees for each of the following propositions.

1. $((A \rightarrow B) \rightarrow B)$ [A p.135]
2. $((A \rightarrow B) \vee (B \rightarrow A))$ [A p.135]
3. $\neg(\neg A \rightarrow (A \vee B))$ [A p.135]
4. $\neg\neg((A \wedge B) \vee (A \wedge \neg B))$ [A p.135]

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Exercises 7.2.3.1

Construct finished trees for each of the following propositions; close paths as appropriate.

1. $\neg(A \rightarrow (B \rightarrow A))$ [A p.136]
2. $((A \rightarrow B) \vee (\neg A \vee B))$ [A p.136]
3. $\neg((A \rightarrow B) \vee (\neg A \vee B))$ [A p.136]
4. $\neg\neg\neg(A \vee B)$ [A p.136]
5. $\neg(A \wedge \neg A)$ [A p.136]
6. $\neg(\neg(A \wedge B) \leftrightarrow (\neg A \vee \neg B))$ [A p.137]

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Exercises 7.3.1.1

Using trees, determine whether the following arguments are valid. For any arguments that are invalid, give a counterexample.

1. A
   $\therefore\ (A \vee B)$ [A p.137]
2. $(A \vee B)$
   $\therefore\ B$ [A p.137]
3. $(A \vee B)$
   $(A \rightarrow C)$
   $(B \rightarrow D)$
   $\therefore\ (C \vee D)$ [A p.138]
4. $((A \vee \neg B) \rightarrow C)$
   $(B \rightarrow \neg D)$
   D
   $\therefore\ C$ [A p.138]
5. B
   $(A \rightarrow B)$
   $\therefore\ A$ [A p.138]

6. A
$(A \rightarrow B)$
$\therefore\ B$ [A p.138]

7. $(A \vee (B \wedge C))$
$(A \rightarrow B)$
$(B \leftrightarrow D)$
$\therefore\ (B \wedge D)$ [A p.139]

8. $\neg(\neg A \rightarrow B)$
$\neg(C \leftrightarrow A)$
$(A \vee C)$
$\neg(C \rightarrow B)$
$\therefore\ \neg(A \rightarrow B)$ [A p.139]

9. $(A \leftrightarrow B)$
$(B \rightarrow C)$
$(\neg B \rightarrow \neg C)$
$(A \vee (B \wedge \neg B))$
$\therefore\ C$ [A p.140]

10. $(A \rightarrow B)$
$(B \rightarrow C)$
$(C \rightarrow D)$
$(D \rightarrow E)$
$\therefore\ \neg(A \wedge \neg E)$ [A p.140]

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Exercises 7.3.2.1

1. Using trees, test whether the following propositions are contradictions. For any proposition that is satisfiable, read off from an open path a scenario in which the proposition is true.

   (i) $A \wedge \neg A$ [A p.141]
   (ii) $(A \vee B) \wedge \neg(A \vee B)$ [A p.141]
   (iii) $(A \rightarrow B) \wedge \neg(A \vee B)$ [A p.141]
   (iv) $(A \rightarrow \neg(A \vee B)) \wedge \neg(\neg(A \vee B) \vee B)$ [A p.141]
   (v) $\neg((\neg B \vee C) \leftrightarrow (B \rightarrow C))$ [A p.142]
   (vi) $(A \leftrightarrow \neg A) \vee (A \rightarrow \neg(B \vee C))$ [A p.142]

2. Using trees, test whether the following sets of propositions are satisfiable. For any set that is satisfiable, read off from an open path a scenario in which all the propositions in the set are true.

   (i) $\{(A \vee B), \neg B, (A \to B)\}$ [A p.142]

   (ii) $\{(A \vee B), (B \vee C), \neg(A \vee C)\}$ [A p.142]

   (iii) $\{\neg(\neg A \to B), \neg(C \leftrightarrow A), (A \vee C), \neg(C \to B), (A \to B)\}$ [A p.143]

   (iv) $\{(A \leftrightarrow B), \neg(A \to C), (C \to A), (A \wedge B) \vee (A \wedge C)\}$ [A p.143]

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Exercises 7.3.3.1

Test whether the following pairs of propositions are contraries, contradictories, or jointly satisfiable.

1. $(\neg A \to B)$ and $(B \to A)$ [A p.144]
2. $(A \to B)$ and $\neg(A \to (A \to B))$ [A p.144]
3. $\neg(A \leftrightarrow \neg B)$ and $\neg(A \vee \neg B)$ [A p.145]
4. $\neg(A \vee \neg B)$ and $(\neg A \to \neg B)$ [A p.146]
5. $(\neg A \wedge (A \to B))$ and $\neg(\neg A \to (A \to B))$ [A p.147]
6. $((A \to B) \leftrightarrow B)$ and $\neg(A \to B)$ [A p.147]

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Exercises 7.3.4.1

Test whether the following propositions are tautologies. (Remember to restore outermost parentheses before adding the negation symbol at the front—recall §2.5.4.) For any proposition that is not a tautology, read off from your tree a scenario in which it is false.

1. $A \to (B \to A)$ [A p.148]
2. $A \to (A \to B)$ [A p.148]

3. $((A \wedge B) \vee \neg(A \rightarrow B)) \rightarrow (C \rightarrow A)$ [A p.148]
4. $(A \wedge (B \vee C)) \leftrightarrow ((A \wedge B) \vee (A \wedge C))$ [A p.149]
5. $\neg A \vee \neg(A \wedge B)$ [A p.149]
6. $A \vee (\neg A \wedge \neg B)$ [A p.149]
7. $(A \rightarrow B) \vee (A \wedge \neg B)$ [A p.150]
8. $(B \wedge \neg A) \leftrightarrow (A \leftrightarrow B)$ [A p.150]
9. $(A \vee (B \vee C)) \leftrightarrow ((A \vee B) \vee C)$ [A p.150]
10. $(A \wedge (B \vee C)) \leftrightarrow ((A \vee B) \wedge C)$ [A p.151]

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Exercises 7.3.5.1

Test whether the following are equivalent. Where the two propositions are not equivalent, read off from your tree a scenario in which they have different truth values.

1. P and $(P \wedge P)$ [A p.151]
2. $(P \rightarrow (Q \vee \neg Q))$ and $(R \rightarrow R)$ [A p.151]
3. $\neg(A \vee B)$ and $(\neg A \wedge \neg B)$ [A p.152]
4. $\neg(A \vee B)$ and $(\neg A \vee \neg B)$ [A p.152]
5. $\neg(A \wedge B)$ and $(\neg A \wedge \neg B)$ [A p.152]
6. $\neg(A \wedge B)$ and $(\neg A \vee \neg B)$ [A p.153]
7. A and $((A \wedge B) \vee (A \wedge \neg B))$ [A p.153]
8. $\neg(P \leftrightarrow Q)$ and $((P \wedge \neg Q) \vee (\neg P \wedge Q))$ [A p.154]
9. $((P \wedge Q) \rightarrow R)$ and $(P \rightarrow (\neg Q \vee R))$ [A p.154]
10. $\neg(P \leftrightarrow Q)$ and $(Q \wedge \neg P)$ [A p.155]

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Chapter 8

The Language of Monadic Predicate Logic

Exercises 8.2.1

Translate the following propositions from English into MPL:

1. The Pacific Ocean is beautiful. [A p.156]
2. New York is heavily populated. [A p.156]
3. Mary is nice. [A p.156]
4. John is grumpy. [A p.157]
5. Seven is a prime number. [A p.157]
6. Pluto is a planet. [A p.157]
7. Bill and Ben are gardeners [A p.157]
8. If Mary is sailing or Jenny is kite flying, then Bill and Ben are grumpy. [A p.157]
9. Mary is neither sailing nor kite flying. [A p.157]
10. Only if Mary is sailing is Jenny kite flying. [A p.157]
11. John is sailing or kite flying but not both. [A p.157]
12. If Mary isn't sailing, then unless he's kite flying, John is sailing. [A p.157]

13. Jenny is sailing only if both Mary and John are. [A p.157]

14. Jenny is sailing if either John or Mary is. [A p.157]

15. If—and only if—Mary is sailing, Jenny is kite flying. [A p.157]

16. If Steve is winning, Mary isn't happy. [A p.157]

17. Two is prime, but it is also even. [A p.157]

18. Canberra is small—but it's not tiny, and it's a capital city. [A p.157]

19. If Rover is kite flying, then two isn't prime. [A p.157]

20. Mary is happy if and only if Jenny isn't. [A p.157]

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Exercises 8.3.2

Translate the following from English into MPL.

1. If Independence Hall is red, then something is red. [A p.158]
2. If everything is red, then Independence Hall is red. [A p.158]
3. Nothing is both green and red. [A p.158]
4. It is not true that nothing is both green and red. [A p.158]
5. Red things aren't green. [A p.158]
6. All red things are heavy or expensive. [A p.158]
7. All red things that are not heavy are expensive. [A p.158]
8. All red things are heavy, but some green things aren't. [A p.158]
9. All red things are heavy, but not all heavy things are red. [A p.158]
10. Some red things are heavy, and furthermore some green things are heavy too. [A p.158]
11. Some red things are not heavy, and some heavy things are not red. [A p.158]

12. If Kermit is green and red, then it is not true that nothing is both green and red. [A p.159]

13. Oscar's piano is heavy, but it is neither red nor expensive. [A p.159]

14. If Spondulix is heavy and expensive, and all expensive things are red and all heavy things are green, then Spondulix is red and green.[1] [A p.159]

15. If Kermit is heavy, then something is green and heavy. [A p.159]

16. If everything is fun, then nothing is worthwhile. [A p.159]

17. Some things are fun and some things are worthwhile, but nothing is both. [A p.159]

18. Nothing is probable unless something is certain. [A p.159]

19. Some things are probable and some aren't, but nothing is certain. [A p.159]

20. If something is certain, then it's probable. [A p.159]

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Exercises 8.3.5

Translate the following propositions from English into MPL.

1. Everyone is happy. [A p.159]
2. Someone is sad. [A p.159]
3. No one is both happy and sad. [A p.159]
4. If someone is sad, then not everyone is happy. [A p.159]
5. No one who isn't happy is laughing. [A p.160]
6. If Gary is laughing, then someone is happy. [A p.160]
7. Whoever is laughing is happy. [A p.160]

[1] "Spondulix" is the name of a famous gold nugget, found in 1872.

8. Everyone is laughing if Gary is. [A p.160]
9. Someone is sad, but not everyone and not Gary. [A p.160]
10. Gary isn't happy unless everyone is sad. [A p.160]
11. All leaves are brown and the sky is gray. [A p.160]
12. Some but not all leaves are brown. [A p.160]
13. Only leaves are brown. [A p.160]
14. Only brown leaves can stay. [A p.160]
15. Everyone is in trouble unless Gary is happy. [A p.160]
16. Everyone who works at this company is in trouble unless Gary is happy. [A p.160]
17. If Stephanie is telling the truth, then someone is lying. [A p.160]
18. If no one is lying, then Stephanie is telling the truth. [A p.160]
19. Either Stephanie is lying, or no-one's telling the truth and everyone is in trouble. [A p.160]
20. If Gary is lying, then not everyone in this room is telling the truth. [A p.160]

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Exercises 8.4.3.1

Write out a construction for each of the following wffs, and state the main operator.

1. $\forall x(Fx \rightarrow Gx)$ [A p.161]
2. $\forall x \neg Gx$ [A p.161]
3. $\neg \exists x(Fx \wedge Gx)$ [A p.161]
4. $(Fa \wedge \neg \exists x \neg Fx)$ [A p.161]
5. $\forall x(Fx \wedge \exists y(Gx \rightarrow Gy))$ [A p.162]

6. $(\forall x(Fx \to Gx) \land Fa)$ [A p.162]
7. $((\neg Fa \land \neg Fb) \to \forall x \neg Fx)$ [A p.162]
8. $\forall x \forall y((Fx \land Fy) \to Gx)$ [A p.163]
9. $\forall x(Fx \to \forall y Fy)$ [A p.163]
10. $(\forall x Fx \to \forall y Fy)$ [A p.163]

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Exercises 8.4.5.1

Identify any free variables in the following formulas. State whether each formula is open or closed.

1. $Tx \land Fx$ [A p.163]
2. $Tx \land Ty$ [A p.163]
3. $\exists x Tx \land \exists x Fx$ [A p.163]
4. $\exists x Tx \land \forall y Fx$ [A p.163]
5. $\exists x Tx \land Fx$ [A p.164]
6. $\exists x(Tx \land Fx)$ [A p.164]
7. $\forall y \exists x Ty$ [A p.164]
8. $\exists x(\forall x Tx \to \exists y Fx)$ [A p.164]
9. $\exists y \forall x Tx \to \exists y Fx$ [A p.164]
10. $\forall x(\exists x Tx \land Fx)$ [A p.164]
11. $\forall x \exists x Tx \land Fx$ [A p.164]
12. $\exists x Ty$ [A p.164]
13. $\forall x Tx \to \exists x Fx$ [A p.164]
14. $\exists x \forall y(Tx \lor Fy)$ [A p.164]
15. $\forall x Fx \land Gx$ [A p.164]

16. $\forall x \forall y Fx \rightarrow Gy$ [A p.164]

17. $\forall x \forall y (Fx \rightarrow \forall x Gy)$ [A p.164]

18. $\exists y Gb \wedge Gc$ [A p.164]

19. $\exists y Gy \wedge \forall x (Fx \rightarrow Gy)$ [A p.164]

20. $\forall x ((Fx \rightarrow \exists x Gx) \wedge Gx)$ [A p.164]

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Chapter 9

Semantics of Monadic Predicate Logic

Exercises 9.1.1

For each of the propositions:

$$\text{(i) } Pa \quad \text{(ii) } \exists x Px \quad \text{(iii) } \forall x Px$$

state whether it is true or false on each of the following models.

1. Domain: $\{1, 2, 3, \ldots\}$[2]
   Referent of a: 1
   Extension of P: $\{1, 3, 5, \ldots\}$[3] [A p.165]
2. Domain: $\{1, 2, 3, \ldots\}$
   Referent of a: 1
   Extension of P: $\{2, 4, 6, \ldots\}$[4] [A p.165]
3. Domain: $\{1, 2, 3, \ldots\}$
   Referent of a: 2
   Extension of P: $\{1, 3, 5, \ldots\}$ [A p.165]
4. Domain: $\{1, 2, 3, \ldots\}$
   Referent of a: 2
   Extension of P: $\{2, 4, 6, \ldots\}$ [A p.165]

[2] That is, the set of positive integers.
[3] That is, the set of odd numbers.
[4] That is, the set of even numbers.

5. Domain: $\{1, 2, 3, \ldots\}$
   Referent of a: 1
   Extension of P: $\{1, 2, 3, \ldots\}$ [A p.165]

6. Domain: $\{1, 2, 3, \ldots\}$
   Referent of a: 2
   Extension of P: $\emptyset$ [A p.165]

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Exercises 9.2.1

State whether each of the following propositions is true or false in each of the six models given in Exercises 9.1.1.

(i) $(\neg Pa \wedge \neg Pa)$

(ii) $(\neg Pa \rightarrow Pa)$

(iii) $(Pa \leftrightarrow \exists x Px)$

(iv) $(\exists x Px \vee \neg Pa)$

(v) $\neg(\forall x Px \wedge \neg \exists x Px)$ [Answers p.165]

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Exercises 9.3.1

1. If $\alpha(x)$ is $(Fx \wedge Ga)$, what is
   (i) $\alpha(a/x)$ [A p.166]
   (ii) $\alpha(b/x)$ [A p.166]

2. If $\alpha(x)$ is $\forall y(Fx \rightarrow Gy)$, what is
   (i) $\alpha(a/x)$ [A p.166]
   (ii) $\alpha(b/x)$ [A p.166]

3. If $\alpha(x)$ is $\forall x(Fx \rightarrow Gx) \wedge Fx$, what is
   (i) $\alpha(a/x)$ [A p.166]

(ii) $\alpha(b/x)$ [A p.166]

4. If $\alpha(x)$ is $\forall x(Fx \wedge Ga)$, what is

   (i) $\alpha(a/x)$ [A p.166]

   (ii) $\alpha(b/x)$ [A p.166]

5. If $\alpha(y)$ is $\exists x(Gx \rightarrow Gy)$, what is

   (i) $\alpha(a/y)$ [A p.166]

   (ii) $\alpha(b/y)$ [A p.166]

6. If $\alpha(x)$ is $\exists y(\forall x(Fx \rightarrow Fy) \vee Fx)$, what is

   (i) $\alpha(a/x)$ [A p.166]

   (ii) $\alpha(b/x)$ [A p.166]

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Exercises 9.4.3

1. Here is a model:
   Domain: $\{1, 2, 3, 4\}$
   Extensions: E: $\{2, 4\}$ O: $\{1, 3\}$
   State whether each of the following propositions is true or false in this model.

   (i) $\forall x Ex$ [A p.167]

   (ii) $\forall x(Ex \vee Ox)$ [A p.167]

   (iii) $\exists x Ex$ [A p.167]

   (iv) $\exists x(Ex \wedge Ox)$ [A p.167]

   (v) $\forall x(\neg Ex \rightarrow Ox)$ [A p.167]

   (vi) $\forall x Ex \vee \exists x \neg Ex$ [A p.167]

2. State whether the given proposition is true or false in the given models.

   (i) $\forall x(Px \vee Rx)$

      (a) Domain: $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
      Extensions: P: $\{1, 2, 3\}$ R: $\{5, 6, 7, 8, 9, 10\}$ [A p.167]

(b) Domain: $\{1,2,3,4,5,6,7,8,9,10\}$
Extensions: P: $\{1,2,3,4\}$ R: $\{4,5,6,7,8,9,10\}$ [A p.167]

(ii) $\exists x(\neg Px \leftrightarrow (Qx \wedge \neg Rx))$

(a) Domain: $\{1,2,3,\ldots\}$
Extensions: P: $\{2,4,6,\ldots\}$ Q: $\{1,3,5,\ldots\}$ R: $\{2,4,6,\ldots\}$
[A p.167]

(b) Domain: $\{1,2,3,\ldots\}$
Extensions: P: $\{2,4,6,\ldots\}$ Q: $\{2,4,6,\ldots\}$ R: $\{1,3,5,\ldots\}$
[A p.167]

(iii) $\exists x Px \wedge Ra$

(a) Domain: $\{1,2,3,\ldots\}$
Referent of a: 7
Extensions: P: $\{2,3,5,7,11,\ldots\}$[5] R: $\{1,3,5,\ldots\}$ [A p.167]

(b) Domain: {Alice, Ben, Carol, Dave}
Referent of a: Alice
Extensions: P: {Alice, Ben} R: {Carol, Dave} [A p.167]

3. Here is a model:

Domain: {Bill, Ben, Alison, Rachel}
Referents: a: Alison r: Rachel
Extensions: M: {Bill, Ben} F: {Alison, Rachel}
J: {Bill, Alison} S: {Ben, Rachel}

State whether each of the following propositions is true or false in this model.

(i) $(Ma \wedge Fr) \rightarrow \exists x(Mx \wedge Fx)$ [A p.167]
(ii) $\forall x \forall y(Mx \rightarrow My)$ [A p.167]
(iii) $(\neg Ma \vee \neg Jr) \rightarrow \exists x \exists y(Mx \wedge Fy)$ [A p.167]
(iv) $\forall x Mx \rightarrow \forall x Jx$ [A p.167]
(v) $\exists x \exists y(Mx \wedge Fy \wedge Sr)$ [A p.167]
(vi) $\exists x(Fx \wedge Sx) \rightarrow \forall x(Fx \rightarrow Sx)$ [A p.167]

[5] That is, the set of prime numbers.

4. For each of the following propositions, describe (a) a model in which it is true, and (b) a model in which it is false. If there is no model of one of these types, explain why.

   (i) $\forall x(Fx \to Gx)$ [A p.167]

   (ii) $\forall xFx \land \neg Fa$ [A p.167]

   (iii) $\exists xFx \land \neg Fa$ [A p.168]

   (iv) $\exists x(Fx \land Gx)$ [A p.168]

   (v) $\forall x(Fx \to Fx)$ [A p.168]

   (vi) $\exists xFx \land \exists xGx$ [A p.168]

   (vii) $\forall xFx \to \exists xFx$ [A p.168]

   (viii) $\exists x(Fx \land \neg Fx)$ [A p.168]

   (ix) $\exists xFx \land \exists x\neg Fx$ [A p.168]

   (x) $\exists x(Fx \to Fx)$ [A p.169]

   (xi) $\exists xFx \to \exists xGx$ [A p.169]

   (xii) $\exists xFx \to \forall xGx$ [A p.169]

   (xiii) $\forall xFx \to Fa$ [A p.169]

   (xiv) $\forall x(Fx \to Fa)$ [A p.169]

   (xv) $Fa \to Fb$ [A p.169]

   (xvi) $\forall x(Fx \lor Gx)$ [A p.170]

   (xvii) $\exists x(Fx \lor Gx)$ [A p.170]

   (xviii) $\forall x(Fx \land \neg Fx)$ [A p.170]

   (xix) $\forall x\exists y(Fx \to Gy)$ [A p.170]

   (xx) $\forall x(Fx \to \exists yGy)$ [A p.170]

5. (i) Is $\forall x(Fx \to Gx)$ true or false in a model in which the extension of F is the empty set? [A p.170]

   (ii) Is $\exists x(Fx \land Gx)$ true in every model in which $\forall x(Fx \to Gx)$ is true? [A p.170]

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Exercises 9.5.1

For each of the following arguments, either produce a countermodel (thereby showing that the argument is invalid) or explain why there cannot be a countermodel (in which case the argument is valid).

1. $\exists x Fx \wedge \exists x Gx$
   $\therefore\ \exists x(Fx \wedge Gx)$ [A p.171]

2. $\exists x(Fx \wedge Gx)$
   $\therefore\ \exists x Fx \wedge \exists x Gx$ [A p.171]

3. $\forall x(Fx \vee Gx)$
   $\neg\forall x Fx$
   $\therefore\ \forall x Gx$ [A p.171]

4. $\forall x(Fx \rightarrow Gx)$
   $\forall x(Gx \rightarrow Hx)$
   $\therefore\ \forall x(Fx \rightarrow Hx)$ [A p.171]

5. $\forall x(Fx \rightarrow Gx)$
   $\forall x(Gx \rightarrow Hx)$
   $\therefore\ \forall x(Hx \rightarrow Fx)$ [A p.171]

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Chapter 10

Trees for Monadic Predicate Logic

Exercises 10.2.2

1. Using trees, determine whether the following propositions are logical truths. For any proposition that is not a logical truth, read off from your tree a model in which it is false.

   (i) $Fa \rightarrow \exists x Fx$ [A p.172]

   (ii) $\exists x Fx \rightarrow \neg\forall x \neg Fx$ [A p.172]

   (iii) $\forall x((Fx \wedge \neg Gx) \rightarrow \exists x Gx)$ [A p.173]

   (iv) $\forall x Fx \rightarrow \exists x Fx$ [A p.173]

   (v) $(Fa \wedge (Fb \wedge Fc)) \rightarrow \forall x Fx$ [A p.173]

   (vi) $\exists x Fx \wedge \exists x \neg Fx$ [A p.174]

   (vii) $\exists x(Fx \rightarrow \forall y Fy)$ [A p.174]

   (viii) $\forall x(Fx \rightarrow Gx) \rightarrow (Fa \rightarrow Ga)$ [A p.174]

   (ix) $\neg\forall x(Fx \wedge Gx) \leftrightarrow \exists x \neg(Fx \wedge Gx)$ [A p.175]

   (x) $\neg\exists x(Fx \wedge Gx) \leftrightarrow \forall x(\neg Fx \wedge \neg Gx)$ [A p.175]

2. Using trees, determine whether the following arguments are valid. For any argument that is not valid, read off from your tree a model in which the premises are true and the conclusion false.

   (i) $\exists x Fx \wedge \exists x Gx$
   $\therefore\ \exists x(Fx \wedge Gx)$ [A p.176]

   (ii) $\exists x \forall y(Fx \rightarrow Gy)$
   $\therefore\ \forall y \exists x(Fx \rightarrow Gy)$ [A p.176]

(iii) $Fa \rightarrow \forall x Gx$
$\therefore\ \forall x(Fa \rightarrow Gx)$ [A p.177]

(iv) $Fa \rightarrow \forall x Gx$
$\therefore\ \exists x(Fa \rightarrow Gx)$ [A p.177]

(v) $\forall x(Fx \vee Gx)$
$\neg\forall x Fx$
$\therefore\ \forall x Gx$ [A p.177]

(vi) $\exists x(Fx \wedge Gx)$
$\therefore\ \exists x Fx \wedge \exists x Gx$ [A p.178]

(vii) $\forall x(Fx \rightarrow Gx)$
Fa
$\therefore\ Ga$ [A p.178]

(viii) $\neg\forall x(Fx \vee Gx)$
$\therefore\ \exists x(\neg Fx \wedge \neg Gx)$ [A p.178]

(ix) $\forall x(Fx \rightarrow Gx)$
$\forall x(Gx \rightarrow Hx)$
$\therefore\ \neg\exists x(\neg Fx \wedge Hx)$ [A p.179]

(x) $\forall x(Fx \vee Gx)$
$\therefore\ \neg\exists x(Fx \wedge Gx)$ [A p.179]

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Exercises 10.3.8

Translate the following arguments into MPL, and then test for validity using trees. For any argument that is not valid, read off from your tree a model in which the premise(s) are true and the conclusion false.

1. All dogs are mammals. All mammals are animals. Therefore, all dogs are animals. [A p.180]

2. If everything is frozen, then everything is cold. So everything frozen is cold. [A p.181]

3. If a thing is conscious, then either there is a divine being, or that thing has a sonic screwdriver. Nothing has a sonic screwdriver. Thus, not everything is conscious. [A p.182]

4. All cows are scientists, no scientist can fly, so no cow can fly. [A p.183]

5. Someone here is not smoking. Therefore, not everyone here is smoking. [A p.184]

6. If Superman rocks up, all cowards will shake. Catwoman is not a coward. So Catwoman will not shake. [A p.185]

7. Each car is either red or blue. All the red cars are defective, but some of the blue cars aren't. Thus, there are some defective cars and some nondefective cars. [A p.186]

8. For each thing, it swims only if there is a fish. Therefore, some things don't swim. [A p.187]

9. All robots built before 1970 run on kerosene. Autovac 23E was built before 1970, but it doesn't run on kerosene. So it's not a robot.

   [A p.188]

10. Everyone who is tall is either an athlete or an intellectual. Some people are athletes and intellectuals, but none of them is tall. Graham is a person. Therefore, if he's an athlete, then either he's not an intellectual, or he isn't tall. [A p.189]

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Chapter 11

Models, Propositions, and Ways the World Could Be

There are no exercises for chapter 11. [Contents]

Chapter 12

General Predicate Logic

Exercises 12.1.3.1

State whether each of the following is a wff of GPL.

1. $\forall x F^1 y$ [A p.191]
2. $\forall x \exists y F^1 y$ [A p.191]
3. $\forall x R^2 xy$ [A p.191]
4. $\forall x \exists x R^2 yy$ [A p.191]
5. $R^2 x$ [A p.191]
6. $\forall x R^2 x$ [A p.191]
7. $\forall x (F^1 x \rightarrow R^2 x)$ [A p.191]
8. $\forall x \exists y (F^1 x \rightarrow R^2 xy)$ [A p.191]
9. $\forall x \exists y (F^1 xy \rightarrow R^2 y)$ [A p.191]
10. $\forall x \exists y \forall x \exists y R^2 xy$ [A p.191]

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Exercises 12.1.6

Translate the following into GPL.

1. Bill heard Alice. [A p.192]
2. Bill did not hear Alice. [A p.192]
3. Bill heard Alice, but Alice did not hear Bill. [A p.192]
4. If Bill heard Alice, then Alice heard Bill. [A p.192]
5. Bill heard Alice if and only if Alice heard Alice. [A p.192]
6. Bill heard Alice, or Alice heard Bill. [A p.192]
7. Clare is taller than Dave, but she's not taller than Edward. [A p.192]
8. Mary prefers Alice to Clare. [A p.192]
9. Mary doesn't prefer Dave to Clare; nor does she prefer Clare to Dave. [A p.192]
10. Edward is taller than Clare, but he's not tall. [A p.192]
11. The Eiffel tower is taller than both Clare and Dave. [A p.192]
12. If Dave is taller than the Eiffel tower, then he's tall. [A p.192]
13. Although the Eiffel tower is taller, Clare prefers Dave. [A p.193]
14. If Alice is taller than Dave, then he prefers himself to her. [A p.193]
15. Dave prefers Edward to Clare only if Edward is taller than the Eiffel tower. [A p.193]
16. Dave prefers Edward to Clare only if she's not tall. [A p.193]
17. Mary has read *Fiesta*, and she likes it. [A p.193]
18. Dave doesn't like *Fiesta*, but he hasn't read it. [A p.193]
19. If Dave doesn't like *The Bell Jar*, then he hasn't read it. [A p.193]
20. Dave prefers *The Bell Jar* to *Fiesta*, even though he hasn't read either. [A p.193]

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Exercises 12.1.9

Translate the following into GPL.

1. (i) Something is bigger than everything. [A p.193]
   (ii) Something is such that everything is bigger than it. [A p.193]
   (iii) If Alice is bigger than Bill, then something is bigger than Bill. [A p.193]
   (iv) If everything is bigger than Bill, then Alice is bigger than Bill. [A p.193]
   (v) If something is bigger than everything, then something is bigger than itself. [A p.193]
   (vi) If Alice is bigger than Bill and Bill is bigger than Alice, then everything is bigger than itself. [A p.193]
   (vii) There is something that is bigger than anything that Alice is bigger than. [A p.193]
   (viii) Anything that is bigger than Alice is bigger than everything that Alice is bigger than. [A p.193]
   (ix) Every room contains at least one chair. [A p.193]
   (x) In some rooms some of the chairs are broken; in some rooms all of the chairs are broken; in no room is every chair unbroken. [A p.194]

2. (i) Every person owns a dog. [A p.194]
   (ii) For every dog, there is a person who owns that dog. [A p.194]
   (iii) There is a beagle that owns a chihuahua. [A p.194]
   (iv) No beagle owns itself. [A p.194]
   (v) No chihuahua is bigger than any beagle. [A p.194]
   (vi) Some chihuahuas are bigger than some beagles. [A p.194]
   (vii) Some dogs are happier than any person. [A p.194]
   (viii) People who own dogs are happier than those who don't. [A p.194]
   (ix) The bigger the dog, the happier it is. [A p.194]
   (x) There is a beagle that is bigger than every chihuahua and smaller than every person. [A p.194]

3. (i) Alice is a timid dog, and some cats are bigger than her. [A p.195]
   (ii) Every dog that is bigger than Alice is bigger than Bill. [A p.195]
   (iii) Bill is a timid cat, and every dog is bigger than him. [A p.195]
   (iv) Every timid dog growls at some gray cat. [A p.195]
   (v) Every dog growls at every timid cat. [A p.195]
   (vi) Some timid dog growls at every gray cat. [A p.195]
   (vii) No timid dog growls at any gray cat. [A p.195]
   (viii) Alice wants to buy something from Woolworths, but Bill doesn't. [A p.195]
   (ix) Alice wants to buy something from Woolworths that Bill doesn't. [A p.195]
   (x) Bill growls at anything that Alice wants to buy from Woolworths. [A p.195]

4. (i) Dave admires everyone. [A p.196]
   (ii) No one admires Dave. [A p.196]
   (iii) Dave doesn't admire himself. [A p.196]
   (iv) No one admires himself.[6] [A p.196]
   (v) Dave admires anyone who doesn't admire himself.[7] [A p.196]
   (vi) Every self-admiring person admires Dave. [A p.196]
   (vii) Frank admires Elvis but he prefers the Rolling Stones. [A p.196]
   (viii) Frank prefers any song recorded by the Rolling Stones to any song recorded by Elvis. [A p.196]
   (ix) The Rolling Stones recorded a top-twenty song, but Elvis didn't. [A p.196]
   (x) Elvis prefers any top-twenty song that the Rolling Stones recorded to any song that he himself recorded. [A p.196]

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[6] Read "himself" here as gender-neutral—that is, the claim is that no one self-admires.
[7] Read "himself" here as gender-neutral—that is, the claim is that Dave admires anyone who doesn't self-admire.

Exercises 12.2.2

1. Here is a model:

Domain:		$\{1, 2, 3, \ldots\}$
Referents:		a: 1 b: 2 c: 3
Extensions:	E:	$\{2, 4, 6, \ldots\}$
	P:	$\{2, 3, 5, 7, 11, \ldots\}$[8]
	L:	$\{\langle 1,2\rangle, \langle 1,3\rangle, \langle 1,4\rangle, \ldots, \langle 2,3\rangle, \langle 2,4\rangle, \ldots, \langle 3,4\rangle, \ldots\}$ [9]

State whether each of the following propositions is true or false in this model.

(i) Lba [A p.196]

(ii) $Lab \vee Lba$ [A p.196]

(iii) Laa [A p.196]

(iv) $\exists x Lxb$ [A p.196]

(v) $\exists x Lxa$ [A p.196]

(vi) $\exists x Lxx$ [A p.197]

(vii) $\forall x \exists y Lxy$ [A p.197]

(viii) $\forall x \exists y Lyx$ [A p.197]

(ix) $\exists x(Px \wedge Lxb)$ [A p.197]

(x) $\exists x(Px \wedge Lcx)$ [A p.197]

(xi) $\forall x \exists y(Ey \wedge Lxy)$ [A p.197]

(xii) $\forall x \exists y(Py \wedge Lxy)$ [A p.197]

(xiii) $\forall x(Lcx \rightarrow Ex)$ [A p.197]

(xiv) $\forall x((Lax \wedge Lxc) \rightarrow Ex)$ [A p.197]

(xv) $\forall x \forall y(Lxy \vee Lyx)$ [A p.197]

(xvi) $\exists x \exists y \exists z(Ex \wedge Py \wedge Ez \wedge Pz \wedge Lxy \wedge Lyz)$ [A p.197]

(xvii) $\exists x \exists y \exists z(Lxy \wedge Lyz \wedge Lzx)$ [A p.197]

(xviii) $\forall x \forall y \forall z((Lxy \wedge Lyz) \rightarrow Lxz)$ [A p.197]

[8] That is, the set of prime numbers.

[9] That is, the set of all pairs $\langle x, y\rangle$ such that x is less than y. A more compact way of writing this set is $\{\langle x, y\rangle : x < y\}$. See §16.1 for an explanation of this kind of notation for sets.

2. Here is a model:

Domain:		$\{1, 2, 3\}$
Referents:		a: 1 b: 2 c: 3
Extensions:	F:	$\{1, 2\}$
	G:	$\{2, 3\}$
	R:	$\{\langle 1, 2\rangle, \langle 2, 1\rangle, \langle 2, 3\rangle\}$
	S:	$\{\langle 1, 2, 3\rangle\}$

State whether each of the following propositions is true or false in this model.

(i) $\forall x \forall y (Rxy \rightarrow Ryx)$ [A p.197]

(ii) $\forall x \forall y (Ryx \rightarrow Rxy)$ [A p.197]

(iii) $\forall x \exists y (Gy \wedge Rxy)$ [A p.197]

(iv) $\forall x (Fx \rightarrow \exists y (Gy \wedge Rxy))$ [A p.197]

(v) $\exists x \exists y \exists z Sxyz$ [A p.197]

(vi) $\exists x \exists y Sxay$ [A p.197]

(vii) $\exists x \exists y Sxby$ [A p.197]

(viii) $\exists x Sxxx$ [A p.197]

(ix) $\exists x \exists y (Fx \wedge Fy \wedge Sxby)$ [A p.197]

(x) $\exists x \exists y (Fx \wedge Gy \wedge Sxby)$ [A p.197]

3. Here is a model:

Domain:		$\{$Alice, Bob, Carol, Dave, Edwina, Frank$\}$
Referents:		a:Alice b: Bob c: Carol d: Dave e: Edwina f: Frank
Extensions:	M:	$\{$Bob, Dave, Frank$\}$
	F:	$\{$Alice, Carol, Edwina$\}$
	L:	$\{\langle$Alice, Carol$\rangle$, $\langle$Alice, Dave$\rangle$, $\langle$Alice, Alice$\rangle$, $\langle$Dave, Carol$\rangle$, $\langle$Edwina, Dave$\rangle$, $\langle$Frank, Bob$\rangle\}$
	S:	$\{\langle$Alice, Bob$\rangle$, $\langle$Alice, Dave$\rangle$, $\langle$Bob, Alice$\rangle$, $\langle$Bob, Dave$\rangle$ $\langle$Dave, Bob$\rangle$, $\langle$Dave, Alice$\rangle\}$

State whether each of the following propositions is true or false in this model.

(i) $\forall x \forall y (Lxy \rightarrow Lyx)$ [A p.197]

(ii) $\exists x Lxx$ [A p.198]

(iii) $\neg \exists x Sxx$ [A p.198]

(iv) $\forall x \forall y(Sxy \to Syx)$ [A p.198]

(v) $\forall x \forall y \forall z((Sxy \wedge Syz) \to Sxz)$ [A p.198]

(vi) $\forall x(Mx \to \exists y Lyx)$ [A p.198]

(vii) $\forall x(Fx \to \exists y Lyx)$ [A p.198]

(viii) $\forall x(Fx \to \exists y Lxy)$ [A p.198]

(ix) $\exists x \exists y(Lax \wedge Lyb)$ [A p.198]

(x) $\forall x((Lxd \vee Ldx) \vee Mx)$ [A p.198]

4. For each of the following propositions, describe (a) a model in which it is true and (b) a model in which it is false. If there is no model of one of these types, explain why.

(i) $\forall x Fxx$ [A p.198]

(ii) $\forall x \forall y(Fxy \to Fyx)$ [A p.198]

(iii) $\forall x \forall y(Fxy \leftrightarrow Fyx)$ [A p.198]

(iv) $\exists x \forall y Fxy$ [A p.198]

(v) $\forall x \exists y Fxy$ [A p.198]

(vi) $\exists x \exists y Fxy$ [A p.198]

(vii) $\forall x \forall y Fxy$ [A p.199]

(viii) $\exists x \exists y Fxy \wedge \neg Faa$ [A p.199]

(ix) $\forall x \forall y Fxy \wedge \neg Faa$ [A p.199]

(x) $\forall x \forall y(Fxy \leftrightarrow Fyx) \wedge Fab \wedge \neg Fba$ [A p.199]

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Exercises 12.3.1

1. Using trees, determine whether the following propositions are logical truths. For any proposition that is not a logical truth, read off from your tree a model in which it is false.

(i) $\forall x(Rxx \to \exists y Rxy)$ [A p.200]

(ii) $\forall x(\exists y Rxy \to \exists z Rzx)$ [A p.200]

(iii) $\forall x Rax \to \forall x \exists y Ryx$ [A p.200]

(iv) $\forall x \exists y \exists z Ryxz \to \exists x \exists y Rxay$ [A p.201]

(v) $\neg\forall x\exists y Rxy$ [A p.201]

(vi) $\forall x\forall y\forall z((Rxy \wedge Ryz) \rightarrow Rxz)$ [A p.202]

(vii) $\exists x\forall y Rxy \rightarrow \forall x\exists y Rxy$ [A p.202]

(viii) $\exists y\forall x Rxy \rightarrow \forall x\exists y Rxy$ [A p.203]

(ix) $\exists x\forall y Rxy \rightarrow \exists x\exists y Rxy$ [A p.203]

(x) $\forall x\forall y\exists z Rxyz \vee \forall x\forall y\forall z\neg Rxyz$ [A p.204]

2. Using trees, determine whether the following arguments are valid. For any argument that is not valid, read off from your tree a model in which the premises are true and the conclusion false.

(i) $\forall x\forall y\forall z((Rxy \wedge Ryz) \rightarrow Rxz)$
Rab
Rba
$\therefore\ \exists x Rxx$ [A p.204]

(ii) $\forall x Fxa \rightarrow \exists x Fax$
$\exists x Fxa$
$\therefore\ \exists x Fax$ [A p.205]

(iii) $\exists x\exists y\exists z(Rxy \wedge Rzy)$
$\therefore\ \exists x Rxx$ [A p.205]

(iv) $\forall x\forall y(Rxy \rightarrow Ryx)$
$\exists x Rxa$
$\therefore\ \exists x Rax$ [A p.206]

(v) $\forall x\forall y(\neg Rxy \rightarrow Ryx)$
$\therefore\ \forall x\exists y Ryx$ [A p.206]

(vi) $\forall x\forall y(Rxy \rightarrow (Fx \wedge Gy))$
$\therefore\ \neg\exists x Rxx$ [A p.206]

(vii) $\forall x(Fx \rightarrow (\forall y Rxy \vee \neg\exists y Rxy))$
Fa
$\neg Rab$
$\therefore\ \neg Raa$ [A p.207]

(viii) $\forall x\forall y(\exists z(Rzx \wedge Rzy) \rightarrow Rxy)$
$\forall x Rax$
$\therefore\ \forall x\forall y Rxy$ [A p.207]

(ix) $\forall x\exists y Rxy$
$\therefore\ \exists x Rxb$ [A p.208]

(x) $\exists x \forall y (Fy \rightarrow Rxy)$
$\exists x \forall y \neg Ryx$
$\therefore\ \exists x \neg Fx$ [A p.208]

3. Translate the following arguments into GPL and then test for validity using trees. For any argument that is not valid, read off from your tree a model in which the premises are true and the conclusion false.

   (i) Alice is older than Bill, and Bill is older than Carol, so Alice must be older than Carol. [A p.209]

   (ii) Alice is older than Bill. Bill is older than Carol. Anything older than something is older than everything that that something is older than. It follows that Alice is older than Carol. [A p.209]

   (iii) I trust everything you trust. You trust all bankers. Dave is a banker. Thus, I trust Dave. [A p.210]

   (iv) Everybody loves somebody, so everybody is loved by somebody. [A p.211]

   (v) Nancy is a restaurateur. She can afford to feed all and only those restaurateurs who can't afford to feed themselves. So Nancy is very wealthy. [A p.212]

   (vi) Everything in Paris is more beautiful than anything in Canberra. The Eiffel tower is in Paris, and Lake Burley Griffin is in Canberra. Therefore, the Eiffel tower is more beautiful than Lake Burley Griffin. [A p.213]

   (vii) Politicians only talk to politicians. No journalist is a politician. So no politician talks to any journalist. [A p.214]

   (viii) There is no object that is smaller than all objects; therefore, there is no object such that every object is smaller than it. [A p.215]

   (ix) Either a movie isn't commercially successful or both Margaret and David like it. There aren't any French movies that Margaret and David both like. So there aren't any commercially successful French movies. [A p.216]

   (x) There's something that causes everything. Thus, there's nothing that is caused by everything. [A p.217]

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Exercises 12.4.1

For each of the following arguments, first translate into GPL and show that the argument is invalid using a tree. Then formulate suitable postulates and show, using a tree, that the argument with these postulates added as extra premises is valid.

1. Roger will eat any food; therefore, Roger will eat that egg. [A p.217]
2. Bill weighs 180 pounds. Ben weighs 170 pounds. So Bill is heavier than Ben. [A p.218]
3. John ran 5 miles; Nancy ran 10 miles; hence, Nancy ran farther than John. [A p.218]
4. Sophie enjoys every novel by Thomas Mann, so she enjoys *Buddenbrooks*. [A p.219]
5. Chris enjoys novels and nothing else; therefore, he does not enjoy anything by Borges. [A p.219]

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Exercises 12.5.4

For each of the following wffs, find an equivalent wff in prenex normal form.

1. $(\forall x Px \vee \forall x Qx)$ [A p.220]
2. $(\exists x Px \vee \exists x Qx)$ [A p.220]
3. $(\forall x Px \rightarrow \forall x Px)$ [A p.220]
4. $(\forall x Px \leftrightarrow \forall x Px)$ [A p.220]
5. $\neg \forall x(Sx \wedge (\exists y Ty \rightarrow \exists z Uxz))$ [A p.220]

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Chapter 13

Identity

Exercises 13.2.2

Translate the following into GPLI.

1. Chris is larger than everything (except himself). [A p.221]
2. All dogs are beagles—except Chris, who is a chihuahua. [A p.221]
3. Ben is happy if he has any dog other than Chris by his side. [A p.221]
4. Chris is happy if he is by anyone's side but Jonathan's. [A p.221]
5. Jonathan is larger than any dog. [A p.222]
6. Everything that Mary wants is owned by someone else. [A p.222]
7. Mary owns something that someone else wants. [A p.222]
8. Mary owns something she doesn't want. [A p.222]
9. If Mary owns a beagle, then no one else does. [A p.222]
10. No one other than Mary owns anything that Mary wants. [A p.222]
11. Everyone prefers *Seinfeld* to *Family Guy*. [A p.222]
12. *Seinfeld* is Adam's most preferred television show. [A p.222]
13. *Family Guy* is Adam's least preferred television show. [A p.222]

14. Jonathon watches *Family Guy*, but he doesn't watch any other television shows. [A p.222]

15. Jonathon is the only person who watches *Family Guy*. [A p.222]

16. Diane is the tallest woman. [A p.222]

17. Edward is the only man who is taller than Diane. [A p.222]

18. Diane isn't the only woman Edward is taller than. [A p.222]

19. No one whom Diane's taller than is taller than Edward. [A p.222]

20. Edward and Diane aren't the only people. [A p.222]

21. You're the only one who knows Ben. [A p.222]

22. I know people other than Ben. [A p.222]

23. Everyone Ben knows (not including Chris and me) is happy. [A p.222]

24. The only happy person I know is Ben. [A p.222]

25. Ben is the tallest happy person I know. [A p.222]

26. Jindabyne is the coldest town between Sydney and Melbourne. [A p.222]

27. There's a colder town than Canberra between Sydney and Melbourne. [A p.222]

28. For every town except Jindabyne, there is a colder town. [A p.222]

29. No town between Sydney and Melbourne is larger than Canberra or colder than Jindabyne. [A p.222]

30. Jindabyne is my most preferred town between Sydney and Melbourne. [A p.223]

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Exercises 13.3.1

1. Here is a model:

   Domain: $\{\text{Clark, Bruce, Peter}\}$
   Referents: a: Clark b: Clark e: Peter f: Peter
   Extensions: F: $\{\text{Bruce, Peter}\}$
   R: $\{\langle \text{Clark, Bruce} \rangle, \langle \text{Clark, Peter} \rangle, \langle \text{Bruce, Bruce} \rangle, \langle \text{Peter, Peter} \rangle\}$

   State whether each of the following propositions is true or false in this model.

   (i) $\forall x(\neg Fx \rightarrow x = a)$ [A p.223]
   (ii) $\forall x(x = a \rightarrow \forall y Rxy)$ [A p.223]
   (iii) $\exists x(x \neq f \wedge Ff \wedge Rxf)$ [A p.223]
   (iv) $\forall x(x \neq b \rightarrow Rax)$ [A p.223]
   (v) $\exists x(x \neq a \wedge \forall y(Fy \rightarrow Rxy))$ [A p.223]
   (vi) $\exists x(x \neq e \wedge Rxx)$ [A p.223]

2. Here is a model:

   Domain: $\{1, 2, 3, \ldots\}$
   Referents: a: 1 b: 1 c: 2 e: 4
   Extensions: F: $\{1, 2, 3\}$
   G: $\{1, 3, 5, \ldots\}$
   R: $\{\langle 1, 2 \rangle, \langle 2, 3 \rangle, \langle 3, 4 \rangle, \langle 4, 5 \rangle, \ldots\}$

   State whether each of the following propositions is true or false in this model.

   (i) $\exists x(Rax \wedge \neg Rbx)$ [A p.223]
   (ii) $\forall x((Fx \wedge \neg Gx) \rightarrow x = c)$ [A p.223]
   (iii) $\forall x(x \neq a \rightarrow \exists y Ryx)$ [A p.223]
   (iv) $\forall x(Gx \rightarrow \exists y \exists z(Rxy \wedge Ryz \wedge Gz))$ [A p.223]
   (v) $\forall x((x = a \vee x = b) \rightarrow x \neq c)$ [A p.223]
   (vi) $\exists x(\neg Fx \wedge x \neq e \wedge \exists y(Fy \wedge Ryx))$ [A p.223]

3. For each of the following propositions, describe (a) a model in which it is true and (b) a model in which it is false. If there is no model of one of these types, explain why.

(i) $\forall x(Fx \to x = a)$ [A p.223]

(ii) $\exists x(x = a \wedge x = b)$ [A p.223]

(iii) $\exists x \forall y(x \neq y \to Rxy)$ [A p.223]

(iv) $\forall x \forall y(Rxy \to x = y)$ [A p.224]

(v) $\forall x \forall y(x \neq y \to \exists z Rxyz)$ [A p.224]

(vi) $\exists x(x = a \wedge a \neq x)$ [A p.224]

(vii) $\forall x \forall y((Fx \wedge Fy) \to x = y)$ [A p.224]

(viii) $\exists x(Fx \wedge \forall y(Gy \to x = y))$ [A p.224]

(ix) $\forall x(Fx \to \exists y(x \neq y \wedge Rxy))$ [A p.224]

(x) $\forall x((Fx \wedge Rax) \to x \neq a)$ [A p.224]

(xi) $\exists x \exists y \exists z(x \neq y \wedge y \neq z \wedge x \neq z \wedge Rxyz)$ [A p.224]

(xii) $\forall x \forall y \forall z(Rxyz \to (x \neq y \wedge y \neq z \wedge x \neq z))$ [A p.224]

(xiii) $\forall x \forall y(x \neq y \to (Fx \vee Fy))$ [A p.225]

(xiv) $\exists x(Fx \wedge \forall y((Fy \wedge x \neq y) \to Rxy))$ [A p.225]

(xv) $\forall x \forall y \forall z(Rxyz \to (Rxxx \wedge Ryyy \wedge Rzzz))$ [A p.225]

(xvi) $\forall x(Rxx \to \forall y(x = y \to Rxy))$ [A p.225]

(xvii) $(Fa \wedge Fb) \wedge \forall x \forall y((Fx \wedge Fy) \to x = y)$ [A p.225]

(xviii) $\exists x \exists y(Fx \wedge Fy \wedge \forall z[Fz \to (x = z \vee y = z)])$ [A p.225]

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Exercises 13.4.3

1. Using trees, determine whether the following sets of propositions are satisfiable. For any set that is satisfiable, read off from your tree a model in which all propositions in the set are true.

(i) $\{Rab \to \neg Rba,\ Rab,\ a = b\}$ [A p.226]

(ii) $\{Rab,\ \neg Rbc,\ a = b\}$ [A p.226]

(iii) $\{\forall x(Fx \to x = a),\ Fa,\ a \neq b\}$ [A p.227]

(iv) $\{\forall x(Fx \to Gx),\ \exists x Fx,\ \neg Ga,\ a = b\}$ [A p.227]

(v) $\{\forall x(x \neq a \to Rax),\ \forall x \neg Rxb,\ a \neq b\}$ [A p.228]

(vi) $\{\exists x \forall y(Fy \to x = y),\ Fa,\ Fb\}$ [A p.228]

(vii) $\{\forall x \forall y(Rxy \to x = y),\ Rab,\ a \neq b\}$ [A p.229]

(viii) $\{\forall x((Fx \wedge Rxa) \to x \neq a),\ Fb \wedge Rba,\ a = b\}$ [A p.229]

(ix) $\{\exists x \exists y \exists z Rxyz,\ \forall x(x = x \to x = a)\}$ [A p.230]

(x) $\{\forall x \neg Rxx,\ \forall x \forall y x = y,\ \exists x Rax\}$ [A p.231]

2. Using trees, determine whether the following arguments are valid. For any argument that is not valid, read off from your tree a model in which the premises are true and the conclusion false.

(i) $\exists x Fx$
$\exists y Gy$
$\forall x \forall y x = y$
$\therefore\ \exists x(Fx \wedge Gx)$ [A p.231]

(ii) $\exists x \exists y(Fx \wedge Gy \wedge \forall z(z = x \vee z = y))$
$\therefore\ \exists x(Fx \wedge Gx)$ [A p.232]

(iii) Rab
$\therefore\ \forall x \forall y \forall z(((Rxy \wedge Ryz) \wedge x = z) \to Ryy)$ [A p.233]

(iv) $\forall x \forall y(Rxy \to Ryx)$
$\exists x(Rax \wedge x \neq b)$
$\therefore\ \exists x(Rxa \wedge x \neq b)$ [A p.233]

(v) $\forall x \forall y x = y$
$\therefore\ \forall x \forall y(Rxy \to Ryx)$ [A p.234]

(vi) $\forall x \forall y \forall z((Rxy \wedge Rxz) \to y = z)$
$Rab \wedge Rcd$
$b \neq d$
$\therefore\ a \neq c$ [A p.234]

(vii) $\exists x \exists y(Rxy \wedge x = y)$
$\therefore\ \neg \forall x Rxx$ [A p.235]

(viii) $\forall x(x = a \vee x = b)$
$\therefore\ \forall x x = a$ [A p.235]

(ix) $\forall x Rax$
$\neg \forall x \forall y x = y$
$\therefore\ \exists x \exists y \exists z(Rxy \wedge Rxz \wedge y \neq z)$ [A p.236]

(x) $\forall x x = a$
$\therefore\ \forall x x = b$ [A p.236]

3. Translate the following propositions into GPLI and then test whether they are logical truths using trees. For any proposition that is not a logical truth, read off from your tree a model in which it is false.

   (i) If Stan is the only firefighter, then nothing else is a firefighter. [A p.237]

   (ii) If Julius Caesar is left-handed but Lewis Carroll isn't, then Lewis Carroll isn't Julius Caesar. [A p.237]

   (iii) If the sun is warming all and only things other than itself, then the sun is warming Apollo. [A p.238]

   (iv) If Kevin Bacon isn't Kevin Bacon, then he's Michael J. Fox. [A p.238]

   (v) If no one who isn't Twain is a witty author, and Clemens is an author, then Clemens is not witty. [A p.239]

   (vi) No spy trusts any other spy. [A p.240]

   (vii) Either everything is identical to this ant, or nothing is. [A p.240]

   (viii) If Doug is afraid of everything but Santa Claus, then either he's afraid of himself, or else he's Santa Claus. [A p.241]

   (ix) If Mark respects Samuel and only Samuel, then Mark doesn't respect himself. [A p.242]

   (x) Either I am a physical body, or I am identical to something that's not a physical body. [A p.243]

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Exercises 13.5.1

1. Translate the following propositions into GPLI and then test whether they are logical truths using trees. For any proposition that is not a logical truth, read off from your tree a model in which it is false.

   (i) There are at most two gremlins. [A p.244]

   (ii) There are at least three Beatles. [A p.245]

   (iii) There is exactly one thing that is identical to Kevin Bacon. [A p.246]

   (iv) If there are at least two oceans, then there is an ocean. [A p.247]

(v) Take any two distinct dogs, the first of which is larger than the second; then the second is not larger than the first. [A p.247]

(vi) If there is exactly one apple, then there is at least one apple. [A p.248]

(vii) It's not the case both that there are at least two apples and that there is at most one apple. [A p.248]

(viii) Either there are no snakes, or there are at least two snakes. [A p.249]

2. Translate the following arguments into GPLI and then test for validity using trees. For any argument that is not valid, read off from your tree a model in which the premises are true and the conclusion false.

   (i) There are at least three things in the room. It follows that there are at least two things in the room. [A p.250]

   (ii) There are at least two bears in Canada, so there are at most two bears in Canada. [A p.251]

   (iii) There is at most one barber. So either every barber cuts his own hair, or no barber cuts any barber's hair. [A p.252]

   (iv) There are at most two things. If you pick a first thing and then pick a second thing (which may or may not be a different object from the first thing), then one of them is heavier than the other. So everything is either the heaviest or the lightest thing. [A p.253]

   (v) Some football players are athletes. Some golfers are athletes. Thus, there are at least two athletes. [A p.254]

   (vi) Everything is a part of itself. So everything has at least two parts. [A p.255]

   (vii) There are at least two things that are identical to the Eiffel tower. Therefore, there is no Eiffel tower. [A p.256]

   (viii) I'm afraid of Jemima and the chief of police. So either Jemima is the chief of police, or I'm afraid of at least two things. [A p.257]

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Exercises 13.6.1.1

Translate the following into GPLI, using Russell's approach to definite descriptions.

1. Joseph Conrad is the author of *The Shadow Line*. [A p.258]
2. The author of *The Shadow Line* authored *Lord Jim*. [A p.258]
3. The author of *The Shadow Line* is the author of *Lord Jim*. [A p.258]
4. Vance reads everything authored by the author of *Lord Jim*. [A p.258]
5. Joseph Conrad authored *The Inheritors*, but it's not the case that he is the author of *The Inheritors*. [A p.258]
6. The author of *The Shadow Line* is taller than any author of *Lord Jim*. [A p.258]
7. There is something taller than the author of *The Shadow Line*. [A p.258]
8. The author of *The Shadow Line* is taller than Joseph Conrad, who is taller than the author of *Lord Jim*. [A p.258]
9. The father of the author of *The Shadow Line* is taller than Joseph Conrad. [A p.258]
10. The father of the author of *The Shadow Line* is taller than the author of *The Shadow Line*. [A p.258]

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Exercises 13.6.2.1

Translate the claims in Exercises 13.6.1.1 into GPLID, using the definite description operator to translate definite descriptions. [A p.259]

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Exercises 13.6.3.1

Translate the claims in Exercises 13.6.1.1 into GPLI, treating definite descriptions as names and stating appropriate uniqueness assumptions as postulates. [A p.260]

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Exercises 13.7.4

1. Translate the following into GPLIF.

   (i) $2 + 2 = 4$ [A p.261]

   (ii) $2 \times 2 = 4$ [A p.261]

   (iii) $2 + 2 = 2 \times 2$ [A p.261]

   (iv) $2^2 = 2 \times 2$ [A p.261]

   (v) $(x + y)^2 = (x + y)(x + y)$ [A p.261]

   (vi) $(x + y)^2 = x^2 + 2xy + y^2$ [A p.262]

   (vii) Whether x is even or odd, $2x$ is even. [A p.262]

   (viii) Tripling an odd number results in an odd number; tripling an even number results in an even number. [A p.262]

   (ix) $5x < 6x$ [A p.262]

   (x) If $x < y$, then $3x < 4y$ [A p.262]

2. Here is a model:

Domain:		{Alison, Bruce, Calvin, Delilah}
Referents:		a: Alison b: Bruce c: Calvin d: Delilah
Extensions:	F:	{Alison, Delilah}
	M:	{Bruce, Calvin}
	S:	{⟨Alison, Bruce⟩, ⟨Alison, Calvin⟩, ⟨Alison,Delilah⟩,⟨Bruce, Calvin⟩, ⟨Bruce, Delilah⟩,⟨Calvin, Delilah⟩}
Values of function symbols:	f:	{⟨Alison, Bruce⟩,⟨Bruce, Calvin⟩, ⟨Calvin, Bruce⟩, ⟨Delilah, Calvin⟩}
	m:	{⟨Alison, Delilah⟩, ⟨Bruce, Alison⟩, {⟨Calvin, Delilah⟩, ⟨Delilah, Alison⟩}
	s :	{⟨Alison, Alison, Bruce⟩, ⟨Alison, Bruce, Calvin⟩, ⟨Alison, Calvin, Delilah⟩, ⟨Alison, Delilah, Alison⟩, ⟨Bruce, Alison, Calvin⟩, ⟨Bruce, Bruce, Calvin⟩, ⟨Bruce, Calvin, Delilah⟩, ⟨Bruce, Delilah, Alison⟩, ⟨Calvin, Alison, Delilah⟩, ⟨Calvin, Bruce,Delilah⟩, ⟨Calvin, Calvin, Delilah⟩, ⟨Calvin, Delilah, Alison⟩, ⟨Delilah, Alison, Alison⟩, ⟨Delilah, Bruce, Alison⟩, ⟨Delilah, Calvin, Alison⟩, ⟨Delilah, Delilah, Alison⟩}

State whether each of the following propositions is true or false in this model.

(i) $\forall x Mf(x)$ [A p.262]

(ii) $\exists x Mm(x)$ [A p.262]

(iii) $s(c,b) = d$ [A p.262]

(iv) $s(a,a) = f(c)$ [A p.262]

(v) $Ff(b) \to Mf(b)$ [A p.262]

(vi) $\forall x \forall y \exists z \forall w (s(x,y) = w \leftrightarrow w = z)$ [A p.262]

(vii) $\exists x \exists y \exists z \exists w (s(x,y) = z \wedge s(x,y) = w \wedge z \neq w)$ [A p.262]

(viii) $s(s(b,a),s(d,a)) = s(b,c)$ [A p.262]

(ix) $\exists x \exists y s(x,y) = m(y)$ [A p.262]

(x) $\forall x \exists y s(y,x) = x$ [A p.262]

3. Here is a model:

Domain:		$\{1,2,3,\ldots\}$
Referents:		a_1: 1 a_2: 2 a_3: 3 $\ldots$
Extensions:	E:	$\{2,4,6,\ldots\}$
	O:	$\{1,2,3,\ldots\}$
	L:	$\{\langle x,y\rangle : x < y\}$ [10]
Values of function symbols:	q:	$\{\langle x,y\rangle : y = x^2\}$ [11]
	s	$\{\langle x,y,z\rangle : z = x+y\}$ [12]
	p	$\{\langle x,y,z\rangle : z = x \times y\}$ [13]

State whether each of the following propositions is true or false in this model.

(i) $s(a_2,a_2) = a_5$ [A p.262]

(ii) $p(a_2,a_2) = a_3$ [A p.262]

(iii) $s(a_2,a_2) = p(a_2,a_2)$ [A p.262]

(iv) $q(a_2) = p(a_1,a_2)$ [A p.262]

(v) $\forall x \forall y q(s(x,y)) = p(s(x,y),s(x,y))$ [A p.262]

(vi) $\forall x \forall y q(s(x,y)) = s(s(q(x),p(a_2,p(x,y))),q(y))$ [A p.262]

(vii) $\forall x E p(a_2,x)$ [A p.262]

(viii) $\forall x((Ox \rightarrow Op(a_3,x)) \wedge (Ex \rightarrow Ep(a_3,x)))$ [A p.262]

(ix) $\exists x L p(a_5,x) p(a_5,x)$ [A p.262]

(x) $\forall x \forall y (Lyx \rightarrow Lp(a_3,x)p(a_4,y))$ [A p.262]

[10] That is, $\{\langle 1,2\rangle, \langle 1,3\rangle, \langle 2,3\rangle, \langle 1,4\rangle, \langle 2,4\rangle, \langle 3,4\rangle, \langle 1,5\rangle, \langle 2,5\rangle, \langle 3,5\rangle, \langle 4,5\rangle, \ldots\}$.

[11] That is, $\{\langle 1,1\rangle, \langle 2,4\rangle, \langle 3,9\rangle, \langle 4,16\rangle, \ldots\}$.

[12] That is, $\{\langle 1,1,2\rangle, \langle 2,1,3\rangle, \langle 2,2,4\rangle, \langle 1,2,3\rangle, \langle 3,1,4\rangle, \langle 3,2,5\rangle, \langle 3,3,6\rangle, \langle 2,3,5\rangle, \langle 1,3,4\rangle, \langle 4,1,5\rangle, \ldots\}$.

[13] That is, $\{\langle 1,1,1\rangle, \langle 2,1,2\rangle, \langle 2,2,4\rangle, \langle 1,2,2\rangle, \langle 3,1,3\rangle, \langle 3,2,6\rangle, \langle 3,3,9\rangle, \langle 2,3,6\rangle, \langle 1,3,3\rangle, \langle 4,1,4\rangle, \ldots\}$.

4. For each of the following propositions, describe (a) a model in which it is true and (b) a model in which it is false. If there is no model of one of these types, explain why.

   (i) $f(a) = f(b)$ [A p.263]

   (ii) $f(a) \neq f(b)$ [A p.263]

   (iii) $f(a) \neq f(a)$ [A p.263]

   (iv) $\forall x \exists y f(x) = y$ [A p.263]

   (v) $\exists x \forall y f(x) = y$ [A p.263]

   (vi) $\forall x \forall y s(x, y) = s(y, x)$ [A p.263]

   (vii) $\forall x \forall y f(s(x, y)) = s(f(x), f(y))$ [A p.263]

   (viii) $\exists x \exists y s(x, y) = f(x) \to \exists x \exists y s(x, y) = f(y)$ [A p.264]

   (ix) $\exists x \exists y s(x, y) = f(x) \to \exists x \exists y f(s(x, y)) = f(x)$ [A p.264]

   (x) $\forall x \forall y \exists z \forall w (s(x, y) = w \leftrightarrow w = z)$ [A p.264]

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Chapter 14

Metatheory

Exercises 14.1.1.1

What is the complexity of each of the following wffs?

1. Fa [A p.265]
2. $(Hx \rightarrow \forall x(Fx \rightarrow Gx))$ [A p.265]
3. $\forall xx = x$ [A p.265]
4. $\forall x \exists y \neg Rxy$ [A p.265]
5. $\neg \forall xa \neq x$ [A p.265]
6. $\forall x(Fx \rightarrow \exists yRxy)$ [A p.265]
7. $(\forall xa = x \wedge \neg \exists xa \neq x)$ [A p.265]
8. $(Fa \wedge (Fa \wedge (Fa \wedge (Fa \wedge (Fa \wedge Fa)))))$ [A p.265]
9. $\forall x(Fx \rightarrow \forall x(Fx \rightarrow \forall x(Fx \rightarrow \forall x(Fx \rightarrow \forall x(Fx \rightarrow Fx)))))$ [A p.265]
10. $(((\neg \exists x(\neg Fx \vee Gx) \wedge a \neq b) \rightarrow \neg Fa) \vee (\neg \exists x(\neg Fx \vee Gx) \vee \neg Fa))$ [A p.265]

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Exercises 14.1.2.1

In §10.1 we showed that the tree rules for (negated and unnegated) disjunction and the quantifiers are truth-preserving (in the precise sense spelled out in §14.1.2), and in §13.4 we showed that the tree rule SI is truth-preserving. Complete the soundness proof by showing that the remaining tree rules are truth-preserving:

1. Unnegated conjunction. [A p.265]
2. Negated conjunction. [A p.266]
3. Unnegated conditional. [A p.266]
4. Negated conditional. [A p.266]
5. Unnegated biconditional. [A p.266]
6. Negated biconditional. [A p.267]
7. Negated negation. [A p.267]

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Exercises 14.1.3.1

Fill in the remaining cases in step (III) of the completeness proof.

1. γ is of the form $\neg\alpha$, and α's main operator is conjunction. [A p.267]
2. γ is of the form $\neg\alpha$, and α's main operator is the conditional. [A p.267]
3. γ is of the form $\neg\alpha$, and α's main operator is the biconditional. [A p.268]
4. γ is of the form $\neg\alpha$, and α's main operator is the existential quantifier. [A p.268]
5. γ's main operator is the biconditional. [A p.268]

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Chapter 15

Other Methods of Proof

Exercises 15.1.5

1. Show the following in A_1 by producing formal proofs.

   (i) $\neg P \to Q, \neg P \vdash Q$ [A p.269]

   (ii) $P \vdash \neg Q \to P$ [A p.269]

   (iii) $\neg Q \vdash (\neg P \to Q) \to P$ [A p.269]

   (iv) $\vdash P \to P$ [A p.269]

   (v) $\neg(P \to \neg Q) \vdash Q$ [A p.270]

   (vi) $P, \neg P \vdash Q$ [A p.270]

   (vii) $P \wedge Q \vdash (P \to \neg Q) \to \neg(P \to \neg Q)$ [A p.270]

2. Show the following in A_1 by producing formal or informal proofs.

   (i) $\vdash \neg(P \to \neg Q) \to Q$ [A p.270]

   (ii) $\vdash P \to (P \vee Q)$ [A p.270]

   (iii) $\vdash ((P \to Q) \to (P \to R)) \to (P \to (Q \to R))$ [A p.271]

   (iv) $\vdash (P \to Q) \to (\neg Q \to \neg P)$ [A p.271]

   (v) $P \to Q,\ P \to \neg Q \vdash \neg P$ [A p.271]

   (vi) $P \to Q, \neg Q \to P \vdash Q$ [A p.272]

   (vii) $\vdash (P \to (Q \to R)) \to (Q \to (P \to R))$ [A p.272]

3. Show the following in A_2 by producing formal or informal proofs.

   (i) $\vdash P \to \neg\neg P$ [A p.272]

(ii) $P \to \neg P \vdash \neg P$ [A p.273]

(iii) $P \to Q \vdash \neg Q \to \neg P$ [A p.273]

(iv) $\vdash \neg Q \to (Q \to P)$ [A p.273]

(v) $P \land Q \vdash P \to Q$ [A p.273]

(vi) $\neg Q \vdash (P \lor Q) \to P$ [A p.274]

(vii) $\neg P \land \neg Q \vdash \neg(P \lor Q)$ [A p.274]

(viii) $\neg(P \lor Q) \vdash \neg P \land \neg Q$ [A p.275]

(ix) $\vdash \neg(P \land \neg P)$ [A p.275]

(x) $\vdash (P \land \neg P) \to Q$ [A p.276]

(xi) $\vdash P \leftrightarrow P$ [A p.276]

(xii) $\vdash P \to (\neg P \to Q)$ [A p.277]

4. Show the following in $A_1^{\forall=}$ by producing formal or informal proofs.

(i) $\forall x(Fx \to Gx), Fa \vdash Ga$ [A p.277]

(ii) $\forall xFx \vdash \forall x(Gx \lor Fx)$ [A p.277]

(iii) $\forall x\forall y(Rxy \to Ryx), Rab \vdash Rba$ [A p.277]

(iv) $\exists xFx \to \neg Ga \vdash Ga \to \forall x\neg Fx$ [A p.278]

(v) $\vdash Fa \to \exists xFx$ [A p.279]

(vi) $Fa, a = b \vdash Fb$ [A p.280]

(vii) $\forall x\forall yx = y \vdash a = b$ [A p.280]

(viii) $a = b, a = c \vdash c = b$ [A p.280]

(ix) $\vdash a = b \to b = a$ [A p.281]

(x) $Fa, \neg Fb \vdash \neg a = b$ [A p.281]

(xi) $\neg b = a, \forall x(\neg Fx \to x = a) \vdash Fb$ [A p.283]

(xii) $\vdash \forall xFx \to \forall yFy$ [A p.283]

5. Explain why the original unrestricted deduction theorem does not hold in $A_1^{\forall=}$ and why the restricted version stated at the end of §15.1.1.1 does hold. [A p.283]

[Contents]

Exercises 15.2.3

1. Show the following in N_1.

   (i) $\vdash (\neg P \rightarrow P) \rightarrow P$ [A p.284]
   (ii) $A \rightarrow C,\ B \rightarrow C,\ A \vee B \vdash C$ [A p.284]
   (iii) $\vdash \neg\neg P \rightarrow P$ [A p.284]
   (iv) $\neg(A \vee B) \vdash \neg A \wedge \neg B$ [A p.285]
   (v) $A, \neg A \vdash B$ [A p.285]
   (vi) $A \rightarrow B,\ B \rightarrow C \vdash A \rightarrow C$ [A p.285]
   (vii) $P \rightarrow Q \vdash \neg Q \rightarrow \neg P$ [A p.286]
   (viii) $A \vee B, \neg A \vdash B$ [A p.286]
   (ix) $P \rightarrow R,\ Q \rightarrow R,\ P \vee Q \vdash R$ [A p.287]
   (x) $P \rightarrow Q \vdash \neg(P \wedge \neg Q)$ [A p.287]

2. Establish each of the following in each of the systems N_2 through N_5.

   (i) $\vdash A \vee \neg A$ [A p.288]
   (ii) $A \wedge \neg A \vdash B$ [A p.289]
   (iii) $\vdash \neg\neg A \rightarrow A$ [A p.289]
   (iv) $\vdash \neg(A \wedge \neg A)$ [A p.291]

3. Show the following in $N_1^{\forall\exists=}$.

   (i) $\vdash \forall x(Fx \rightarrow Fx)$ [A p.291]
   (ii) $\exists x(Fx \wedge Gx) \vdash \exists x Fx \wedge \exists x Gx$ [A p.291]
   (iii) $\forall x(Fx \rightarrow Gx), \neg\exists x Gx \vdash \neg\exists x Fx$ [A p.292]
   (iv) $\forall x(Fx \rightarrow x = a) \vdash Fb \rightarrow a = b$ [A p.292]
   (v) $\forall x \forall y x = y, Raa \vdash \forall x \forall y Rxy$ [A p.293]
   (vi) $\vdash \forall x Rxx \rightarrow \forall x \exists y Rxy$ [A p.293]
   (vii) $\vdash \exists x Fx \rightarrow \neg\forall x \neg Fx$ [A p.294]
   (viii) $\neg\exists x Fx \vdash \forall x \neg Fx$ [A p.294]
   (ix) $\forall x x = a \vdash b = c$ [A p.294]
   (x) $\vdash \forall x \forall y((Fx \wedge \neg Fy) \rightarrow \neg x = y)$ [A p.295]

4. (i) Reformulate the rules of system N_1 in list style. Re-present your answers to Question 1 above as proofs in the list style.

   [A p.295]

   (ii) Reformulate the rules of system N_1 in stack style. Re-present your answers to Question 1 above as proofs in the stack style.

   [A p.299]

5. State natural deduction rules (i.e., introduction and elimination rules) for $\leftrightarrow$. [A p.302]

[Contents]

Exercises 15.3.3

1. Define the following notions in terms of sequents.

   (i) The proposition α is:

      (a) a contradiction [A p.303]

      (b) satisfiable [A p.303]

   (ii) Propositions α and β are:

      (a) jointly satisfiable [A p.303]

      (b) equivalent [A p.303]

2. Redo some of Exercise 7.3.1.1 and Exercise 7.3.2.1 using the sequent calculus S_1 instead of trees. [A p.303]

3. Redo some of Exercise 10.2.2, Exercise 12.3.1 and Exercise 13.4.3 using the sequent calculus $S_1^{\forall\exists=}$ instead of trees. [A p.303]

4. State sequent rules (i.e., left and right introduction rules) for $\leftrightarrow$.

   [A p.303]

5. State a (new) tree rule that is the analogue of Cut. [A p.303]

[Contents]

Chapter 16

Set Theory

There are no exercises for chapter 16. [Contents]

Answers

Chapter 1

Propositions and Arguments

Answers 1.2.1

1. Proposition [Q p.2]
2. Non-proposition (Exhortation) [Q p.2]
3. Non-proposition (Exclamation) [Q p.2]
4. Non-proposition (Wish) [Q p.2]
5. Proposition (Not a wish: the speaker is making a statement about what she wishes.) [Q p.2]
6. Proposition [Q p.2]
7. Proposition [Q p.2]
8. Proposition [Q p.2]
9. Non-proposition (Wish) [Q p.2]
10. Non-proposition (Command) [Q p.2]

[Contents]

Answers 1.3.1

1. If the stock market crashes, thousands of experienced investors will lose a lot of money.

   The stock market won't crash. [Q p.3]

2. Diamond is harder than topaz.
Topaz is harder than quartz.
Quartz is harder than calcite.
Calcite is harder than talc.

Diamond is harder than talc. [Q p.3]

3. Any friend of yours is a friend of mine.
You're friends with everyone on the volleyball team.

If Sally's on the volleyball team, she's a friend of mine. [Q p.3]

4. When a politician engages in shady business dealings, it ends up on page one of the newspapers
No South Australian senator has ever appeared on page one of a newspaper.

No South Australian senator engages in shady business dealings. [Q p.3]

[Contents]

Answers 1.4.1

1. Valid. [Q p.3]
2. Invalid. [Q p.3]
3. Valid. [Q p.3]
4. Valid. [Q p.4]

[Contents]

Answers 1.5.1

1. Arguments 1 and 3. [Q p.4]
2. Argument 2. [Q p.4]
3. Argument 4. [Q p.4]

[Contents]

Answers 1.6.1.1

1. (i) Bob is a good student [Q p.4]
   (ii) I have decided not to go to the party. [Q p.4]
   (iii) Mars is the closest planet to the sun. [Q p.4]
   (iv) Alice is late. [Q p.4]
   (v) I like scrambled eggs. [Q p.4]
   (vi) Scrambled eggs are good for you. [Q p.4]
2. True. [Q p.4]
3. False. [Q p.4]

[Contents]

Answers 1.6.2.1

1. The sun is shining. I am happy. [Q p.5]
2. Maisie is my friend. Rosie is my friend. [Q p.5]
3. Sailing is fun. Snowboarding is fun. [Q p.5]
4. We watched the movie. We ate popcorn. [Q p.5]
5. Sue does not want the red bicycle. Sue does not like the blue bicycle. [Q p.5]
6. The road to the campsite is long. The road to the campsite is uneven. [Q p.5]

[Contents]

Answers 1.6.4.1

1. (a) That's pistachio ice cream.
   (b) That doesn't taste the way it should. [Q p.5]
2. (a) That tastes the way it should.
   (b) That isn't pistachio ice cream. [Q p.5]

3. (a) That is supposed to taste that way.
   (b) That isn't pistachio ice cream. [Q p.5]
4. (a) You pressed the red button.
   (b) Your cup contains coffee. [Q p.5]
5. (a) You pressed the green button.
   (b) Your cup does not contain coffee. [Q p.5]
6. (a) Your cup contains hot chocolate.
   (b) You pressed the green button. [Q p.5]

[Contents]

Answers 1.6.6

1. This is a conditional with antecedent 'It will be sunny and windy tomorrow' and consequent 'We shall go sailing or kite flying tomorrow'. The antecedent is a conjunction with conjuncts 'It will be sunny tomorrow' and 'It will be windy tomorrow'. The consequent is a disjunction with disjuncts 'We shall go sailing tomorrow' and 'We shall go kite flying tomorrow'. [Q p.6]

2. This is a conditional with antecedent 'It will rain or snow tomorrow' and consequent 'We shall not go sailing or kite flying tomorrow'. The antecedent is a disjunction with disjuncts 'It will rain tomorrow' and 'It will snow tomorrow'. The consequent is a negation with negand 'We shall go sailing or kite flying tomorrow'. The negand, as mentioned in answer to the previous question, is a disjunction with disjuncts 'We shall go sailing tomorrow' and 'We shall go kite flying tomorrow'. [Q p.6]

3. This is a disjunction with disjuncts 'He'll stay here and we'll come back and collect him later' and 'He'll come with us and we'll all come back together'. The first of these disjuncts is a conjunction with conjuncts 'He'll stay here' and 'We'll come back and collect him later'; the second of the disjuncts is also a conjunction, with conjuncts 'He'll come with us' and 'We'll all come back together'. [Q p.6]

4. This is a conjunction with conjuncts 'Jane is a talented painter and a wonderful sculptor' and 'If she remains interested in art, her work

will one day be of the highest quality.' The first of these conjuncts is itself a conjunction, with conjuncts 'Jane is a talented painter' and 'Jane is a wonderful sculptor'; the second conjunct is a conditional, with antecedent 'Jane remains interested in art' and consequent 'Jane's work will one day be of the highest quality'. [Q p.6]

5. This is a negation with negand 'The unemployment rate will both increase and decrease in the next quarter'. The negand is a conjunction with conjuncts 'The unemployment rate will increase in the next quarter' and 'The unemployment rate will decrease in the next quarter'. [Q p.6]

6. This is a conditional with antecedent 'You don't stop swimming during the daytime' and consequent 'Your sunburn will get worse and become painful'. The antecedent is a negation with negand 'You stop swimming during the daytime'; the consequent is a conjunction with conjuncts 'Your sunburn will get worse' and 'Your sunburn will become painful'. [Q p.6]

7. This is a disjunction with disjuncts 'Steven won't get the job' and 'I'll leave and all my clients will leave'. The first of these disjuncts is a negation with negand 'Steven will get the job'; the second disjunct is a conjunction with conjuncts 'I'll leave' and 'All my clients will leave'. [Q p.6]

8. This is a biconditional with components 'The Tigers will not lose' and 'Both Thompson and Thomson will get injured'. The first is a negation with negand 'The Tigers will lose'; the second is a conjunction with conjuncts 'Thompson will get injured' and 'Thomson will get injured'. [Q p.6]

9. This is a conjunction with conjuncts 'Fido will wag his tail if you give him dinner at 6 this evening' and 'Fido will bark if you do not give him dinner at 6 this evening'. The first of these conjuncts is a conditional with antecedent 'You will give Fido dinner at 6 this evening' and consequent 'Fido will wag his tail [at 6 this evening]'; the second conjunct is a conditional with antecedent 'You do not give Fido dinner at 6 this evening' and consequent 'Fido will bark [at 6 this evening]'. Finally, the antecedent of this last conditional is a negation with negand 'You give Fido dinner at 6 this evening'.

   [Q p.6]

10. This is a disjunction with disjuncts 'It will rain or snow today' and 'It will not rain or snow today'. The first of these disjuncts is itself a disjunction, with disjuncts 'It will rain today' and 'It will snow today'. The second of these disjuncts is a negation with negand 'It will rain or snow today'. The latter, as already mentioned, is a disjunction, with disjuncts 'It will rain today' and 'It will snow today'. [Q p.6]

[Contents]

Chapter 2

The Language of Propositional Logic

Answers 2.3.3

1. Aristotle was not a philosopher. [Q p.7]
2. Aristotle was a philosopher and paper burns. [Q p.7]
3. Aristotle was a philosopher and paper doesn't burn. [Q p.7]
4. Fire is not hot and paper does not burn. [Q p.7]
5. It's not true *both* that fire is hot *and* that paper burns. [Q p.7]

[Contents]

Answers 2.3.5

1. Either Aristotle was a philosopher and paper burns, or fire is hot. [Q p.8]
2. Either Aristotle wasn't a philosopher, or paper doesn't burn. [Q p.8]
3. Aristotle was a philosopher or paper burns—but not both. [Q p.8]
4. It's not the case either that Aristotle was a philosopher or that fire is hot. [Q p.8]

5. Aristotle was a philosopher, and either paper burns or fire is hot.

[Q p.8]

[Contents]

Answers 2.3.8

1. (i) If snow is white, then the sky is blue. [Q p.8]

   (ii) Snow is white if and only if both snow is white and roses are not red. [Q p.8]

   (iii) It's not the case that if roses are red then snow is not white. [Q p.8]

   (iv) If roses are red or snow is white, then roses are red and snow is not white. [Q p.8]

   (v) Either snow is white and snow is white, or roses are red and the sky is not blue. [Q p.8]

   (vi) Either grass is green, or if snow is white then roses are red. [Q p.8]

   (vii) Bananas are yellow if and only if they're yellow; and they're not if and only if they're not. [Q p.8]

   (viii) If, if the sky is blue then snow is white, then if snow isn't white then the sky isn't blue. [Q p.8]

   (ix) If roses are red, snow is white and the sky is blue, then either bananas are yellow or grass is green. [Q p.8]

   (x) It's not the case both that roses aren't red and that either snow isn't white or grass is green. [Q p.9]

2. Glossary:

B:	The sky is blue
E:	Snow is red
J:	Jim is tall
M:	Maisy is tall
N:	Nora is tall
R:	Roses are red
W:	Snow is white

(i) $(W \rightarrow B)$ [Q p.9]

(ii) $(B \leftrightarrow (W \wedge \neg R))$ [Q p.9]

(iii) $\neg(R \rightarrow \neg W)$ [Q p.9]

(iv) $((E \wedge R) \rightarrow (R \vee \neg E))$ [Q p.9]

(v) $((J \leftrightarrow M) \wedge (M \rightarrow \neg N))$ [Q p.9]

(vi) $(J \rightarrow (N \vee M))$ [Q p.9]

(vii) $(J \rightarrow (M \vee \neg N))$ [Q p.9]

(viii) $((W \wedge M) \vee (W \wedge \neg M))$ [Q p.9]

(ix) $((J \wedge \neg J) \rightarrow (B \wedge \neg B))$ [Q p.9]

(x) $((M \wedge B) \rightarrow (J \wedge \neg B))$ [Q p.9]

3. Glossary:

G: We are skiing
K: We are kite flying
L: We are sailing
S: It is snowing
U: It is sunny
W: It is windy

(i) $(S \rightarrow \neg K)$ [Q p.9]
(ii) $((U \wedge W) \rightarrow (L \vee K))$ [Q p.9]
(iii) $((K \rightarrow W) \wedge (L \rightarrow W))$ [Q p.9]
(iv) $((L \vee K) \vee G)$ [Q p.9]
(v) $(W \leftrightarrow L)$ [Q p.9]
(vi) $(G \rightarrow (W \vee S))$ [Q p.9]
(vii) $(G \rightarrow (W \wedge S))$ [Q p.9]
(viii) $(U \rightarrow (W \rightarrow K))$ [Q p.9]
(ix) $(L \rightarrow ((U \wedge W) \wedge \neg S))$ [Q p.9]
(x) $(((U \wedge W) \rightarrow L) \wedge ((S \wedge \neg W) \rightarrow G))$ [Q p.10]

[Contents]

Answers 2.5.1

1. (i) No [Q p.10]
(ii) No [Q p.10]
(iii) Yes [Q p.10]
(iv) No [Q p.10]
(v) No [Q p.10]
(vi) No [Q p.10]
(vii) No [Q p.10]
(viii) No [Q p.10]
(ix) No [Q p.10]
(x) Yes [Q p.10]

2. (i) (a) 1 is an odd number.
   (b) If x is an odd number then so is $x + 2$.
   (c) Nothing else is an odd number.

   Note: We are assuming here that 'number' means 'positive integer'. If it is taken to mean 'integer' (i.e. positive, negative or zero) then the answer is:

   (a) 1 is an odd number.
   (b) If x is an odd number then so are $x + 2$ and $x - 2$.
   (c) Nothing else is an odd number. [Q p.10]

(ii) (a) 5 is divisible by five.
   (b) If x is divisible by five then so is $x + 5$.
   (c) Nothing else is divisible by five.

   Note: We are assuming here that 'number' means 'positive integer'. If it is taken to mean 'integer' (i.e. positive, negative or zero) then the answer is:

   (a) 5 is divisible by five.
   (b) If x is divisible by five then so are $x + 5$ and $x - 5$.
   (c) Nothing else is divisible by five. [Q p.10]

(iii) (a) *a* is such a word; *b* is such a word.
   (b) If x is such a word then so are xa and xb.
   (c) Nothing else is such a word. [Q p.10]

(iv) (a) Bob's mother is in the set; Bob's father is in the set.
   (b) If x is in the set then so are x's mother and x's father.
   (c) Nothing else is in the set. [Q p.10]

(v) (a) *hah hah hah* is a cackle.
   (b) If x is a cackle then so is x *hah*.
   (c) Nothing else is a cackle. [Q p.10]

[Contents]

Answers 2.5.3.1

1.

1. P	/ (2i)
2. Q	/ (2i)
3. R	/ (2i)
4. $\neg P$	1 / (2ii$\neg$)
5. $(Q \wedge R)$	2, 3 / (2ii$\wedge$)
6. $(\neg P \vee (Q \wedge R))$	4, 5 / (2ii$\vee$)

Main connective is $\vee$. [Q p.11]

2.

1. P	/ (2i)
2. Q	/ (2i)
3. R	/ (2i)
4. $(Q \vee R)$	2, 3 / (2ii$\vee$)
5. $(P \wedge (Q \vee R))$	1, 4 / (2ii$\wedge$)
6. $\neg(P \wedge (Q \vee R))$	5 / (2ii$\neg$)

Main connective is $\neg$. [Q p.11]

3.

1. P	/ (2i)
2. Q	/ (2i)
3. R	/ (2i)
4. $\neg P$	1 / (2ii$\neg$)
5. $\neg Q$	2 / (2ii$\neg$)
6. $\neg R$	3 / (2ii$\neg$)
7. $(\neg P \wedge \neg Q)$	4, 5 / (2ii$\wedge$)
8. $((\neg P \wedge \neg Q) \vee \neg R)$	6, 7 / (2ii$\vee$)

Main connective is $\vee$. [Q p.11]

4.

1. P	/ (2i)
2. Q	/ (2i)
3. R	/ (2i)
4. S	/ (2i)
5. $(P \rightarrow Q)$	1, 2 / (2ii$\rightarrow$)
6. $(R \rightarrow S)$	3, 4 / (2ii$\rightarrow$)
7. $((P \rightarrow Q) \vee (R \rightarrow S))$	5, 6 / (2ii$\vee$)

Main connective is $\vee$. [Q p.11]

5.

1. P	/ (2i)
2. Q	/ (2i)
3. R	/ (2i)
4. S	/ (2i)
5. $(P \leftrightarrow Q)$	1, 2 / (2ii$\leftrightarrow$)
6. $((P \leftrightarrow Q) \leftrightarrow R)$	3, 5 / (2ii$\leftrightarrow$)
7. $(((P \leftrightarrow Q) \leftrightarrow R) \leftrightarrow S)$	4, 6 / (2ii$\leftrightarrow$)

Main connective is $\leftrightarrow$. [Q p.11]

6.

1. P	/ (2i)
2. $\neg P$	1 / (2ii$\neg$)
3. $\neg\neg P$	2 / (2ii$\neg$)
4. $(\neg P \wedge \neg\neg P)$	2, 3 / (2ii$\wedge$)
5. $(P \wedge \neg P)$	1, 2 / (2ii$\wedge$)
6. $((\neg P \wedge \neg\neg P) \rightarrow (P \wedge \neg P))$	4, 5 / (2ii$\rightarrow$)

Main connective is $\rightarrow$. [Q p.11]

[Contents]

Answers 2.5.4.1

1. [Q p.11]

ordering	disambiguation
2	2
3	3
4	4
5	2
6	5

[Contents]

Answers 2.5.5.1

1. (i) $(\neg P \vee (Q \wedge R))$ [Q p.11]
   (ii) $\neg((P \vee Q) \wedge R)$ [Q p.11]
   (iii) $(\neg(P \vee Q) \wedge R)$ [Q p.11]
   (iv) $((\neg P \wedge \neg Q) \vee \neg R)$ [Q p.11]
   (v) $([(P \leftrightarrow Q) \leftrightarrow R] \leftrightarrow S)$ [Q p.11]

2. (i) $\neg \wedge P \vee QR$ [Q p.11]

(ii) $\rightarrow \rightarrow P \vee QRS$ [Q p.12]

(iii) $\vee \rightarrow PQ \rightarrow RS$ [Q p.12]

(iv) $\rightarrow P \rightarrow \vee QRS$ [Q p.12]

(v) $\rightarrow \wedge \neg P \neg \neg P \wedge P \neg P$ [Q p.12]

[Contents]

Chapter 3

Semantics of Propositional Logic

Answers 3.2.1

1. [Q p.13]

	$(\neg$	P	$\wedge$	$(Q$	$\vee$	$R))$
phase 0:		T		T		F
phase 1:	F				T	
phase 2:			F			

2. [Q p.13]

	$\neg$	$(P$	$\vee$	$(Q$	$\rightarrow$	$R))$
phase 0:		T		T		F
phase 1:					F	
phase 2:			T			
phase 3:	F					

3. [Q p.13]

	$(\neg$	$\neg$	P	$\wedge$	$(Q$	$\rightarrow$	$(R$	$\vee$	$P)))$
phase 0:			F		T		T		F
phase 1:		T						T	
phase 2:	F					T			
phase 3:				F					

4. [Q p.13]

	$(\neg$	$\neg$	P	$\wedge$	$(Q$	$\rightarrow$	$(R$	$\vee$	$P)))$
phase 0:			T		F		F		T
phase 1:		F						T	
phase 2:	T					T			
phase 3:				T					

5. [Q p.13]

	$((P$	$\vee$	$Q)$	$\rightarrow$	$(P$	$\vee$	$P))$
phase 0:	F		T		F		F
phase 1:		T				F	
phase 2:				F			

6. [Q p.13]

	$((P$	$\vee$	$Q)$	$\rightarrow$	$(P$	$\vee$	$P))$
phase 0:	T		F		T		T
phase 1:		T				T	
phase 2:				T			

7. [Q p.13]

	$(P$	$\rightarrow$	$(Q$	$\rightarrow$	$(R$	$\rightarrow$	$S)))$
phase 0:	T		T		T		F
phase 1:						F	
phase 2:				F			
phase 3:		F					

8. [Q p.13]

	$(P$	$\rightarrow$	$(Q$	$\rightarrow$	$(R$	$\rightarrow$	$S)))$
phase 0:	F		T		F		T
phase 1:						T	
phase 2:				T			
phase 3:		T					

9. [Q p.13]

	$\neg$	$(((\neg$	P	$\leftrightarrow$	$P)$	$\leftrightarrow$	$Q)$	$\rightarrow$	$R)$
phase 0:			F		F		F		F
phase 1:		T							
phase 2:				F					
phase 3:						T			
phase 4:								F	
phase 5:	T								

10. [Q p.14]

	$\neg$	$(((\neg$	P	$\leftrightarrow$	$P)$	$\leftrightarrow$	$Q)$	$\rightarrow$	$R)$
phase 0:			T		T		T		T
phase 1:		F							
phase 2:				F					
phase 3:						F			
phase 4:								T	
phase 5:	F								

[Contents]

Answers 3.3.1

1. [Q p.14]

P	Q	$((P$	$\wedge$	$Q)$	$\vee$	$P)$
T	T		$\cancel{T}$		T	
T	F		$\cancel{F}$		T	
F	T		$\cancel{F}$		F	
F	F		$\cancel{F}$		F	

2.

P	$(P$	$\wedge$	$(P$	$\vee$	$P))$
T		T		$\cancel{T}$	
F		F		$\cancel{F}$	

[Q p.14]

3.

P	Q	$\neg$	$(\neg P$	$\wedge$	$\neg Q)$
T	T	T	$\cancel{F}$	$\cancel{F}$	$\cancel{F}$
T	F	T	$\cancel{F}$	$\cancel{F}$	$\cancel{T}$
F	T	T	$\cancel{T}$	$\cancel{F}$	$\cancel{F}$
F	F	F	$\cancel{T}$	$\cancel{T}$	$\cancel{T}$

[Q p.14]

4.

Q	$(Q$	$\rightarrow$	$(Q$	$\wedge$	$\neg Q))$
T		F		$\cancel{F}$	$\cancel{F}$
F		T		$\cancel{F}$	$\cancel{T}$

[Q p.14]

5.

P	Q	R	$(P$	$\rightarrow$	$(Q$	$\rightarrow$	$R))$
T	T	T		T		$\cancel{T}$	
T	T	F		F		$\cancel{F}$	
T	F	T		T		$\cancel{T}$	
T	F	F		T		$\cancel{T}$	
F	T	T		T		$\cancel{T}$	
F	T	F		T		$\cancel{F}$	
F	F	T		T		$\cancel{T}$	
F	F	F		T		$\cancel{T}$	

[Q p.14]

6.

P	Q	((P	$\vee$	$Q)$	$\leftrightarrow$	$(P$	$\wedge$	$Q))$
T	T		$\cancel{T}$		T		$\cancel{T}$	
T	F		$\cancel{T}$		F		$\cancel{F}$	
F	T		$\cancel{T}$		F		$\cancel{F}$	
F	F		$\cancel{F}$		T		$\cancel{F}$	

[Q p.14]

7.

P	Q	$\neg$	$((P$	$\wedge$	$Q)$	$\leftrightarrow$	$Q)$
T	T	F		$\cancel{T}$		$\cancel{T}$	
T	F	F		$\cancel{F}$		$\cancel{T}$	
F	T	T		$\cancel{F}$		$\cancel{F}$	
F	F	F		$\cancel{F}$		$\cancel{T}$	

[Q p.14]

8.

P	$(((P$	$\rightarrow$	$\neg P)$	$\rightarrow$	$\neg P)$	$\rightarrow$	$\neg P)$
T		$\cancel{F}$	$\cancel{F}$	$\cancel{T}$	$\cancel{F}$	F	$\cancel{F}$
F		$\cancel{T}$	$\cancel{T}$	$\cancel{T}$	$\cancel{T}$	T	$\cancel{T}$

[Q p.14]

9.

P	Q	R	$\neg(P$	$\wedge$	$(Q \wedge R))$
T	T	T	F	$\cancel{T}$	$\cancel{T}$
T	T	F	T	$\cancel{F}$	$\cancel{F}$
T	F	T	T	$\cancel{F}$	$\cancel{F}$
T	F	F	T	$\cancel{F}$	$\cancel{F}$
F	T	T	T	$\cancel{F}$	$\cancel{T}$
F	T	F	T	$\cancel{F}$	$\cancel{F}$
F	F	T	T	$\cancel{F}$	$\cancel{F}$
F	F	F	T	$\cancel{F}$	$\cancel{F}$

[Q p.14]

10.

R	S	T	$((\neg R$	$\vee$	$S)$	$\wedge$	$(S$	$\vee$	$\neg T))$
T	T	T	$\cancel{F}$	$\cancel{T}$		T		$\cancel{T}$	$\cancel{F}$
T	T	F	$\cancel{F}$	$\cancel{T}$		T		$\cancel{T}$	$\cancel{T}$
T	F	T	$\cancel{F}$	$\cancel{F}$		F		$\cancel{F}$	$\cancel{F}$
T	F	F	$\cancel{F}$	$\cancel{F}$		F		$\cancel{T}$	$\cancel{T}$
F	T	T	$\cancel{T}$	$\cancel{T}$		T		$\cancel{T}$	$\cancel{F}$
F	T	F	$\cancel{T}$	$\cancel{T}$		T		$\cancel{T}$	$\cancel{T}$
F	F	T	$\cancel{T}$	$\cancel{T}$		F		$\cancel{F}$	$\cancel{F}$
F	F	F	$\cancel{T}$	$\cancel{T}$		T		$\cancel{T}$	$\cancel{T}$

[Q p.14]

[Contents]

Answers 3.4.1

1.

P	Q	$(P \to Q)$	$(Q \to P)$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

[Q p.14]

2.

P	Q	$\neg$	$(P \leftrightarrow Q)$	$((P \vee Q)$	$\wedge$	$\neg$	$(P \wedge Q))$
T	T	F	$\cancel{T}$	$\cancel{T}$	F	$\cancel{F}$	$\cancel{T}$
T	F	T	$\cancel{F}$	$\cancel{T}$	T	$\cancel{T}$	$\cancel{F}$
F	T	T	$\cancel{F}$	$\cancel{T}$	T	$\cancel{T}$	$\cancel{F}$
F	F	F	$\cancel{T}$	$\cancel{F}$	F	$\cancel{T}$	$\cancel{F}$

[Q p.14]

3.

P	Q	$\neg(P$	$\wedge$	$\neg Q)$	$\neg Q$
T	T	T	$\cancel{F}$	$\cancel{F}$	F
T	F	F	$\cancel{T}$	$\cancel{T}$	T
F	T	T	$\cancel{F}$	$\cancel{F}$	F
F	F	T	$\cancel{F}$	$\cancel{T}$	T

[Q p.14]

4.

P	Q	R	$((P$	$\rightarrow$	$Q)$	$\wedge$	$R))$	$(P$	$\vee$	$(Q$	$\vee$	$R))$
T	T	T		$\cancel{T}$		T			T		$\cancel{T}$	
T	T	F		$\cancel{T}$		F			T		$\cancel{T}$	
T	F	T		$\cancel{F}$		F			T		$\cancel{T}$	
T	F	F		$\cancel{F}$		F			T		$\cancel{F}$	
F	T	T		$\cancel{T}$		T			T		$\cancel{T}$	
F	T	F		$\cancel{T}$		F			T		$\cancel{T}$	
F	F	T		$\cancel{T}$		T			T		$\cancel{T}$	
F	F	F		$\cancel{T}$		F			F		$\cancel{F}$	

[Q p.14]

5.

P	Q	R	S	$((P \wedge Q)$	$\wedge$	$(\neg R$	$\wedge$	$\neg S))$	$((P$	$\vee$	$(R$	$\rightarrow$	$Q)) \wedge S)$
T	T	T	T	$\cancel{T}$	F	$\cancel{F}$	$\cancel{F}$	$\cancel{F}$		$\cancel{T}$		$\cancel{T}$	T
T	T	T	F	$\cancel{T}$	F	$\cancel{F}$	$\cancel{F}$	$\cancel{T}$		$\cancel{T}$		$\cancel{T}$	F
T	T	F	T	$\cancel{T}$	F	$\cancel{T}$	$\cancel{F}$	$\cancel{F}$		$\cancel{T}$		$\cancel{T}$	T
T	T	F	F	$\cancel{T}$	T	$\cancel{T}$	$\cancel{T}$	$\cancel{T}$		$\cancel{T}$		$\cancel{T}$	F
T	F	T	T	$\cancel{F}$	F	$\cancel{F}$	$\cancel{F}$	$\cancel{F}$		$\cancel{T}$		$\cancel{F}$	T
T	F	T	F	$\cancel{F}$	F	$\cancel{F}$	$\cancel{F}$	$\cancel{T}$		$\cancel{T}$		$\cancel{F}$	F
T	F	F	T	$\cancel{F}$	F	$\cancel{T}$	$\cancel{F}$	$\cancel{F}$		$\cancel{T}$		$\cancel{T}$	T
T	F	F	F	$\cancel{F}$	F	$\cancel{T}$	$\cancel{T}$	$\cancel{T}$		$\cancel{T}$		$\cancel{T}$	F
F	T	T	T	$\cancel{F}$	F	$\cancel{F}$	$\cancel{F}$	$\cancel{F}$		$\cancel{T}$		$\cancel{T}$	T
F	T	T	F	$\cancel{F}$	F	$\cancel{F}$	$\cancel{F}$	$\cancel{T}$		$\cancel{T}$		$\cancel{T}$	F
F	T	F	T	$\cancel{F}$	F	$\cancel{T}$	$\cancel{F}$	$\cancel{F}$		$\cancel{T}$		$\cancel{T}$	T
F	T	F	F	$\cancel{F}$	F	$\cancel{T}$	$\cancel{T}$	$\cancel{T}$		$\cancel{T}$		$\cancel{T}$	F
F	F	T	T	$\cancel{F}$	F	$\cancel{F}$	$\cancel{F}$	$\cancel{F}$		$\cancel{F}$		$\cancel{F}$	F
F	F	T	F	$\cancel{F}$	F	$\cancel{F}$	$\cancel{F}$	$\cancel{T}$		$\cancel{F}$		$\cancel{F}$	F
F	F	F	T	$\cancel{F}$	F	$\cancel{T}$	$\cancel{F}$	$\cancel{F}$		$\cancel{T}$		$\cancel{T}$	T
F	F	F	F	$\cancel{F}$	F	$\cancel{T}$	$\cancel{T}$	$\cancel{T}$		$\cancel{T}$		$\cancel{T}$	F

[Q p.14]

[Q p.15]

6.

P	Q	$(P$	$\wedge$	$\neg P)$	$(Q$	$\wedge$	$\neg Q)$
T	T		F	$\cancel{F}$		F	$\cancel{F}$
T	F		F	$\cancel{F}$		F	$\cancel{T}$
F	T		F	$\cancel{T}$		F	$\cancel{F}$
F	F		F	$\cancel{T}$		F	$\cancel{T}$

[Q p.15]

7.

P	Q	R	$(P$	$\vee$	$(Q \leftrightarrow R))$	$((Q \rightarrow P)$	$\wedge$	$Q)$
T	T	T		T	$\cancel{T}$	$\cancel{T}$	T	
T	T	F		T	$\cancel{F}$	$\cancel{T}$	T	
T	F	T		T	$\cancel{F}$	$\cancel{T}$	F	
T	F	F		T	$\cancel{T}$	$\cancel{T}$	F	
F	T	T		T	$\cancel{T}$	$\cancel{F}$	F	
F	T	F		F	$\cancel{F}$	$\cancel{F}$	F	
F	F	T		F	$\cancel{F}$	$\cancel{T}$	F	
F	F	F		T	$\cancel{T}$	$\cancel{T}$	F	

8.

P	Q	R	$\neg$	$((P \wedge Q)$	$\wedge$	$R)$	$((P \to Q)$	$\leftrightarrow$	$(P \to R))$
T	T	T	F	$\cancel{T}$	$\cancel{T}$		$\cancel{T}$	T	$\cancel{T}$
T	T	F	T	$\cancel{T}$	$\cancel{F}$		$\cancel{T}$	F	$\cancel{F}$
T	F	T	T	$\cancel{F}$	$\cancel{F}$		$\cancel{F}$	F	$\cancel{T}$
T	F	F	T	$\cancel{F}$	$\cancel{F}$		$\cancel{F}$	T	$\cancel{F}$
F	T	T	T	$\cancel{F}$	$\cancel{F}$		$\cancel{T}$	T	$\cancel{T}$
F	T	F	T	$\cancel{F}$	$\cancel{F}$		$\cancel{T}$	T	$\cancel{T}$
F	F	T	T	$\cancel{F}$	$\cancel{F}$		$\cancel{T}$	T	$\cancel{T}$
F	F	F	T	$\cancel{F}$	$\cancel{F}$		$\cancel{T}$	T	$\cancel{T}$

[Q p.15]

9.

P	Q	$(P \vee Q)$	$\neg P$	$(Q \vee Q)$
T	T	T	F	T
T	F	T	F	F
F	T	T	T	T
F	F	F	T	F

[Q p.15]

10.

P	Q	R	S	$(P$	$\to$	$(Q$	$\to$	$(R \to S)))$	$\neg S$
T	T	T	T		T		$\cancel{T}$	$\cancel{T}$	F
T	T	T	F		F		$\cancel{F}$	$\cancel{F}$	T
T	T	F	T		T		$\cancel{T}$	$\cancel{T}$	F
T	T	F	F		T		$\cancel{T}$	$\cancel{T}$	T
T	F	T	T		T		$\cancel{T}$	$\cancel{T}$	F
T	F	T	F		T		$\cancel{T}$	$\cancel{F}$	T
T	F	F	T		T		$\cancel{T}$	$\cancel{T}$	F
T	F	F	F		T		$\cancel{T}$	$\cancel{T}$	T
F	T	T	T		T		$\cancel{T}$	$\cancel{T}$	F
F	T	T	F		T		$\cancel{F}$	$\cancel{F}$	T
F	T	F	T		T		$\cancel{T}$	$\cancel{T}$	F
F	T	F	F		T		$\cancel{T}$	$\cancel{T}$	T
F	F	T	T		T		$\cancel{T}$	$\cancel{T}$	F
F	F	T	F		T		$\cancel{T}$	$\cancel{F}$	T
F	F	F	T		T		$\cancel{T}$	$\cancel{T}$	F
F	F	F	F		T		$\cancel{T}$	$\cancel{T}$	T

[Q p.15]

[Contents]

Answers 3.5.1

1. No. None of our connectives has a truth table which matches the outputs of this truth function in all cases. [Q p.15]

2. [Q p.15]

input	output of function f_4^2	output of function f_5^2
(T,T)	T	T
(T,F)	F	F
(F,T)	F	T
(F,F)	F	T

3. T [Q p.15]

4. (iv) You do not need to know any truth values. Whether $(A \rightarrow B)$ is T or F, $\star(A \rightarrow B)$ is T, because $\star$ sends both possible inputs (T and F) to T. [Q p.15]

5. $\rightarrow$. The outputs of g are as follows:

input	output of g
(T,T)	T
(T,F)	F
(F,T)	T
(F,F)	T

These outputs match the truth table for $\rightarrow$ in every case.

NB To get the output of g where the input is (x, y), we first take x as input to f_2^1, and then take the output of *this*, and y—in that order—as the inputs to f_3^2. The output of *that* is the output of g for input (x, y):

x	y	$f_2^1(x)$	$f_3^2(f_2^1(x), y)$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

The rightmost column of this table gives us the outputs of g for all possible values of x and y, since $g(x, y) = f_3^2(f_2^1(x), y)$. [Q p.15]

[Contents]

Chapter 4

Uses of Truth Tables

Answers 4.1.2

1. Valid [Q p.16]

A	B	C	$A \lor B$	$A \to C$	$(B \to C)$	$\to C$
T	T	T	T	T	$\cancel{T}$	T
T	T	F	T	F	$\cancel{F}$	T
T	F	T	T	T	$\cancel{T}$	T
T	F	F	T	F	$\cancel{T}$	F
F	T	T	T	T	$\cancel{T}$	T
F	T	F	T	T	$\cancel{F}$	T
F	F	T	F	T	$\cancel{T}$	T
F	F	F	F	T	$\cancel{T}$	F

2. Invalid. Counterexample: A false, B false, C false (row 8). [Q p.16]

A	B	C	$\neg A$	$\neg$	$((A \to B)$	$\land$	$(B \to C))$	$\lor\ C$
T	T	T	F	$\cancel{F}$	$\cancel{T}$	$\cancel{T}$	$\cancel{T}$	T
T	T	F	F	$\cancel{T}$	$\cancel{T}$	$\cancel{F}$	$\cancel{F}$	T
T	F	T	F	$\cancel{T}$	$\cancel{F}$	$\cancel{F}$	$\cancel{T}$	T
T	F	F	F	$\cancel{T}$	$\cancel{F}$	$\cancel{F}$	$\cancel{T}$	T
F	T	T	T	$\cancel{F}$	$\cancel{T}$	$\cancel{T}$	$\cancel{T}$	T
F	T	F	T	$\cancel{T}$	$\cancel{T}$	$\cancel{F}$	$\cancel{F}$	T
F	F	T	T	$\cancel{F}$	$\cancel{T}$	$\cancel{T}$	$\cancel{T}$	T
$*$ F	F	F	T	$\cancel{F}$	$\cancel{T}$	$\cancel{T}$	$\cancel{T}$	F

3. Invalid. Counterexample: A true, B true, C false (row 2). [Q p.16]

	A	B	C	$(A$	$\wedge$	$\neg B)$	$\rightarrow$	C	$\neg C$	$\neg A$
	T	T	T		$\cancel{F}$	$\cancel{F}$	T		F	F
$*$	T	T	F		$\cancel{F}$	$\cancel{F}$	T		T	F
	T	F	T		$\cancel{T}$	$\cancel{T}$	T		F	F
	T	F	F		$\cancel{T}$	$\cancel{T}$	F		T	F
	F	T	T		$\cancel{F}$	$\cancel{F}$	T		F	T
	F	T	F		$\cancel{F}$	$\cancel{F}$	T		T	T
	F	F	T		$\cancel{F}$	$\cancel{T}$	T		F	T
	F	F	F		$\cancel{F}$	$\cancel{T}$	T		T	T

4. Valid. [Q p.16]

A	B	C	$(A$	$\wedge$	$B)$	$\leftrightarrow$	C	C	$\rightarrow$	B
T	T	T		$\cancel{T}$		T			T	
T	T	F		$\cancel{T}$		F			T	
T	F	T		$\cancel{F}$		F			F	
T	F	F		$\cancel{F}$		T			T	
F	T	T		$\cancel{F}$		F			T	
F	T	F		$\cancel{F}$		T			T	
F	F	T		$\cancel{F}$		F			F	
F	F	F		$\cancel{F}$		T			T	

5. Valid. [Q p.16]

A	B	C	$(\neg A$	$\wedge$	$\neg B)$	$\leftrightarrow$	$\neg C$	$\neg(A$	$\vee$	$B)$	$C \rightarrow$	$\neg C$
T	T	T	$\cancel{F}$	$\cancel{F}$	$\cancel{F}$	T	$\cancel{F}$	F	$\cancel{T}$		F	$\cancel{F}$
T	T	F	$\cancel{F}$	$\cancel{F}$	$\cancel{F}$	F	$\cancel{T}$	F	$\cancel{T}$		T	$\cancel{T}$
T	F	T	$\cancel{F}$	$\cancel{F}$	$\cancel{T}$	T	$\cancel{F}$	F	$\cancel{T}$		F	$\cancel{F}$
T	F	F	$\cancel{F}$	$\cancel{F}$	$\cancel{T}$	F	$\cancel{T}$	F	$\cancel{T}$		T	$\cancel{T}$
F	T	T	$\cancel{T}$	$\cancel{F}$	$\cancel{F}$	T	$\cancel{F}$	F	$\cancel{T}$		F	$\cancel{F}$
F	T	F	$\cancel{T}$	$\cancel{F}$	$\cancel{F}$	F	$\cancel{T}$	F	$\cancel{T}$		T	$\cancel{T}$
F	F	T	$\cancel{T}$	$\cancel{T}$	$\cancel{T}$	F	$\cancel{F}$	T	$\cancel{F}$		F	$\cancel{F}$
F	F	F	$\cancel{T}$	$\cancel{T}$	$\cancel{T}$	T	$\cancel{T}$	T	$\cancel{F}$		T	$\cancel{T}$

6. Valid [Q p.16]

A	B	C	$A \lor B$	$\neg A$	$\lor$	C	$B \to C$
T	T	T	T	~~F~~	T		T
T	T	F	T	~~F~~	F		F
T	F	T	T	~~F~~	T		T
T	F	F	T	~~F~~	F		T
F	T	T	T	~~T~~	T		T
F	T	F	T	~~T~~	T		F
F	F	T	F	~~T~~	T		T
F	F	F	F	~~T~~	T		T

7. Invalid. Counterexample: A false, B false, C false (row 8). [Q p.17]

A	B	C	$\neg$	$(A \lor B)$	$\leftrightarrow$	$\neg C$	$\neg A$	$\land$	$\neg B$	C	$\land$	$\neg C$
T	T	T	~~F~~	~~T~~	T	~~F~~	~~F~~	F	~~F~~		F	~~F~~
T	T	F	~~F~~	~~T~~	F	~~T~~	~~F~~	F	~~F~~		F	~~T~~
T	F	T	~~F~~	~~T~~	T	~~F~~	~~F~~	F	~~T~~		F	~~F~~
T	F	F	~~F~~	~~T~~	F	~~T~~	~~F~~	F	~~T~~		F	~~T~~
F	T	T	~~F~~	~~T~~	T	~~F~~	~~T~~	F	~~F~~		F	~~F~~
F	T	F	~~F~~	~~T~~	F	~~T~~	~~T~~	F	~~F~~		F	~~T~~
F	F	T	~~T~~	~~F~~	F	~~F~~	~~T~~	T	~~T~~		F	~~F~~
$*$ F	F	F	~~T~~	~~F~~	T	~~T~~	~~T~~	T	~~T~~		F	~~T~~

8. Invalid. Counterexample: A true, B false, C true (row 3). [Q p.17]

A	B	C	$\neg$	$(A \land B)$	$\to$	$(C \lor A)$	$\neg A$	$\lor$	$\neg B$	$\neg$	$(C$	$\lor$	$\neg C)$
T	T	T	~~F~~	~~T~~	T	~~T~~	~~F~~	F	~~F~~	F		~~T~~	~~F~~
T	T	F	~~F~~	~~T~~	T	~~T~~	~~F~~	F	~~F~~	F		~~T~~	~~T~~
$*$ T	F	T	~~T~~	~~F~~	T	~~T~~	~~F~~	T	~~T~~	F		~~T~~	~~F~~
T	F	F	~~T~~	~~F~~	T	~~T~~	~~F~~	T	~~T~~	F		~~T~~	~~T~~
F	T	T	~~T~~	~~F~~	T	~~T~~	~~T~~	T	~~F~~	F		~~T~~	~~F~~
F	T	F	~~T~~	~~F~~	F	~~F~~	~~T~~	T	~~F~~	F		~~T~~	~~T~~
F	F	T	~~T~~	~~F~~	T	~~T~~	~~T~~	T	~~T~~	F		~~T~~	~~F~~
F	F	F	~~T~~	~~F~~	F	~~F~~	~~T~~	T	~~T~~	F		~~T~~	~~T~~

9. Valid. [Q p.17]

A	B	C	A	$\rightarrow$	$(B \wedge C)$	B	$\leftrightarrow$	$\neg C$	$\neg A$
T	T	T		T	$\cancel{T}$		F	$\cancel{F}$	F
T	T	F		F	$\cancel{F}$		T	$\cancel{T}$	F
T	F	T		F	$\cancel{F}$		T	$\cancel{F}$	F
T	F	F		F	$\cancel{F}$		F	$\cancel{T}$	F
F	T	T		T	$\cancel{T}$		F	$\cancel{F}$	T
F	T	F		T	$\cancel{F}$		T	$\cancel{T}$	T
F	F	T		T	$\cancel{F}$		T	$\cancel{F}$	T
F	F	F		T	$\cancel{F}$		F	$\cancel{T}$	T

10. Valid. [Q p.17]

A	B	C	$A \rightarrow B$	$B \rightarrow C$	$\neg C$	$\neg A$
T	T	T	T	T	F	F
T	T	F	T	F	T	F
T	F	T	F	T	F	F
T	F	F	F	T	T	F
F	T	T	T	T	F	T
F	T	F	T	F	T	T
F	F	T	T	T	F	T
F	F	F	T	T	T	T

[Contents]

Answers 4.2.1

1. Neither [Q p.17]

P	Q	$((P$	$\vee$	$Q)$	$\rightarrow$	$P)$
T	T		$\cancel{T}$		T	
T	F		$\cancel{T}$		T	
F	T		$\cancel{T}$		F	
F	F		$\cancel{F}$		T	

2. Neither [Q p.17]

P	Q	R	$(\neg P$	$\wedge$	$(Q$	$\vee$	$R))$
T	T	T	$\cancel{F}$	F		$\cancel{T}$	
T	T	F	$\cancel{F}$	F		$\cancel{T}$	
T	F	T	$\cancel{F}$	F		$\cancel{T}$	
T	F	F	$\cancel{F}$	F		$\cancel{F}$	
F	T	T	$\cancel{T}$	T		$\cancel{T}$	
F	T	F	$\cancel{T}$	T		$\cancel{T}$	
F	F	T	$\cancel{T}$	T		$\cancel{T}$	
F	F	F	$\cancel{T}$	F		$\cancel{F}$	

3. Contradiction [Q p.17]

P	Q	$((\neg P$	$\vee$	$Q)$	$\leftrightarrow$	$(P$	$\wedge$	$\neg Q))$
T	T	$\cancel{F}$	$\cancel{T}$		F		$\cancel{F}$	$\cancel{F}$
T	F	$\cancel{F}$	$\cancel{F}$		F		$\cancel{T}$	$\cancel{T}$
F	T	$\cancel{T}$	$\cancel{T}$		F		$\cancel{F}$	$\cancel{F}$
F	F	$\cancel{T}$	$\cancel{T}$		F		$\cancel{F}$	$\cancel{T}$

4. Tautology [Q p.17]

P	Q	R	$(P$	$\rightarrow$	$(Q$	$\rightarrow$	$(R$	$\rightarrow$	$P)))$
T	T	T		T		$\cancel{T}$		$\cancel{T}$	
T	T	F		T		$\cancel{T}$		$\cancel{T}$	
T	F	T		T		$\cancel{T}$		$\cancel{T}$	
T	F	F		T		$\cancel{T}$		$\cancel{T}$	
F	T	T		T		$\cancel{F}$		$\cancel{F}$	
F	T	F		T		$\cancel{T}$		$\cancel{T}$	
F	F	T		T		$\cancel{T}$		$\cancel{F}$	
F	F	F		T		$\cancel{T}$		$\cancel{T}$	

5. Tautology [Q p.17]

P	Q	$(P$	$\rightarrow$	$((P$	$\rightarrow$	$Q)$	$\rightarrow$	$Q))$
T	T		T		$\cancel{T}$		$\cancel{T}$	
T	F		T		$\cancel{F}$		$\cancel{T}$	
F	T		T		$\cancel{T}$		$\cancel{T}$	
F	F		T		$\cancel{T}$		$\cancel{F}$	

6. Neither [Q p.17]

P	Q	$(P \rightarrow$	$((Q \rightarrow P)$	$\rightarrow Q))$
T	T	T	$\cancel{T}$	$\cancel{T}$
T	F	F	$\cancel{T}$	$\cancel{F}$
F	T	T	$\cancel{F}$	$\cancel{T}$
F	F	T	$\cancel{T}$	$\cancel{F}$

7. Tautology [Q p.17]

P	Q	$((P \rightarrow Q)$	$\vee$	$\neg(Q$	$\wedge$	$\neg Q))$
T	T	$\cancel{T}$	T	$\cancel{T}$	$\cancel{F}$	$\cancel{F}$
T	F	$\cancel{F}$	T	$\cancel{T}$	$\cancel{F}$	$\cancel{T}$
F	T	$\cancel{T}$	T	$\cancel{T}$	$\cancel{F}$	$\cancel{F}$
F	F	$\cancel{T}$	T	$\cancel{T}$	$\cancel{F}$	$\cancel{T}$

8. Tautology [Q p.17]

P	Q	$((P \rightarrow Q)$	$\vee$	$\neg(Q$	$\wedge$	$\neg P))$
T	T	$\cancel{T}$	T	$\cancel{T}$	$\cancel{F}$	$\cancel{F}$
T	F	$\cancel{F}$	T	$\cancel{T}$	$\cancel{F}$	$\cancel{F}$
F	T	$\cancel{T}$	T	$\cancel{F}$	$\cancel{T}$	$\cancel{T}$
F	F	$\cancel{T}$	T	$\cancel{T}$	$\cancel{F}$	$\cancel{T}$

9. Neither [Q p.17]

P	Q	$((P \wedge Q)$	$\leftrightarrow$	$(Q \leftrightarrow P))$
T	T	$\cancel{T}$	T	$\cancel{T}$
T	F	$\cancel{F}$	T	$\cancel{F}$
F	T	$\cancel{F}$	T	$\cancel{F}$
F	F	$\cancel{F}$	F	$\cancel{T}$

10. Contradiction [Q p.17]

P	Q	$\neg$	$((P \wedge Q)$	$\rightarrow$	$(Q \leftrightarrow P))$
T	T	F	$\cancel{T}$	$\cancel{T}$	$\cancel{T}$
T	F	F	$\cancel{F}$	$\cancel{T}$	$\cancel{F}$
F	T	F	$\cancel{F}$	$\cancel{T}$	$\cancel{F}$
F	F	F	$\cancel{F}$	$\cancel{T}$	$\cancel{T}$

[Contents]

Answers 4.3.1

1.

	P	Q	$(P$	$\to$	$Q)$	$\neg(P$	$\wedge$	$\neg Q)$
$*$	T	T		T		T	$\cancel{F}$	$\cancel{F}$
	T	F		F		F	$\cancel{T}$	$\cancel{T}$
	F	T		T		T	$\cancel{F}$	$\cancel{F}$
	F	F		T		T	$\cancel{F}$	$\cancel{T}$

(a) jointly satisfiable, because both true on e.g. $*$'ed row.

(b) equivalent, because same truth value on every row

(c) not contradictory, because jointly satisfiable

(d) not contrary, because jointly satisfiable [Q p.18]

2.

	P	Q	$(P$	$\wedge$	$Q)$	$(P$	$\wedge$	$\neg Q)$
$*$	T	T		T			F	$\cancel{F}$
	T	F		F			T	$\cancel{T}$
$\dagger$	F	T		F			F	$\cancel{F}$
	F	F		F			F	$\cancel{T}$

(a) jointly unsatisfiable, because no row on which both true

(b) not equivalent, because different truth values on e.g. $*$'ed row.

(c) not contradictory, because both false on e.g. $\dagger$'ed row.

(d) contrary because jointly unsatisfiable and not contradictory

[Q p.18]

3.

	P	Q	$\neg(P$	$\leftrightarrow$	$Q)$	$\neg(P$	$\to$	$Q)$	$\vee$	$\neg(P$	$\vee$	$\neg Q)$
	T	T	F	$\cancel{T}$		$\cancel{F}$	$\cancel{T}$		F	$\cancel{F}$	$\cancel{T}$	$\cancel{F}$
$*$	T	F	T	$\cancel{F}$		$\cancel{T}$	$\cancel{F}$		T	$\cancel{F}$	$\cancel{T}$	$\cancel{T}$
	F	T	T	$\cancel{F}$		$\cancel{F}$	$\cancel{T}$		T	$\cancel{T}$	$\cancel{F}$	$\cancel{F}$
	F	F	F	$\cancel{T}$		$\cancel{F}$	$\cancel{T}$		F	$\cancel{F}$	$\cancel{T}$	$\cancel{T}$

(a) jointly satisfiable, because both true on e.g. $*$'ed row.

(b) equivalent because same truth value on every row

(c) not contradictory, because jointly satisfiable

(d) not contrary, because jointly satisfiable [Q p.18]

4.

	P	Q	R	$(P$	$\to$	$(Q$	$\to$	$R))$	$((P$	$\to$	$Q)$	$\to$	$R)$
$*$	T	T	T		T		$\cancel{\text{T}}$			$\cancel{\text{T}}$		T	
	T	T	F		F		$\cancel{\text{F}}$			$\cancel{\text{T}}$		F	
	T	F	T		T		$\cancel{\text{T}}$			$\cancel{\text{F}}$		T	
	T	F	F		T		$\cancel{\text{T}}$			$\cancel{\text{F}}$		T	
	F	T	T		T		$\cancel{\text{T}}$			$\cancel{\text{T}}$		T	
†	F	T	F		T		$\cancel{\text{F}}$			$\cancel{\text{T}}$		F	
	F	F	T		T		$\cancel{\text{T}}$			$\cancel{\text{T}}$		T	
	F	F	F		T		$\cancel{\text{T}}$			$\cancel{\text{T}}$		F	

(a) jointly satisfiable because both true on e.g. $*$'ed row.

(b) not equivalent because different truth values on e.g. †'ed row.

(c) not contradictory, because jointly satisfiable

(d) not contrary, because jointly satisfiable [Q p.18]

5.

	P	Q	R	$(P$	$\wedge$	$(Q$	$\wedge$	$\neg Q))$	$\neg$	$(Q$	$\to$	$\neg$	$(R$	$\wedge$	$\neg Q))$
$*$	T	T	T		F		$\cancel{\text{F}}$	$\cancel{\text{F}}$	F		$\cancel{\text{T}}$	$\cancel{\text{T}}$		$\cancel{\text{F}}$	$\cancel{\text{F}}$
	T	T	F		F		$\cancel{\text{F}}$	$\cancel{\text{F}}$	F		$\cancel{\text{T}}$	$\cancel{\text{T}}$		$\cancel{\text{F}}$	$\cancel{\text{F}}$
	T	F	T		F		$\cancel{\text{F}}$	$\cancel{\text{T}}$	F		$\cancel{\text{T}}$	$\cancel{\text{F}}$		$\cancel{\text{T}}$	$\cancel{\text{T}}$
	T	F	F		F		$\cancel{\text{F}}$	$\cancel{\text{T}}$	F		$\cancel{\text{T}}$	$\cancel{\text{T}}$		$\cancel{\text{F}}$	$\cancel{\text{T}}$
	F	T	T		F		$\cancel{\text{F}}$	$\cancel{\text{F}}$	F		$\cancel{\text{T}}$	$\cancel{\text{T}}$		$\cancel{\text{F}}$	$\cancel{\text{F}}$
	F	T	F		F		$\cancel{\text{F}}$	$\cancel{\text{F}}$	F		$\cancel{\text{T}}$	$\cancel{\text{T}}$		$\cancel{\text{F}}$	$\cancel{\text{F}}$
	F	F	T		F		$\cancel{\text{F}}$	$\cancel{\text{T}}$	F		$\cancel{\text{T}}$	$\cancel{\text{F}}$		$\cancel{\text{T}}$	$\cancel{\text{T}}$
	F	F	F		F		$\cancel{\text{F}}$	$\cancel{\text{T}}$	F		$\cancel{\text{T}}$	$\cancel{\text{T}}$		$\cancel{\text{F}}$	$\cancel{\text{T}}$

(a) jointly unsatisfiable, because no row on which both true

(b) equivalent because same truth value on every row

(c) not contradictory, because both false on e.g. $*$'ed row.

(d) contrary, because jointly unsatisfiable and not contradictory

[Q p.18]

6.

	P	R	$(P$	$\land$	$\neg P)$	$(R$	$\lor$	$\neg R)$
*	T	T		F	$\cancel{F}$		T	$\cancel{F}$
	T	F		F	$\cancel{F}$		T	$\cancel{T}$
	F	T		F	$\cancel{T}$		T	$\cancel{F}$
	F	F		F	$\cancel{T}$		T	$\cancel{T}$

(a) jointly unsatisfiable, because no row on which both true

(b) not equivalent because different truth values on e.g. $*$'ed row.

(c) contradictory, because jointly unsatisfiable and no row on which both false

(d) not contrary, because no row on which both false [Q p.18]

7.

	P	Q	$(P$	$\land$	$\neg P)$	$\neg($	Q	$\to$	$Q)$
*	T	T		F	$\cancel{F}$	F		$\cancel{T}$	
	T	F		F	$\cancel{F}$	F		$\cancel{T}$	
	F	T		F	$\cancel{T}$	F		$\cancel{T}$	
	F	F		F	$\cancel{T}$	F		$\cancel{T}$	

(a) jointly unsatisfiable, because no row on which both true

(b) equivalent because same truth value on every row

(c) not contradictory, because both false on e.g. $*$'ed row.

(d) contrary, because jointly unsatisfiable and not contradictory

[Q p.18]

8.

P	Q	R	$((P \to Q)$	$\to$	$R)$	$\neg(P$	$\vee$	$\neg(Q$	$\wedge$	$\neg R))$
$*$ T	T	T	$\cancel{T}$	T		F	$\cancel{T}$	$\cancel{T}$	$\cancel{F}$	$\cancel{F}$
† T	T	F	$\cancel{T}$	F		F	$\cancel{T}$	$\cancel{F}$	$\cancel{T}$	$\cancel{T}$
T	F	T	$\cancel{F}$	T		F	$\cancel{T}$	$\cancel{T}$	$\cancel{F}$	$\cancel{F}$
T	F	F	$\cancel{F}$	T		F	$\cancel{T}$	$\cancel{T}$	$\cancel{F}$	$\cancel{T}$
F	T	T	$\cancel{T}$	T		F	$\cancel{T}$	$\cancel{T}$	$\cancel{F}$	$\cancel{F}$
F	T	F	$\cancel{T}$	F		T	$\cancel{F}$	$\cancel{F}$	$\cancel{T}$	$\cancel{T}$
F	F	T	$\cancel{T}$	T		F	$\cancel{T}$	$\cancel{T}$	$\cancel{F}$	$\cancel{F}$
F	F	F	$\cancel{T}$	F		F	$\cancel{T}$	$\cancel{T}$	$\cancel{F}$	$\cancel{T}$

(a) jointly unsatisfiable, because no row on which both true

(b) not equivalent because different truth values on e.g. $*$'ed row.

(c) not contradictory, because both false on e.g. †'ed row.

(d) contrary, because jointly unsatisfiable and not contradictory

[Q p.18]

9.

P	Q	$(P \leftrightarrow Q)$	$((P \wedge Q)$	$\vee$	$(\neg P$	$\wedge$	$\neg Q))$
$*$ T	T	T	$\cancel{T}$	T	$\cancel{F}$	$\cancel{F}$	$\cancel{F}$
T	F	F	$\cancel{F}$	F	$\cancel{F}$	$\cancel{F}$	$\cancel{T}$
F	T	F	$\cancel{F}$	F	$\cancel{T}$	$\cancel{F}$	$\cancel{F}$
F	F	T	$\cancel{F}$	T	$\cancel{T}$	$\cancel{T}$	$\cancel{T}$

(a) jointly satisfiable, because both true on e.g. row 1

(b) equivalent because same truth value on every row

(c) not contradictory, because jointly satisfiable

(d) not contrary, because jointly satisfiable [Q p.18]

10.

	P	Q	$(P \leftrightarrow Q)$	$((P \wedge Q)$	$\vee$	$(\neg P$	$\wedge$	$\neg Q))$
$*$	T	T	T	$\cancel{T}$	T	$\cancel{F}$	$\cancel{F}$	$\cancel{F}$
	T	F	F	$\cancel{F}$	F	$\cancel{F}$	$\cancel{F}$	$\cancel{T}$
	F	T	F	$\cancel{F}$	F	$\cancel{T}$	$\cancel{F}$	$\cancel{F}$
	F	F	T	$\cancel{F}$	T	$\cancel{T}$	$\cancel{T}$	$\cancel{T}$

(a) jointly satisfiable, because both true on e.g. row 1

(b) equivalent because same truth value on every row

(c) not contradictory, because jointly satisfiable

(d) not contrary, because jointly satisfiable [Q p.18]

[Contents]

Answers 4.4.1

1. Satisfiable ($*$'ed row) [Q p.18]

	P	Q	$(P \vee Q)$	$\neg$	$(P \wedge Q)$
	T	T	T	F	$\cancel{T}$
$*$	T	F	T	T	$\cancel{F}$
	F	T	T	T	$\cancel{F}$
	F	F	F	T	$\cancel{F}$

2. Unsatisfiable [Q p.18]

P	Q	$\neg$	$(P \rightarrow Q)$	$(P \leftrightarrow Q)$	$(\neg P$	$\vee$ $Q)$
T	T	F	$\cancel{T}$	T	$\cancel{F}$	T
T	F	T	$\cancel{F}$	F	$\cancel{F}$	F
F	T	F	$\cancel{T}$	F	$\cancel{T}$	T
F	F	F	$\cancel{T}$	T	$\cancel{T}$	T

3. Unsatisfiable [Q p.18]

P	$(P \rightarrow$	$\neg P)$	$(P \vee$	$\neg P)$	$(\neg P$	$\rightarrow P)$
T	F	$\cancel{F}$	T	$\cancel{F}$	$\cancel{F}$	T
F	T	$\cancel{T}$	T	$\cancel{T}$	$\cancel{T}$	F

4. Unsatisfiable [Q p.18]

P	Q	R	$(P \lor Q)$	$\lor$	R	$(\neg P$	$\to$	$\neg Q)$	$(\neg Q$	$\to$	$\neg R)$	$\neg P$
T	T	T	$\cancel{\text{T}}$	T		$\cancel{\text{F}}$	T	$\cancel{\text{F}}$	$\cancel{\text{F}}$	T	$\cancel{\text{F}}$	F
T	T	F	$\cancel{\text{T}}$	T		$\cancel{\text{F}}$	T	$\cancel{\text{F}}$	$\cancel{\text{F}}$	T	$\cancel{\text{T}}$	F
T	F	T	$\cancel{\text{T}}$	T		$\cancel{\text{F}}$	T	$\cancel{\text{T}}$	$\cancel{\text{T}}$	F	$\cancel{\text{F}}$	F
T	F	F	$\cancel{\text{T}}$	T		$\cancel{\text{F}}$	T	$\cancel{\text{T}}$	$\cancel{\text{T}}$	T	$\cancel{\text{T}}$	F
F	T	T	$\cancel{\text{T}}$	T		$\cancel{\text{T}}$	F	$\cancel{\text{F}}$	$\cancel{\text{F}}$	T	$\cancel{\text{F}}$	T
F	T	F	$\cancel{\text{T}}$	T		$\cancel{\text{T}}$	F	$\cancel{\text{F}}$	$\cancel{\text{F}}$	T	$\cancel{\text{T}}$	T
F	F	T	$\cancel{\text{F}}$	T		$\cancel{\text{T}}$	T	$\cancel{\text{T}}$	$\cancel{\text{T}}$	F	$\cancel{\text{F}}$	T
F	F	F	$\cancel{\text{F}}$	F		$\cancel{\text{T}}$	T	$\cancel{\text{T}}$	$\cancel{\text{T}}$	T	$\cancel{\text{T}}$	T

5. Satisfiable ($*$'ed row) [Q p.18]

	P	Q	R	$(P \leftrightarrow Q)$	$(Q \lor R)$	$(R \to P)$
$*$	T	T	T	T	T	T
	T	T	F	T	T	T
	T	F	T	F	T	T
	T	F	F	F	F	T
	F	T	T	F	T	F
	F	T	F	F	T	T
	F	F	T	T	T	F
	F	F	F	T	F	T

6. Satisfiable ($*$'ed row) [Q p.18]

	P	Q	$(\neg P$	$\to$	$\neg Q)$	$(P \leftrightarrow Q)$
$*$	T	T	$\cancel{\text{F}}$	T	$\cancel{\text{F}}$	T
	T	F	$\cancel{\text{F}}$	T	$\cancel{\text{T}}$	F
	F	T	$\cancel{\text{T}}$	F	$\cancel{\text{F}}$	F
	F	F	$\cancel{\text{T}}$	T	$\cancel{\text{T}}$	T

7. Unsatisfiable [Q p.18]

P	$\neg P$	$(P$	$\to$	$(P \to P))$	$(\neg P$	$\leftrightarrow$	$P)$
T	F		T	$\cancel{\text{T}}$	F	F	
F	T		T	$\cancel{\text{T}}$	T	F	

8. Unsatisfiable [Q p.19]

P	Q	R	$(P$	$\vee$	$\neg Q)$	$(P \to R)$	$\neg R$	$(\neg R$	$\to$	$Q)$
T	T	T		T	$\cancel{F}$	T	F	$\cancel{F}$	T	
T	T	F		T	$\cancel{F}$	F	T	$\cancel{T}$	T	
T	F	T		T	$\cancel{T}$	T	F	$\cancel{F}$	T	
T	F	F		T	$\cancel{T}$	F	T	$\cancel{T}$	F	
F	T	T		F	$\cancel{F}$	T	F	$\cancel{F}$	T	
F	T	F		F	$\cancel{F}$	T	T	$\cancel{T}$	T	
F	F	T		T	$\cancel{T}$	T	F	$\cancel{F}$	T	
F	F	F		T	$\cancel{T}$	T	T	$\cancel{T}$	F	

9. Satisfiable ($*$'ed row) [Q p.19]

	P	Q	R	$\neg R$	$\neg P$	$((Q$	$\to$	$\neg Q)$	$\to R)$
	T	T	T	F	F		$\cancel{F}$	$\cancel{F}$	T
	T	T	F	T	F		$\cancel{F}$	$\cancel{F}$	T
	T	F	T	F	F		$\cancel{T}$	$\cancel{T}$	T
	T	F	F	T	F		$\cancel{T}$	$\cancel{T}$	F
	F	T	T	F	T		$\cancel{F}$	$\cancel{F}$	T
$*$	F	T	F	T	T		$\cancel{F}$	$\cancel{F}$	T
	F	F	T	F	T		$\cancel{T}$	$\cancel{T}$	T
	F	F	F	T	T		$\cancel{T}$	$\cancel{T}$	F

10. Unsatisfiable [Q p.19]

P	Q	$(\neg P$	$\vee$	$\neg Q)$	$\neg$	$(P$	$\wedge$	$\neg Q)$	$(P$	$\vee$	$\neg Q)$	$\neg$	$(\neg P$	$\wedge$	$\neg Q)$
T	T	$\cancel{F}$	F	$\cancel{F}$	T		$\cancel{F}$	$\cancel{F}$		T	$\cancel{F}$	T	$\cancel{F}$	$\cancel{F}$	$\cancel{F}$
T	F	$\cancel{F}$	T	$\cancel{T}$	F		$\cancel{T}$	$\cancel{T}$		T	$\cancel{T}$	T	$\cancel{F}$	$\cancel{F}$	$\cancel{T}$
F	T	$\cancel{T}$	T	$\cancel{F}$	T		$\cancel{F}$	$\cancel{F}$		F	$\cancel{F}$	T	$\cancel{T}$	$\cancel{F}$	$\cancel{F}$
F	F	$\cancel{T}$	T	$\cancel{T}$	T		$\cancel{F}$	$\cancel{T}$		T	$\cancel{T}$	F	$\cancel{T}$	$\cancel{T}$	$\cancel{T}$

[Contents]

Chapter 5

Logical Form

Answers 5.1.1

Note: There are also other correct answers to questions 1–4.

1. (i) $\neg(\alpha \rightarrow \beta)$
   (ii) $\neg(\alpha \rightarrow (\beta \rightarrow \gamma))$
   (iii) $\neg(\alpha \rightarrow (\alpha \rightarrow \beta))$ [Q p.20]
2. (i) $(\alpha \rightarrow \beta)$
   (ii) $(\alpha \rightarrow \alpha)$
   (iii) $((\alpha \vee \beta) \rightarrow (\alpha \vee \beta))$ [Q p.20]
3. (i) α
   (ii) $\alpha \wedge \beta$
   (iii) $\alpha \wedge (\beta \rightarrow \gamma)$ [Q p.20]
4. (i) α
   (ii) $\alpha \leftrightarrow \beta$
   (iii) $(\alpha \wedge \beta) \leftrightarrow \gamma$ [Q p.20]

[Contents]

Answers 5.2.1

1. First form: $\neg\neg\alpha$

   (i) $\alpha : C$

   (ii) $\alpha : (A \wedge B)$

   (iii) $\alpha : (C \wedge \neg D)$

   Second form: $\neg\alpha$

   (i) $\alpha : \neg C$

   (ii) $\alpha : \neg(A \wedge B)$

   (iii) $\alpha : \neg(C \wedge \neg D)$

   Third form: α

   (i) $\alpha : \neg\neg C$

   (ii) $\alpha : \neg\neg(A \wedge B)$

   (iii) $\alpha : \neg\neg(C \wedge \neg D)$ [Q p.20]

2. (i) (a) Yes: $\alpha : P$; $\beta : Q$. [Q p.21]

   (b) Yes: $\alpha : R$; $\beta : Q$. [Q p.21]

   (c) Yes: $\alpha : R$; $\beta : (R \to Q)$. [Q p.21]

   (ii) (a) Yes: $\alpha : P$; $\beta : Q$. [Q p.21]

   (b) Yes: $\alpha : P$; $\beta : P$. [Q p.21]

   (c) No. [Q p.21]

   (iii) (a) Yes: $\alpha : \neg P$; $\beta : Q$. [Q p.21]

   (b) Yes: $\alpha : P$; $\beta : \neg P$. [Q p.21]

   (c) No. [Q p.21]

   (iv) (a) No. [Q p.21]

   (b) No. [Q p.21]

   (c) Yes: $\alpha : \neg P$; $\beta : \neg P$ [Q p.21]

[Contents]

Answers 5.3.1

Note: There are also other correct answers to questions 1–4.

1. (i) $\neg(\alpha \to (\alpha \to \beta))$
   $\therefore (\alpha \vee (\alpha \to \beta))$
   replacements: $\alpha : R \,;\, \beta : Q$

   (ii) $\neg(\alpha \to \beta)$
   $\therefore (\alpha \vee \beta)$
   replacements: $\alpha : R \,;\, \beta : (R \to Q)$

   (iii) $\neg(\alpha \to (\beta \to \gamma))$
   $\therefore (\alpha \vee (\beta \to \gamma))$
   replacements: $\alpha : R \,;\, \beta : R \,;\, \gamma : Q$

   (iv) α
   $\therefore \beta$
   replacements: $\alpha : \neg(R \to (R \to Q)) \,;\, \beta : R \vee (R \to Q)$ [Q p.21]

2. (i) $(\alpha \wedge \beta) \to \beta$
   $\neg\beta$
   $\therefore\ \neg(\alpha \wedge \beta)$
   replacements: $\alpha : P \,;\, \beta : Q$

   (ii) $\alpha \to \beta$
   $\neg\beta$
   $\therefore\ \neg\alpha$
   replacements: $\alpha : (P \wedge Q) \,;\, \beta : Q$

   (iii) $\alpha \to \beta$
   γ
   $\therefore\ \neg\alpha$
   replacements: $\alpha : (P \wedge Q) \,;\, \beta : Q \,;\, \gamma : \neg Q$

   (iv) α
   β
   $\therefore\ \gamma$
   replacements: $\alpha : (P \wedge Q) \to Q \,;\, \beta : \neg Q \,;\, \gamma : \neg(P \wedge Q)$ [Q p.22]

3. (i) $\neg\alpha \rightarrow (\beta \rightarrow \gamma)$
$\neg\alpha$
$\therefore\ \beta \rightarrow \gamma$
replacements: $\alpha : Q\ ; \beta : R\ ; \gamma : S$

(ii) $\neg\alpha \rightarrow \beta$
$\neg\alpha$
$\therefore\ \beta$
replacements: $\alpha : Q\ ; \beta : (R \rightarrow S)$

(iii) $\alpha \rightarrow \beta$
α
$\therefore\ \beta$
replacements: $\alpha : \neg Q\ ; \beta : (R \rightarrow S)$

(iv) α
β
$\therefore\ \gamma$
replacements: $\alpha : \neg Q \rightarrow (R \rightarrow S)\ ; \beta : \neg Q\ ; \gamma : (R \rightarrow S)$

[Q p.22]

4. (i) $(\alpha \rightarrow \neg\beta) \vee (\neg\beta \rightarrow \alpha)$
$\neg(\neg\beta \rightarrow \alpha)$
$\therefore \alpha \rightarrow \neg\beta$
replacements: $\alpha : P\ ; \beta : Q$

(ii) $(\alpha \rightarrow \beta) \vee (\beta \rightarrow \alpha)$
$\neg(\beta \rightarrow \alpha)$
$\therefore \alpha \rightarrow \beta$
replacements: $\alpha : P\ ; \beta : \neg Q$

(iii) $\alpha \vee \beta$
$\neg\beta$
$\therefore \alpha$
replacements: $\alpha : (P \rightarrow \neg Q)\ ; \beta : (\neg Q \rightarrow P)$

(iv) α
β
$\therefore\ \gamma$
replacements: $\alpha : (P \rightarrow \neg Q) \vee (\neg Q \rightarrow P)\ ;$
$\beta : \neg(\neg Q \rightarrow P)\ ; \gamma : (P \rightarrow \neg Q)$

[Q p.22]

[Contents]

Answers 5.4.1

1. (i) $\alpha : P$; $\beta : Q$

   (ii)

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

[Q p.22]

2. (i) $\alpha : (A \wedge B)$; $\beta : (B \vee C)$

   (ii)

A	B	C	$A \wedge B$	$(A \wedge B) \to (B \vee C)$	$B \vee C$
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	T	T
T	F	F	F	T	F
F	T	T	F	T	T
F	T	F	F	T	T
F	F	T	F	T	T
F	F	F	F	T	F

[Q p.22]

3. (i) $\alpha : (A \vee \neg A)$; $\beta : (A \wedge \neg A)$

   (ii)

P	$(A \vee \neg A)$	$(A \vee \neg A) \to (A \wedge \neg A)$	$(A \wedge \neg A)$
T	T	F	F
F	T	F	F

[Q p.22]

4. (i) $\alpha : (P \to \neg P)$; $\beta : (P \to (Q \wedge \neg R))$

   (ii)

P	Q	R	$(P \to \neg P)$	$(P \to \neg P) \quad \to \quad (P \to (Q \wedge \neg R))$	$(P \quad \to \quad (Q \wedge \neg R))$
T	T	T	F	T	F
T	T	F	F	T	T
T	F	T	F	T	F
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	T	T
F	F	T	T	T	T
F	F	F	T	T	T

[Q p.22]

[Contents]

Answers 5.5.1

1. (i)

	α	β
	T	T
$*$	T	F
	F	T
	F	F

Invalid: in $*$'ed row, the premise is T and the conclusion is F.

[Q p.23]

(ii) Instance: P

$\therefore P$

Truth table:

P
T
F

Valid: there is no row in which the premise (P) is true and the conclusion (P) is false. [Q p.23]

2. $\alpha \vee \neg\alpha$

$\therefore \alpha \wedge \neg\alpha$ [Q p.23]

[Contents]

Chapter 6

Connectives: Translation and Adequacy

Answers 6.5.1

1. Glossary:

B:	Bob is happy.
R:	It is raining.
S:	The sun is shining.

   Translation:

   $(B \leftrightarrow R)$
   $(R \vee S)$
   $\therefore (B \rightarrow \neg S)$

   Truth Table:

B	R	S	$(B \leftrightarrow R)$	$(R \vee S)$	$(B \rightarrow \neg S)$
$*$ T	T	T	T	T	F
T	T	F	T	T	T
T	F	T	F $\times$		
T	F	F	F $\times$		
F	T	T	F $\times$		
F	T	F	F $\times$		
F	F	T	T	T	T
F	F	F	T	F $\times$	

   [Q p.24]

   Invalid. Counterexample ($*$'ed row), where B is T, R is T and S is T.

2. Glossary:

M:	I have money.
C:	I have a card.
W:	I shall walk.
T:	I shall get tired.
R:	I shall have a rest.

Translation:

$(\neg M \wedge \neg C) \rightarrow W$
$W \rightarrow (T \vee R)$
$\therefore (R \rightarrow M)$

Truth Table:

M	C	W	T	R	$(\neg M \wedge \neg C)$	$\rightarrow$	W	W	$\rightarrow$	$(T \vee R)$	$R \rightarrow M$
T	T	T	T	T	$\cancel{\text{F}}$	T			T	$\cancel{\text{T}}$	T
T	T	T	T	F	$\cancel{\text{F}}$	T			T	$\cancel{\text{T}}$	T
T	T	T	F	T	$\cancel{\text{F}}$	T			T	$\cancel{\text{T}}$	T
T	T	T	F	F	$\cancel{\text{F}}$	T			F	$\cancel{\text{F}}$	T
T	T	F	T	T	$\cancel{\text{F}}$	T			T	$\cancel{\text{T}}$	T
T	T	F	T	F	$\cancel{\text{F}}$	T			T	$\cancel{\text{T}}$	T
T	T	F	F	T	$\cancel{\text{F}}$	T			T	$\cancel{\text{T}}$	T
T	T	F	F	F	$\cancel{\text{F}}$	T			T	$\cancel{\text{F}}$	T
T	F	T	T	T	$\cancel{\text{F}}$	T			T	$\cancel{\text{T}}$	T
T	F	T	T	F	$\cancel{\text{F}}$	T			T	$\cancel{\text{T}}$	T
T	F	T	F	T	$\cancel{\text{F}}$	T			T	$\cancel{\text{T}}$	T
T	F	T	F	F	$\cancel{\text{F}}$	T			F	$\cancel{\text{F}}$	T
T	F	F	T	T	$\cancel{\text{F}}$	T			T	$\cancel{\text{T}}$	T
T	F	F	T	F	$\cancel{\text{F}}$	T			T	$\cancel{\text{T}}$	T
T	F	F	F	T	$\cancel{\text{F}}$	T			T	$\cancel{\text{T}}$	T
T	F	F	F	F	$\cancel{\text{F}}$	T			T	$\cancel{\text{F}}$	T
F	T	T	T	T	$\cancel{\text{F}}$	T			T	$\cancel{\text{T}}$	F
F	T	T	T	F	$\cancel{\text{F}}$	T			T	$\cancel{\text{T}}$	T
F	T	T	F	T	$\cancel{\text{F}}$	T			T	$\cancel{\text{T}}$	F
F	T	T	F	F	$\cancel{\text{F}}$	T			F	$\cancel{\text{F}}$	T
F	T	F	T	T	$\cancel{\text{F}}$	T			T	$\cancel{\text{T}}$	F
F	T	F	T	F	$\cancel{\text{F}}$	T			T	$\cancel{\text{T}}$	T
F	T	F	F	T	$\cancel{\text{F}}$	T			T	$\cancel{\text{T}}$	F
F	T	F	F	F	$\cancel{\text{F}}$	T			T	$\cancel{\text{F}}$	T
$*$ F	F	T	T	T	$\cancel{\text{T}}$	T			T	$\cancel{\text{T}}$	F
F	F	T	T	F	$\cancel{\text{T}}$	T			T	$\cancel{\text{T}}$	T
F	F	T	F	T	$\cancel{\text{T}}$	T			T	$\cancel{\text{T}}$	F
F	F	T	F	F	$\cancel{\text{T}}$	T			F	$\cancel{\text{F}}$	T
F	F	F	T	T	$\cancel{\text{T}}$	F			T	$\cancel{\text{T}}$	F
F	F	F	T	F	$\cancel{\text{T}}$	F			T	$\cancel{\text{T}}$	T
F	F	F	F	T	$\cancel{\text{T}}$	F			T	$\cancel{\text{T}}$	F
F	F	F	F	F	$\cancel{\text{T}}$	F			T	$\cancel{\text{F}}$	T

Invalid. Counterexample ($*$'ed row), where M is F, C is F, W is T, T is T and R is T. [Q p.24]

3. Glossary:

M: Maisy is upset.
T: There is thunder.
L: There is lightning.

Translation:

$M \to T$
$T \to L$
$\therefore\ \neg M \vee L$

Truth Table:

M	T	L	$M \to T$	$T \to L$	$\neg M$	$\vee$	L
T	T	T	T	T	$\cancel{\text{F}}$	T	
T	T	F	T	F	$\cancel{\text{F}}$	F	
T	F	T	F	T	$\cancel{\text{F}}$	T	
T	F	F	F	T	$\cancel{\text{F}}$	F	
F	T	T	T	T	$\cancel{\text{T}}$	T	
F	T	F	T	F	$\cancel{\text{T}}$	T	
F	F	T	T	T	$\cancel{\text{T}}$	T	
F	F	F	T	T	$\cancel{\text{T}}$	T	

Valid. [Q p.24]

4. Glossary:

C: The car started.
K: You turned the key.
A: You pressed the accelerator.

Translation:

$C \to (K \land A)$
$(K \land \neg A) \to \neg C$
$\neg C$
$\therefore (A \land \neg K) \lor (\neg K \land \neg A)$

Truth Table:

C	K	A	$C \to (K \land A)$	$(K \land \neg A) \to \neg C$	$\neg C$	$(A \land \neg K) \lor (\neg K \land \neg A)$
T	T	T	T	T	F$\times$	
T	T	F	F$\times$			
T	F	T	F$\times$			
T	F	F	F$\times$			
$*$ F	T	T	T	T	T	F
F	T	F	T	T	T	F
F	F	T	T	T	T	T
F	F	F	T	T	T	T

Invalid. Counterexample ($*$'ed row), where C is F, K is T and A is T.

[Q p.24]

5. Glossary:

B: Maisy is barking.
R: There is a robber outside.
A: Maisy is asleep.
D: Maisy is depressed.

Translation:

$(\neg B \vee R)$
$(R \wedge \neg B) \rightarrow (A \vee D)$
$\neg A \wedge \neg D$
$\therefore (B \leftrightarrow R)$

Truth Table:

B	R	A	D	$\neg B \vee R$	$(R \wedge \neg B) \rightarrow (A \vee D)$	$\neg A \wedge \neg D$	$B \leftrightarrow R$
T	T	T	T	T	T	F	T
T	T	T	F	T	T	F	T
T	T	F	T	T	T	F	T
T	T	F	F	T	T	T	T
T	F	T	T	F	T	F	F
T	F	T	F	F	T	F	F
T	F	F	T	F	T	F	F
T	F	F	F	F	T	T	F
F	T	T	T	T	T	F	F
F	T	T	F	T	T	F	F
F	T	F	T	T	T	F	F
F	T	F	F	T	F	T	F
F	F	T	T	T	T	F	T
F	F	T	F	T	T	F	T
F	F	F	T	T	T	F	T
F	F	F	F	T	T	T	T

Valid. [Q p.24]

6. Glossary:

S: It is sunny.
W: It is too windy.
L: We are sailing.
F: We are having fun.

Translation:

$\neg S \rightarrow (W \vee L)$
$L \rightarrow F$
$\neg S \wedge \neg W$
$\therefore F$

Truth Table:

S	W	L	F	$\neg S$	$\rightarrow$	$(W \vee L)$	$(L \rightarrow F)$	$(\neg S$	$\wedge$	$\neg W)$
T	T	T	T	$\cancel{F}$	T	$\cancel{T}$	T		F	
T	T	T	F	$\cancel{F}$	T	$\cancel{T}$	F$\times$			
T	T	F	T	$\cancel{F}$	T	$\cancel{T}$	T		F	
T	T	F	F	$\cancel{F}$	T	$\cancel{T}$	T		F	
T	F	T	T	$\cancel{F}$	T	$\cancel{T}$	T		F	
T	F	T	F	$\cancel{F}$	T	$\cancel{T}$	F$\times$			
T	F	F	T	$\cancel{F}$	T	$\cancel{F}$	T		F	
T	F	F	F	$\cancel{F}$	T	$\cancel{F}$	T		F	
F	T	T	T	$\cancel{T}$	T	$\cancel{T}$	T		F	
F	T	T	F	$\cancel{T}$	T	$\cancel{T}$	F			
F	T	F	T	$\cancel{T}$	T	$\cancel{T}$	T		F	
F	T	F	F	$\cancel{T}$	T	$\cancel{T}$	T		F	
F	F	T	T	$\cancel{T}$	T	$\cancel{T}$	T		T	
F	F	T	F	$\cancel{T}$	T	$\cancel{T}$	F$\times$			
F	F	F	T	$\cancel{T}$	F$\times$	$\cancel{F}$				
F	F	F	F	$\cancel{T}$	F$\times$	$\cancel{F}$				

Valid. [Q p.25]

7. Glossary:

S:	You came through Singleton.
M:	You came through Maitland.
N:	You came through Newcastle.
C:	You came through Cessnock.

Translation:

$(S \wedge M) \ \vee N$
$\neg(S \vee M) \wedge C$
$\therefore (N \wedge C)$

Truth Table:

S	M	N	C	$(S \wedge M)$	$\vee$	N	$\neg$	$(S \vee M)$	$\wedge$	C	$(N \wedge C)$
T	T	T	T	$\cancel{T}$	T		$\cancel{F}$	$\cancel{T}$	F$\times$		
T	T	T	F	$\cancel{T}$	T		$\cancel{F}$	$\cancel{T}$	F$\times$		
T	T	F	T	$\cancel{T}$	T		$\cancel{F}$	$\cancel{T}$	F$\times$		
T	T	F	F	$\cancel{T}$	T		$\cancel{F}$	$\cancel{T}$	F$\times$		
T	F	T	T	$\cancel{F}$	T		$\cancel{F}$	$\cancel{T}$	F$\times$		
T	F	T	F	$\cancel{F}$	T		$\cancel{F}$	$\cancel{T}$	F$\times$		
T	F	F	T	$\cancel{F}$	F$\times$						
T	F	F	F	$\cancel{F}$	F$\times$						
F	T	T	T	$\cancel{F}$	T		$\cancel{F}$	$\cancel{T}$	F$\times$		
F	T	T	F	$\cancel{F}$	T		$\cancel{F}$	$\cancel{T}$	F$\times$		
F	T	F	T	$\cancel{F}$	F$\times$						
F	T	F	F	$\cancel{F}$	F$\times$						
F	F	T	T	$\cancel{F}$	T		$\cancel{T}$	$\cancel{F}$	T		T
F	F	T	F	$\cancel{F}$	T		$\cancel{T}$	$\cancel{F}$	F$\times$		
F	F	F	T	$\cancel{F}$	F$\times$						
F	F	F	F	$\cancel{F}$	F$\times$						

Valid. [Q p.25]

8. Glossary:

S: The shop will be open.
L: We shall have lobster for lunch.
T: It is Sunday.
R: We shall go to a restaurant.

Translation:

$S \to L$
$S \vee T$
$T \to (R \wedge L)$
$\therefore L$

Truth Table:

S	L	T	R	$S \to L$	$S \vee T$	T	$\to$	$(R \wedge L)$
T	T	T	T	T	T		T	$\cancel{\text{T}}$
T	T	T	F	T	T		F	$\cancel{\text{F}}$
T	T	F	T	T	T		T	$\cancel{\text{T}}$
T	T	F	F	T	T		T	$\cancel{\text{F}}$
T	F	T	T	F$\times$				
T	F	T	F	F$\times$				
T	F	F	T	F$\times$				
T	F	F	F	F$\times$				
F	T	T	T	T	T		T	$\cancel{\text{T}}$
F	T	T	F	T	T		F	$\cancel{\text{F}}$
F	T	F	T	T	F$\times$			
F	T	F	F	T	F$\times$			
F	F	T	T	T	T		F	$\cancel{\text{F}}$
F	F	T	F	T	T		F	$\cancel{\text{F}}$
F	F	F	T	T	F$\times$			
F	F	F	F	T	F$\times$			

Valid. [Q p.25]

9. Glossary:

C: You will catch Billy a fish.
D: You will feed Billy for a day.
T: You will teach Billy to fish.
L: You will feed Billy for life.

Translation:

$C \to D$
$T \to L$
$\therefore \neg L \lor T$

Truth Table:

	C	D	T	L	$C \to D$	$T \to L$	$\neg L$	$\lor\ T$
	T	T	T	T	T	T	$\cancel{\text{F}}$	T
	T	T	T	F	T	F$\times$		
$*$	T	T	F	T	T	T	$\cancel{\text{F}}$	F
	T	T	F	F	T	T	$\cancel{\text{T}}$	T
	T	F	T	T	F$\times$			
	T	F	T	F	F$\times$			
	T	F	F	T	F$\times$			
	T	F	F	F	F$\times$			
	F	T	T	T	T	T	$\cancel{\text{F}}$	T
	F	T	T	F	T	F$\times$		
	F	T	F	T	T	T	$\cancel{\text{F}}$	F
	F	T	F	F	T	T	$\cancel{\text{T}}$	T
	F	F	T	T	T	T	$\cancel{\text{F}}$	T
	F	F	T	F	T	F$\times$		
	F	F	F	T	T	T	$\cancel{\text{F}}$	F
	F	F	F	F	T	T	$\cancel{\text{T}}$	T

Invalid. Counterexample ($*$'ed row), where C is T, D is T and T is F and L is T. [Q p.25]

10. Glossary:

    H: I shall be happy.
    W: The Tigers will win.
    D: It will be a draw.

    Translation:

    $W \to H$
    $W \vee \neg W$
    $\neg W \to D$
    $\therefore (\neg D \wedge \neg W) \to H$

    Truth Table:

W	H	D	$W \to H$	W	$\vee$	$\neg W$	$\neg W$	$\to$	D	$(\neg D$	$\wedge$	$\neg W)$	$\to$	H
T	T	T	T		T	$\cancel{\text{F}}$	$\cancel{\text{F}}$	T		$\cancel{\text{F}}$	$\cancel{\text{F}}$	$\cancel{\text{F}}$	T	
T	T	F	T		T	$\cancel{\text{F}}$	$\cancel{\text{F}}$	T		$\cancel{\text{T}}$	$\cancel{\text{F}}$	$\cancel{\text{F}}$	T	
T	F	T	F×											
T	F	F	F×											
F	T	T	T		T	$\cancel{\text{T}}$	$\cancel{\text{T}}$	T		$\cancel{\text{F}}$	$\cancel{\text{F}}$	$\cancel{\text{T}}$	T	
F	T	F	T		T	$\cancel{\text{T}}$	$\cancel{\text{T}}$	F×						
F	F	T	T		T	$\cancel{\text{T}}$	$\cancel{\text{T}}$	T		$\cancel{\text{F}}$	$\cancel{\text{F}}$	$\cancel{\text{T}}$	T	
F	F	F	T		T	$\cancel{\text{T}}$	$\cancel{\text{T}}$	F×						

    Valid.

[Q p.25]

[Contents]

Answers 6.6.3

1. (i) Functionally complete:

α	β	$\neg(\alpha \to \neg\beta)$	$(\neg\alpha \to \beta)$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	F

The second last column is the same as the truth table for $\alpha \wedge \beta$, and the last column is the same as the truth table for $\alpha \vee \beta$, so we have defined $\wedge$ and $\vee$ in terms of $\neg$ and $\to$. We already know

that $\{\wedge, \vee, \neg\}$ is functionally complete, so this establishes that $\{\neg, \rightarrow\}$ is functionally complete.

(Where do the columns come from? That is, it is easy to see, once the formula $\neg(\alpha \rightarrow \neg\beta)$ is given, that this formula is equivalent to $(\alpha \wedge \beta)$—and similarly for $(\neg\alpha \rightarrow \beta)$ and $(\alpha \vee \beta)$—but where do these formulas come from in the first place? The answer is: they come by trial and error, guided by knowledge of the relevant truth tables. We know what truth table we want to end up with—say, the truth table for $(\alpha \wedge \beta)$—and we know what connectives we are allowed to use—in this case $\neg$ and $\rightarrow$—and what their truth tables are; we then play around with formulas involving different combinations of the allowed connectives until we find one that has the desired truth table.)

[A p.25]

(ii) Not functionally complete: we cannot define any connective which has an odd number of T's in its truth table; e.g. $\rightarrow$ (one T), $\vee$ (three T's) or $\wedge$ (one T).

Consider the truth table for $\alpha \leftrightarrow \beta$. (We make no assumptions about how complex α and β are—i.e. about how many connectives and basic propositions they contain—hence no assumptions about how many rows there are in this truth table.) $\alpha \leftrightarrow \beta$ is T iff α and β have the *same* truth value (both T or both F); $\alpha \leftrightarrow \beta$ is F iff α and β have *opposite* truth values (one T and the other F). Let us call a row in which $\alpha \leftrightarrow \beta$ is true a 'T row' and a row in which it is false an 'F row', and let us say that a T is worth 1 point and an F is worth 0 points (this has no deep significance: it simply allows the following discussion to be presented in a simple way). If we sum the number of points in the α and β columns (combined), each F row contributes 1 point, and each T row contributes either 2 points or 0 points. Now suppose there is an *odd* number of T rows (and hence an odd number of F rows, as there is an even number of rows in total in any truth table). Then the total number of points in the α and β columns (combined) is an odd number (the number of F rows) plus some 2's and 0's—i.e. an odd number. That means that either α is true in an odd number of rows and β is true in an even number of rows, or vice versa. Either way, it follows that $\alpha \leftrightarrow \beta$ is true in an odd number of rows iff one of α and β is true in an odd number of rows.

Consider the truth table for $\alpha \veebar \beta$. $\alpha \veebar \beta$ is T iff α and β have *op-*

posite truth values (one T and the other F); $\alpha \veebar \beta$ is F iff α and β have the *same* truth value (both T or both F). If we sum the number of points in the α and β columns (combined), each T row contributes 1 point, and each F row contributes either 2 points or 0 points. Now suppose there is an *odd* number of T rows (and hence an odd number of F rows). Then the total number of points in the α and β columns (combined) is an odd number (the number of T rows) plus some 2's and 0's—i.e. an odd number. That means that either α is true in an odd number of rows and β is true in an even number of rows, or vice versa. Either way, it follows that $\alpha \veebar \beta$ is true in an odd number of rows iff one of α and β is true in an odd number of rows.

Now think about formulas that we can define using only $\leftrightarrow$, $\veebar$ and basic propositions. Each basic proposition is true in an even number of rows (half the rows in the table: recall how the matrix is laid out); and as we have just seen, any proposition built from $\leftrightarrow$ or $\veebar$ and propositions which are true in an even number of rows, is itself true in an even number of rows. So *every* proposition that we can define using only $\leftrightarrow$, $\veebar$ and basic propositions has an even number of T's in its truth table.

[Q p.25]

(iii) Functionally complete:

α	β	$\alpha \downarrow \alpha$	$(\alpha \downarrow \alpha) \downarrow (\beta \downarrow \beta)$	$(\alpha \downarrow \beta) \downarrow (\alpha \downarrow \beta)$
T	T	F	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	F

The third last column is the same as the truth table for $\neg\alpha$, the second last column is the same as the truth table for $\alpha \wedge \beta$, and the last column is the same as the truth table for $\alpha \vee \beta$, so we have defined $\neg$, $\wedge$ and $\vee$ in terms of $\downarrow$. We already know that $\{\wedge, \vee, \neg\}$ is functionally complete, so this establishes that $\{\downarrow\}$ is functionally complete. [Q p.25]

(iv) Not functionally complete: we cannot define any connective which has an F in the top row. When α and β are both T, $(\alpha \rightarrow \beta)$ is T and $(\alpha \wedge \beta)$ is T—hence so is *any* formula, however complex, built up from α's, β's, $\rightarrow$'s and $\wedge$'s. Hence no such formula is equivalent to $\neg\alpha$, which is F when α is T. [Q p.25]

(v) Functionally complete:

α	β	$\neg(\neg\alpha \textcircled{2}_{12} \beta)$	$\alpha \textcircled{2}_{12} \neg\beta$
T	T	T	T
T	F	T	F
F	T	T	F
F	F	F	F

The second last column is the same as the truth table for $\alpha \vee \beta$, and the last column is the same as the truth table for $\alpha \wedge \beta$, so we have defined $\wedge$ and $\vee$ in terms of $\neg$ and $\textcircled{2}_{12}$. We already know that $\{\wedge, \vee, \neg\}$ is functionally complete, so this establishes that $\{\neg, \textcircled{2}_{12}\}$ is functionally complete. [Q p.25]

(vi) Not functionally complete: $(\alpha \textcircled{2}_4 \beta)$ and $(\alpha \textcircled{2}_4 \alpha)$ are both equivalent to α, and $(\beta \textcircled{2}_4 \beta)$ and $(\beta \textcircled{2}_4 \alpha)$ are both equivalent to β, so we cannot express anything more with α's, β's, $\vee$'s and $\textcircled{2}_4$'s than we can with just α's, β's and $\vee$'s. But $\{\vee\}$ is not a functionally complete set of connectives (why not?), hence neither is $\{\vee, \textcircled{2}_4\}$. [Q p.25]

2. (i) [Q p.25]

A	B	$(B \textcircled{2}_{14} A)$
T	T	F
T	F	T
F	T	F
F	F	F

(ii) [Q p.26]

A	B	$((A \textcircled{2}_{11} B)$	$\textcircled{2}_{15}\ B)$
T	T	~~F~~	F
T	F	~~T~~	F
F	T	~~F~~	F
F	F	~~T~~	F

(iii) [Q p.26]

A	B	$\neg$	$(A$	$\vee$	$(A \textcircled{2}_6 B))$
T	T	F		~~T~~	~~T~~
T	F	F		~~T~~	~~F~~
F	T	F		~~T~~	~~T~~
F	F	T		~~F~~	~~F~~

(iv) [Q p.26]

A	B	A	$\leftrightarrow$	$(A$	$\textcircled{2}_3$	$\neg B)$
T	T		T		~~T~~	~~F~~
T	F		T		~~T~~	~~T~~
F	T		F		~~T~~	~~F~~
F	F		T		~~F~~	~~T~~

(v) [Q p.26]

A	B	$(A \textcircled{2}_{12} B)$	$\veebar$	$(B \textcircled{2}_{12} A)$
T	T	$\cancel{\text{F}}$	F	$\cancel{\text{F}}$
T	F	$\cancel{\text{T}}$	T	$\cancel{\text{F}}$
F	T	$\cancel{\text{F}}$	T	$\cancel{\text{T}}$
F	F	$\cancel{\text{F}}$	F	$\cancel{\text{F}}$

(vi) [Q p.26]

A	B	$(A \textcircled{2}_{12} B)$	$\veebar$	$(B \textcircled{2}_{16} A)$
T	T	$\cancel{\text{F}}$	F	$\cancel{\text{F}}$
T	F	$\cancel{\text{T}}$	T	$\cancel{\text{F}}$
F	T	$\cancel{\text{F}}$	F	$\cancel{\text{F}}$
F	F	$\cancel{\text{F}}$	F	$\cancel{\text{F}}$

3. Here are two ways to answer each part.

   First, we can form a disjunction of row descriptions, in the way explained in §6.6.2, pp.129–131. This gives:

   (i) $(\alpha \wedge \beta \wedge \gamma) \vee (\alpha \wedge \neg\beta \wedge \gamma) \vee (\alpha \wedge \neg\beta \wedge \neg\gamma) \vee (\neg\alpha \wedge \beta \wedge \gamma) \vee (\neg\alpha \wedge \neg\beta \wedge \gamma) \vee (\neg\alpha \wedge \neg\beta \wedge \neg\gamma)$ [Q p.26]

   (ii) $(\alpha \wedge \neg\beta \wedge \gamma) \vee (\alpha \wedge \neg\beta \wedge \neg\gamma) \vee (\neg\alpha \wedge \beta \wedge \gamma) \vee (\neg\alpha \wedge \beta \wedge \neg\gamma)$ [Q p.26]

   Alternatively, we can use the following hints, to get:

   (i) $(\neg\beta \vee \gamma)$ or $\neg(\beta \wedge \neg\gamma)$ [Q p.26]
   (Hint: Look at the truth table for $\sharp(\alpha, \beta, \gamma)$, and note that it is equivalent to $(\beta \rightarrow \gamma)$.)

   (ii) $(\alpha \vee \beta) \wedge \neg(\alpha \wedge \beta)$ [Q p.26]
   (Hint: Look at the truth table for $\natural(\alpha, \beta, \gamma)$, and note that it is equivalent to $(\alpha \veebar \beta)$.)

   There are also other correct answers to (i) and (ii).

4. (i) $(A \wedge \neg B)$ [Q p.26]
   (ii) $(\neg A \wedge \neg B)$ [Q p.26]
   (iii) $\neg(A \wedge B)$ [Q p.26]
   (iv) $A \wedge \neg B$ [Q p.26]
   (v) $\neg(A \wedge \neg B) \wedge \neg(B \wedge \neg A)$ [Q p.26]
   (vi) $\neg(A \wedge \neg A)$ [Q p.26]

5. (i) $\textcircled{1}_4$ [Q p.26]

(ii) ②$_{14}$ [Q p.26]

(iii) ①$_2$ and $\neg$ [Q p.26]

(iv) ②$_4$, ②$_6$, ②$_{11}$ and ②$_{13}$ [Q p.26]

[Contents]

Chapter 7

Trees for Propositional Logic

Answers 7.2.1.1

1. $$(\neg A \vee \neg B) \checkmark$$
$$\neg A \qquad \neg B$$

[Q p.27]

2. $$(\neg A \rightarrow B) \checkmark$$
$$\neg\neg A \qquad B$$

[Q p.27]

3. $$(A \rightarrow B) \wedge B \checkmark$$
$$(A \rightarrow B)$$
$$B$$

[Q p.27]

4. $$((A \leftrightarrow B) \leftrightarrow B) \checkmark$$
$$(A \leftrightarrow B) \qquad \neg(A \leftrightarrow B)$$
$$B \qquad \neg B$$

[Q p.27]

5. $$\neg(A \leftrightarrow \neg\neg A) \checkmark$$
$$A \qquad \neg A$$
$$\neg\neg\neg A \qquad \neg\neg A$$

[Q p.27]

6.
$$\begin{array}{c} \neg(\neg A \vee B)\ \checkmark \\ \neg\neg A \\ \neg B \end{array}$$

[Q p.27]

[Contents]

Answers 7.2.2.1

1.
$$\begin{array}{cc} \multicolumn{2}{c}{(A \to B) \to B\ \checkmark} \\ \neg(A \to B)\ \checkmark & B \\ A & \\ \neg B & \end{array}$$

[Q p.27]

2.
$$\begin{array}{cccc} \multicolumn{4}{c}{(A \to B) \vee (B \to A)\ \checkmark} \\ \multicolumn{2}{c}{(A \to B)\ \checkmark} & \multicolumn{2}{c}{(B \to A)\ \checkmark} \\ \neg A & B & \neg B & A \end{array}$$

[Q p.27]

3.
$$\begin{array}{c} \neg(\neg A \to (A \vee B))\ \checkmark \\ \neg A \\ \neg(A \vee B)\ \checkmark \\ \neg A \\ \neg B \end{array}$$

[Q p.27]

4.
$$\begin{array}{cc} \multicolumn{2}{c}{\neg\neg((A \wedge B) \vee (A \wedge \neg B))\ \checkmark} \\ \multicolumn{2}{c}{((A \wedge B) \vee (A \wedge \neg B))\ \checkmark} \\ (A \wedge B)\ \checkmark & (A \wedge \neg B)\ \checkmark \\ A & A \\ B & \neg B \end{array}$$

[Q p.27]

[Contents]

Answers 7.2.3.1

1.
$$\begin{array}{c}
\neg(A \to (B \to A)) \checkmark \\
A \\
\neg(B \to A) \checkmark \\
B \\
\neg A \\
\times
\end{array}$$

[Q p.28]

2.
$$\begin{array}{cc}
\multicolumn{2}{c}{(A \to B) \vee (\neg A \vee B) \checkmark} \\
(A \to B) \checkmark & (\neg A \vee B) \checkmark \\
\begin{array}{cc} \neg A & B \end{array} & \begin{array}{cc} \neg A & B \end{array}
\end{array}$$

[Q p.28]

3.
$$\begin{array}{c}
\neg((A \to B) \vee (\neg A \vee B)) \checkmark \\
\neg(A \to B) \checkmark \\
\neg(\neg A \vee B) \checkmark \\
A \\
\neg B \\
\neg\neg A \checkmark \\
\neg B \\
A
\end{array}$$

[Q p.28]

4.
$$\begin{array}{c}
\neg\neg\neg(A \vee B) \checkmark \\
\neg(A \vee B) \checkmark \\
\neg A \\
\neg B
\end{array}$$

[Q p.28]

5.
$$\begin{array}{cc}
\multicolumn{2}{c}{\neg(A \wedge \neg A) \checkmark} \\
\neg A & \neg\neg A \checkmark \\
 & A
\end{array}$$

[Q p.28]

6. $\neg(\neg(A \wedge B) \leftrightarrow (\neg A \vee \neg B))$ ✓

$\neg(A \wedge B)$ ✓	$\neg\neg(A \wedge B)$ ✓
$\neg(\neg A \vee \neg B)$ ✓	$(\neg A \vee \neg B)$ ✓
$\neg\neg A$ ✓	$(A \wedge B)$ ✓
$\neg\neg B$ ✓	A
A	B
B	$\neg A$ $\neg B$
$\neg A$ $\neg B$	$\times$ $\times$
$\times$ $\times$	

[Q p.28]

[Contents]

Answers 7.3.1.1

1. Valid. [Q p.28]

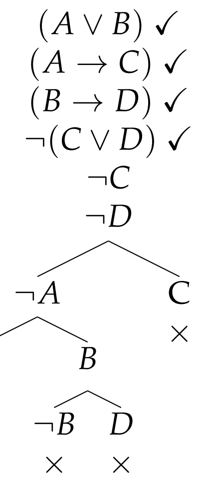
$$A$$
$$\neg(A \vee B) \checkmark$$
$$\neg A$$
$$\neg B$$
$$\times$$

2. Invalid. Counterexample: A is T, B is F. [Q p.28]

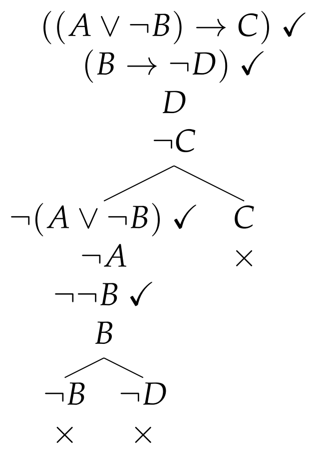
$$(A \vee B) \checkmark$$
$$\neg B$$

A	B
$\uparrow$	$\times$

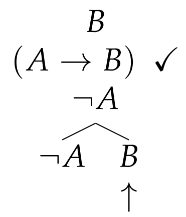
3. Valid. [Q p.28]



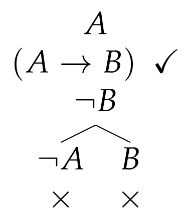
4. Valid. [Q p.28]



5. Invalid. Counterexample: A is F, B is T. [Q p.28]



6. Valid. [Q p.29]



7. Valid. [Q p.29]

$(A \lor (B \land C))$ ✓
$(A \to B)$ ✓
$(B \leftrightarrow D)$ ✓
$\neg(B \land D)$ ✓

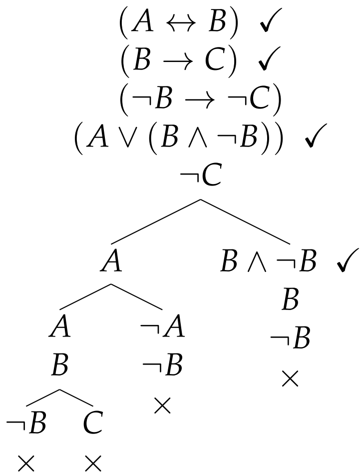
- B
  D
  - $\neg B$
    ×
  - $\neg D$
    ×
- $\neg B$
  $\neg D$
  - $\neg B$
    - $\neg A$
      - A
        ×
      - $B \land C$ ✓
        B
        C
        ×
    - B
      ×
  - $\neg D$
    - $\neg A$
      - A
        ×
      - $B \land C$ ✓
        B
        C
        ×
    - B
      ×

8. Invalid. Counterexample: A is F, B is F, C is T. [Q p.29]

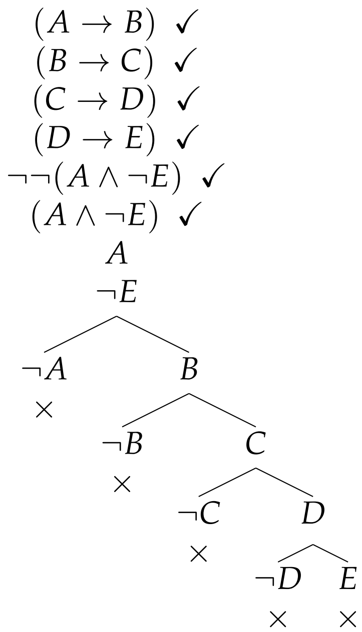
$\neg(\neg A \to B)$ ✓
$\neg(C \leftrightarrow A)$ ✓
$(A \lor C)$ ✓
$\neg(C \to B)$ ✓
$\neg\neg(A \to B)$ ✓
$A \to B$ ✓
$\neg A$
$\neg B$
C
$\neg B$

- A
  ×
- C
  - C
    $\neg A$
    - $\neg A$
      ↑
    - B
      ×
  - $\neg C$
    A
    ×

9. Valid. [Q p.29]



10. Valid. [Q p.29]



[Contents]

Answers 7.3.2.1

1.

(i) Contradiction. [Q p.29]

$$\begin{array}{c} (A \wedge \neg A) \checkmark \\ A \\ \neg A \\ \times \end{array}$$

(ii) Contradiction. [Q p.29]

$$\begin{array}{c} (A \vee B) \wedge \neg(A \vee B) \checkmark \\ (A \vee B) \checkmark \\ \neg(A \vee B) \checkmark \\ \neg A \\ \neg B \\ \begin{array}{cc} A & B \\ \times & \times \end{array} \end{array}$$

(iii) Satisfiable. True when A is F and B is F. [Q p.29]

$$\begin{array}{c} (A \rightarrow B) \wedge \neg(A \vee B) \checkmark \\ (A \rightarrow B) \checkmark \\ \neg(A \vee B) \checkmark \\ \neg A \\ \neg B \\ \begin{array}{cc} \neg A & B \\ \uparrow & \times \end{array} \end{array}$$

(iv) Contradiction. [Q p.29]

$$\begin{array}{c} (A \rightarrow \neg(A \vee B)) \wedge \neg(\neg(A \vee B) \vee B) \ \checkmark \\ (A \rightarrow \neg(A \vee B)) \ \checkmark \\ \neg(\neg(A \vee B) \vee B) \ \checkmark \\ \neg\neg(A \vee B) \ \checkmark \\ \neg B \\ (A \vee B) \ \checkmark \\ \begin{array}{cc} \neg A & \neg(A \vee B) \\ \begin{array}{cc} A & B \\ \times & \times \end{array} & \times \end{array} \end{array}$$

(v) Contradiction. [Q p.29]

$$\neg((\neg B \vee C) \leftrightarrow (B \to C)) \ \checkmark$$

$$\begin{array}{cc}
\neg B \vee C \ \checkmark & \neg(\neg B \vee C) \ \checkmark \\
\neg(B \to C) \ \checkmark & B \to C \ \checkmark \\
B & \neg\neg B \ \checkmark \\
\neg C & \neg C \\
\begin{array}{cc} \neg B & C \\ \times & \times \end{array} & B \\
 & \begin{array}{cc} \neg B & C \\ \times & \times \end{array}
\end{array}$$

(vi) Satisfiable. True when A is F, B is F and C is F. [Q p.29]

$$(A \leftrightarrow \neg A) \vee (A \to \neg(B \vee C)) \ \checkmark$$

$$\begin{array}{cc}
(A \leftrightarrow \neg A) \ \checkmark & A \to \neg(B \vee C) \ \checkmark \\
\begin{array}{cc} A & \neg A \\ \neg A & \neg\neg A \ \checkmark \\ \times & A \\ & \times \end{array} & \begin{array}{cc} \neg A & \neg(B \vee C) \ \checkmark \\ & \neg B \\ & \neg C \\ & \uparrow \end{array}
\end{array}$$

2.

(i) Unsatisfiable. [Q p.30]

$$\begin{array}{c}
(A \vee B) \ \checkmark \\
\neg B \\
(A \to B) \ \checkmark \\
\begin{array}{cc} \mathrm{A} & B \\ \begin{array}{cc} \neg A & B \\ \times & \times \end{array} & \times \end{array}
\end{array}$$

(ii) Satisfiable. All true when A is F, B is T and C is F. [Q p.30]

$$\begin{array}{c}
(A \vee B) \ \checkmark \\
(B \vee C) \ \checkmark \\
\neg(A \vee C) \ \checkmark \\
\neg A \\
\neg C \\
\begin{array}{cc} A & B \\ \times & \begin{array}{cc} B & C \\ \uparrow & \times \end{array} \end{array}
\end{array}$$

(iii) Satisfiable. All true when A is F, B is F and C is T. [Q p.30]

$$
\begin{array}{c}
\neg(\neg A \rightarrow B)\ \checkmark \\
\neg(C \leftrightarrow A)\ \checkmark \\
(A \vee C)\ \checkmark \\
\neg(C \rightarrow B)\ \checkmark \\
(A \rightarrow B)\ \checkmark \\
\neg A \\
\neg B \\
C \\
\neg B
\end{array}
$$

- A, $\times$
- C
  - $\neg A$
    - C, $\neg A$, $\uparrow$
    - $\neg C$, A, $\times$
  - B, $\times$

(iv) Satisfiable. All true when A is T, B is T and C is F. [Q p.30]

$$
\begin{array}{c}
A \leftrightarrow B\ \checkmark \\
\neg(A \rightarrow C)\ \checkmark \\
(C \rightarrow A)\ \checkmark \\
(A \wedge B) \vee (A \wedge C)\ \checkmark \\
A \\
\neg C
\end{array}
$$

- A, B
  - $\neg C$
    - $(A \wedge B)\ \checkmark$, A, B, $\uparrow$
    - $(A \wedge C)\ \checkmark$, A, C, $\times$
  - A
    - $(A \wedge B)\ \checkmark$, A, B
    - $(A \wedge C)\ \checkmark$, A, C, $\times$
- $\neg A$, $\neg B$, $\times$

[Contents]

Answers 7.3.3.1

1. Can both be true, e.g. when A is T and B is F:

$$\begin{array}{c} (\neg A \to B)\ \checkmark \\ (B \to A)\ \checkmark \\ \begin{array}{cc} \begin{array}{c} \neg\neg A\ \checkmark \\ A \\ \begin{array}{cc} \neg B & A \\ \uparrow & \end{array} \end{array} & \begin{array}{c} B \\ \begin{array}{cc} \neg B & A \\ \times & \end{array} \end{array} \end{array} \end{array}$$

Therefore, jointly satisfiable. [Q p.30]

2. Cannot both be true:

$$\begin{array}{c} (A \to B)\ \checkmark \\ \neg(A \to (A \to B))\ \checkmark \\ A \\ \neg(A \to B)\ \checkmark \\ A \\ \neg B \\ \begin{array}{cc} \neg A & B \\ \times & \times \end{array} \end{array}$$

Cannot both be false:

$$\begin{array}{c} \neg(A \to B)\ \checkmark \\ \neg\neg(A \to (A \to B))\ \checkmark \\ (A \to (A \to B))\ \checkmark \\ A \\ \neg B \\ \begin{array}{cc} \begin{array}{c} \neg A \\ \times \end{array} & \begin{array}{c} (A \to B)\ \checkmark \\ \begin{array}{cc} \neg A & B \\ \times & \times \end{array} \end{array} \end{array} \end{array}$$

Therefore, contradictories. [Q p.30]

3. Cannot both be true:

$$\begin{array}{c}
\neg(A \leftrightarrow \neg B) \checkmark \\
\neg(A \lor \neg B) \checkmark \\
\neg A \\
\neg\neg B \checkmark \\
B \\
\begin{array}{cc}
A & \neg A \\
\neg\neg B & \neg B \\
\times & \times
\end{array}
\end{array}$$

Can both be false, e.g. when A is T and B is F:

$$\begin{array}{c}
\neg\neg(A \leftrightarrow \neg B) \checkmark \\
\neg\neg(A \lor \neg B) \checkmark \\
(A \leftrightarrow \neg B) \checkmark \\
(A \lor \neg B) \checkmark \\
\begin{array}{cc}
\begin{array}{c}
A \\
\neg B \\
\begin{array}{cc}
A & \neg B \\
\uparrow &
\end{array}
\end{array}
&
\begin{array}{c}
\neg A \\
\neg\neg B \checkmark \\
B \\
\begin{array}{cc}
A & \neg B \\
\times & \times
\end{array}
\end{array}
\end{array}
\end{array}$$

Therefore, contraries. [Q p.30]

4. Cannot both be true:

$$
\begin{array}{c}
\neg(A \vee \neg B)\ \checkmark \\
(\neg A \rightarrow \neg B)\ \checkmark \\
\neg A \\
\neg\neg B\ \checkmark \\
B \\
\begin{array}{cc}
\neg\neg A\ \checkmark & \neg B \\
A & \times \\
\times &
\end{array}
\end{array}
$$

Cannot both be false:

$$
\begin{array}{c}
\neg\neg(A \vee \neg B)\ \checkmark \\
\neg(\neg A \rightarrow \neg B)\ \checkmark \\
(A \vee \neg B)\ \checkmark \\
\neg A \\
\neg\neg B\ \checkmark \\
B \\
\begin{array}{cc}
A & \neg B \\
\times & \times
\end{array}
\end{array}
$$

Therefore, contradictories. [Q p.30]

5. Cannot both be true:

$$\begin{array}{c}
(\neg A \wedge (A \to B)) \ \checkmark \\
\neg(\neg A \to (A \to B)) \ \checkmark \\
\neg A \\
(A \to B) \ \checkmark \\
\neg A \\
\neg(A \to B) \ \checkmark \\
A \\
\neg B \\
\times
\end{array}$$

Can both be false, e.g. when A is T and B is T:

$$\begin{array}{c}
\neg(\neg A \wedge (A \to B)) \ \checkmark \\
\neg\neg(\neg A \to (A \to B)) \ \checkmark \\
(\neg A \to (A \to B)) \ \checkmark \\
\begin{array}{cc}
\begin{array}{c}
\neg\neg A \ \checkmark \\
A \\
\begin{array}{cc}
\begin{array}{c} \neg\neg A \ \checkmark \\ A \end{array} &
\begin{array}{c} (A \to B) \ \checkmark \\ \begin{array}{cc} \neg A & B \\ \times & \uparrow \end{array} \end{array}
\end{array}
\end{array} &
\begin{array}{c}
\neg(A \to B) \ \checkmark \\
A \\
\neg B \\
\begin{array}{cc}
\begin{array}{c} \neg\neg A \ \checkmark \\ A \end{array} &
\begin{array}{c} (A \to B) \ \checkmark \\ \begin{array}{cc} \neg A & B \\ \times & \times \end{array} \end{array}
\end{array}
\end{array}
\end{array}
\end{array}$$

Therefore, contraries. [Q p.30]

6. Can both be true, e.g. when A is T and B is F:

$$\begin{array}{c}
(A \to B) \leftrightarrow B \ \checkmark \\
\neg(A \to B) \ \checkmark \\
A \\
\neg B \\
\begin{array}{cc}
\begin{array}{c} (A \to B) \ \checkmark \\ B \\ \times \end{array} &
\begin{array}{c} \neg(A \to B) \ \checkmark \\ \neg B \\ A \\ \neg B \\ \uparrow \end{array}
\end{array}
\end{array}$$

Therefore, jointly satisfiable. [Q p.30]

[Contents]

Answers 7.3.4.1

1. Tautology: [Q p.30]

$$\begin{array}{c} \neg(A \to (B \to A)) \checkmark \\ A \\ \neg(B \to A) \checkmark \\ B \\ \neg A \\ \times \end{array}$$

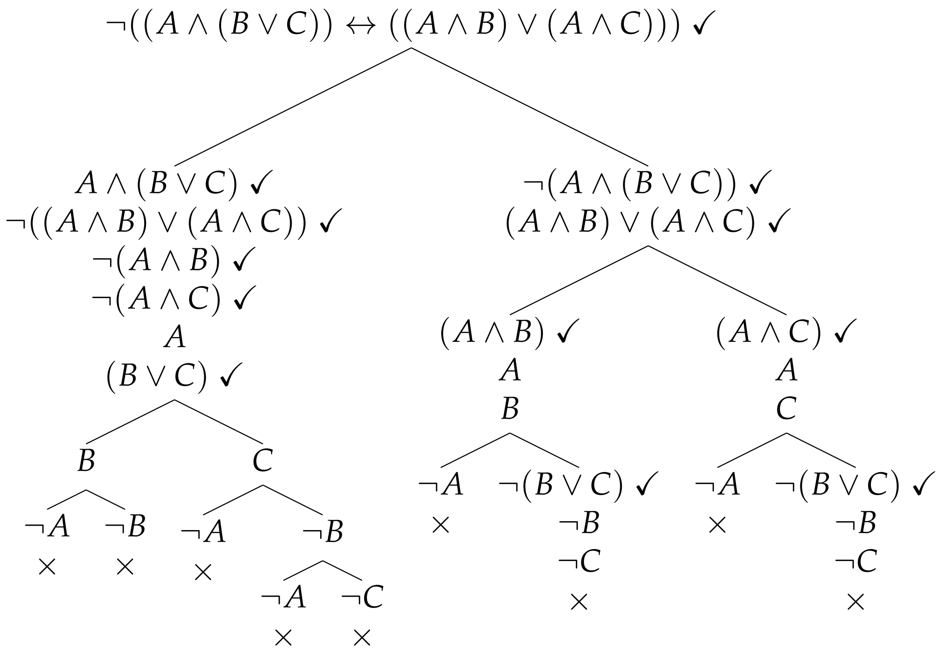
2. Not a tautology. False when A is T and B is F: [Q p.30]

$$\begin{array}{c} \neg(A \to (A \to B)) \checkmark \\ A \\ \neg(A \to B) \checkmark \\ A \\ \neg B \\ \uparrow \end{array}$$

3. Tautology: [Q p.31]

$$\begin{array}{c} \neg(((A \land B) \lor \neg(A \to B)) \to (C \to A)) \checkmark \\ ((A \land B) \lor \neg(A \to B)) \checkmark \\ \neg(C \to A) \checkmark \\ C \\ \neg A \\ \begin{array}{cc} (A \land B) \checkmark & \neg(A \to B) \checkmark \\ A & A \\ B & \neg B \\ \times & \times \end{array} \end{array}$$

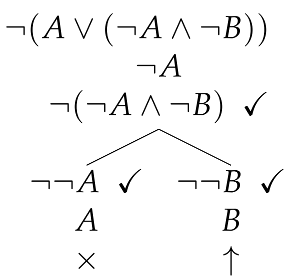
4. Tautology: [Q p.31]



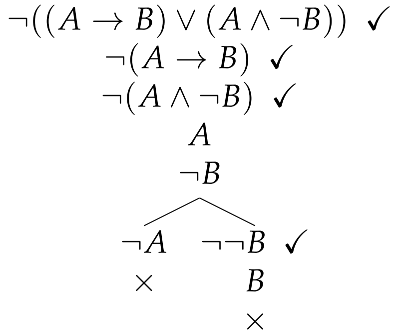
5. Not a tautology. False when A is T and B is T: [Q p.31]

$$
\begin{array}{c}
\neg(\neg A \vee \neg(A \wedge B)) \checkmark \\
\neg\neg A \checkmark \\
\neg\neg(A \wedge B) \checkmark \\
A \\
(A \wedge B) \checkmark \\
A \\
B \\
\uparrow
\end{array}
$$

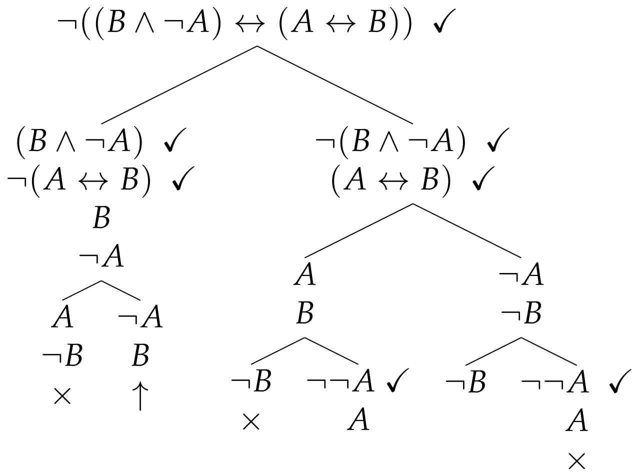
6. Not a tautology. False when A is F and B is T: [Q p.31]



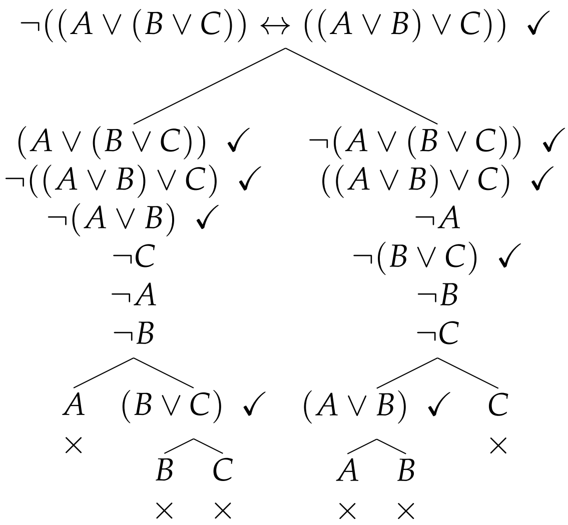
7. Tautology. [Q p.31]



8. Not a tautology. False when A is F and B is T: [Q p.31]



9. Tautology. [Q p.31]



10. Not a tautology. False when, e.g., A is T, B is T and C is F: [Q p.31]

$$
\begin{array}{c}
\neg((A \wedge (B \vee C)) \leftrightarrow ((A \vee B) \wedge C)) \checkmark \\
\begin{array}{cc}
\begin{array}{c}
(A \wedge (B \vee C)) \checkmark \\
\neg((A \vee B) \wedge C) \checkmark \\
A \\
(B \vee C) \checkmark \\
\begin{array}{cc}
\begin{array}{c} \neg(A \vee B) \checkmark \\ \neg A \\ \neg B \\ \times \end{array} &
\begin{array}{c} \neg C \\ \begin{array}{cc} B & C \\ \uparrow & \times \end{array} \end{array}
\end{array}
\end{array}
&
\begin{array}{c}
\neg(A \wedge (B \vee C)) \checkmark \\
((A \vee B) \wedge C) \checkmark \\
(A \vee B) \checkmark \\
C \\
\begin{array}{cc}
\begin{array}{c} \neg A \\ \begin{array}{cc} A & B \\ \times & \end{array} \end{array} &
\begin{array}{c} \neg(B \vee C) \checkmark \\ \neg B \\ \neg C \\ \times \end{array}
\end{array}
\end{array}
\end{array}
\end{array}
$$

[Contents]

Answers 7.3.5.1

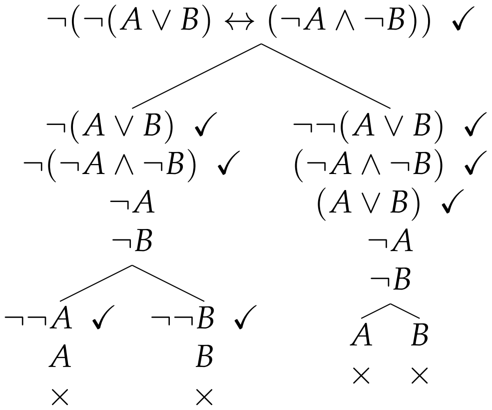
1. Equivalent: [Q p.31]

$$
\begin{array}{c}
\neg(P \leftrightarrow (P \wedge P)) \checkmark \\
\begin{array}{cc}
\begin{array}{c} \neg P \\ (P \wedge P) \checkmark \\ P \\ P \\ \times \end{array} &
\begin{array}{c} P \\ \neg(P \wedge P) \checkmark \\ \begin{array}{cc} \neg P & \neg P \\ \times & \times \end{array} \end{array}
\end{array}
\end{array}
$$

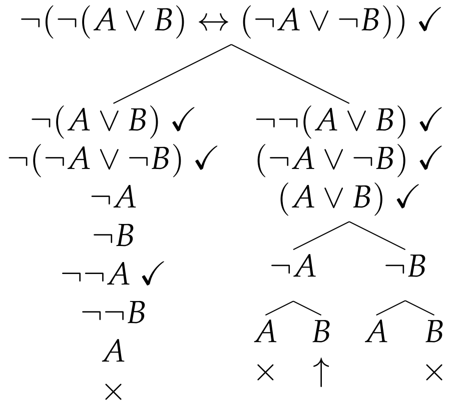
2. Equivalent: [Q p.31]

$$
\begin{array}{c}
\neg((P \rightarrow (Q \vee \neg Q)) \leftrightarrow (R \rightarrow R)) \checkmark \\
\begin{array}{cc}
\begin{array}{c} (P \rightarrow (Q \vee \neg Q)) \\ \neg(R \rightarrow R) \checkmark \\ R \\ \neg R \\ \times \end{array} &
\begin{array}{c} \neg(P \rightarrow (Q \vee \neg Q)) \checkmark \\ (R \rightarrow R) \\ P \\ \neg(Q \vee \neg Q) \checkmark \\ \neg Q \\ \neg\neg Q \checkmark \\ Q \\ \times \end{array}
\end{array}
\end{array}
$$

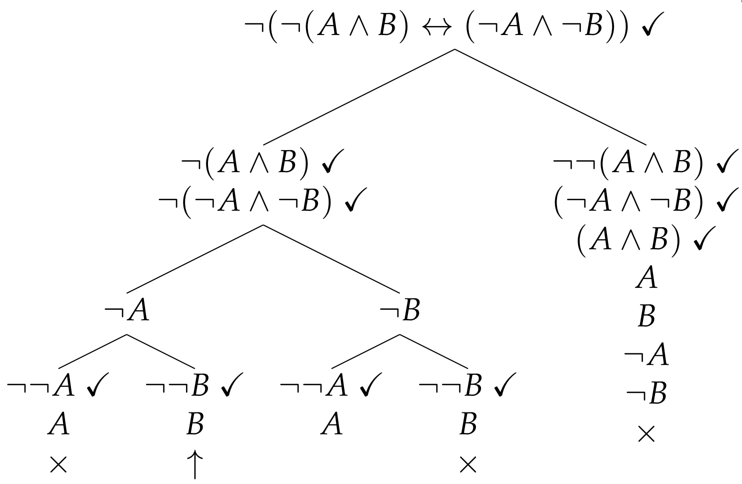
3. Equivalent: [Q p.31]



4. Not equivalent. Different truth values when, e.g., A is F and B is T: [Q p.31]



5. Not equivalent. Different truth values when, e.g., A is F and B is T: [Q p.31]



6. Equivalent. [Q p.31]

$\neg(\neg(A \wedge B) \leftrightarrow (\neg A \vee \neg B))$ ✓

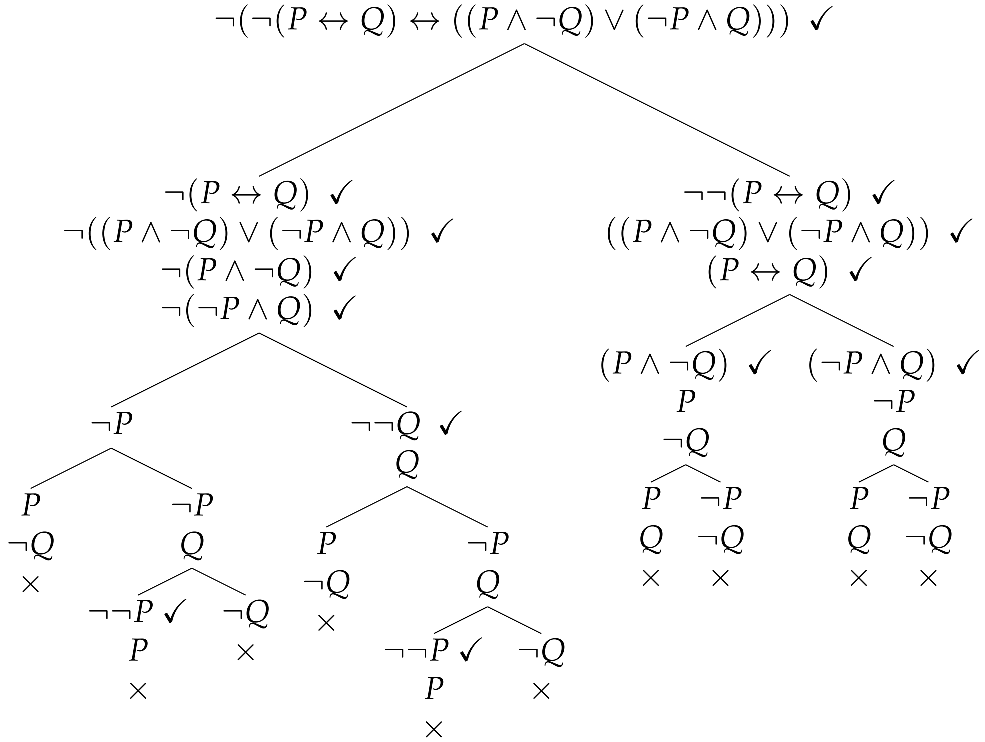
- $\neg(A \wedge B)$ ✓
  $\neg(\neg A \vee \neg B)$ ✓
  $\neg\neg A$ ✓
  $\neg\neg B$ ✓
  A
  B
  - $\neg A$
    $\times$
  - $\neg B$
    $\times$
- $\neg\neg(A \wedge B)$ ✓
  $(\neg A \vee \neg B)$ ✓
  $(A \wedge B)$ ✓
  A
  B
  - $\neg A$
    $\times$
  - $\neg B$
    $\times$

7. Equivalent. [Q p.31]

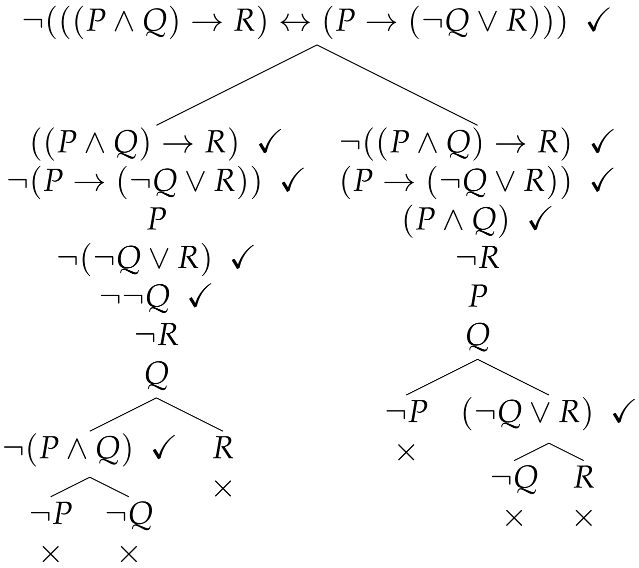
$\neg(A \leftrightarrow ((A \wedge B) \vee (A \wedge \neg B)))$ ✓

- A
  $\neg((A \wedge B) \vee (A \wedge \neg B))$ ✓
  $\neg(A \wedge B)$ ✓
  $\neg(A \wedge \neg B)$ ✓
  - $\neg A$
    $\times$
  - $\neg B$
    - $\neg A$
      $\times$
    - $\neg\neg B$ ✓
      B
      $\times$
- $\neg A$
  $((A \wedge B) \vee (A \wedge \neg B))$ ✓
  - $(A \wedge B)$ ✓
    A
    B
    $\times$
  - $(A \wedge \neg B)$ ✓
    A
    $\neg B$
    $\times$

8. Equivalent. [Q p.31]

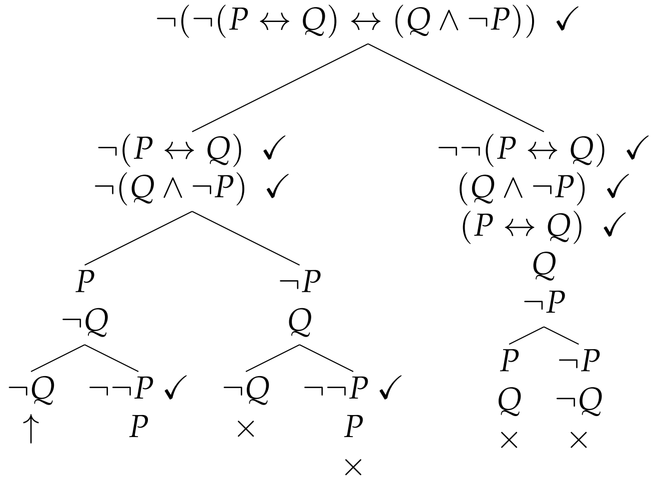


9. Equivalent. [Q p.31]



10. Not equivalent. Different truth values when P is T and Q is F:

[Q p.31]



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Chapter 8

The Language of Monadic Predicate Logic

Answers 8.2.1

Glossary:

b:	Bill	B:	is beautiful
c:	Canberra	C:	is a capital city
d:	Ben	E:	is even
e:	Jenny	G:	is a gardener
j:	John	H:	is happy
l:	Pluto	K:	is kite flying
m:	Mary	N:	is nice
n:	New York	P:	is a prime number
p:	The Pacific Ocean	R:	is grumpy
q:	seven (7)	S:	is small
r:	Rover	T:	is a planet
s:	Steve	V:	is heavily populated
t:	two (2)	W:	is winning
A:	is sailing	Y:	is tiny

Translations:

1. Bp [Q p.32]

2. Vn [Q p.32]

3. Nm [Q p.32]

4. Rj [Q p.32]
5. Pq [Q p.32]
6. Tl [Q p.32]
7. $Gb \wedge Gd$ [Q p.32]
8. $(Am \vee Ke) \rightarrow (Rb \wedge Rd)$ [Q p.32]
9. $\neg(Am \vee Km)$ [Q p.32]
10. $Ke \rightarrow Am$ [Q p.32]
11. $(Aj \vee Kj) \wedge \neg(Aj \wedge Kj)$ [Q p.32]
12. $\neg Am \rightarrow (\neg Kj \rightarrow Aj)$ [Q p.32]
13. $Ae \rightarrow (Am \wedge Aj)$ [Q p.33]
14. $(Aj \vee Am) \rightarrow Ae$ [Q p.33]
15. $Am \leftrightarrow Ke$ [Q p.33]
16. $Ws \rightarrow \neg Hm$ [Q p.33]
17. $Pt \wedge Et$ [Q p.33]
18. $Sc \wedge \neg Yc \wedge Cc$ [Q p.33]
19. $Kr \rightarrow \neg Pt$ [Q p.33]
20. $Hm \leftrightarrow \neg He$ [Q p.33]

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Answers 8.3.2

Glossary:

Cx:	x is certain	Rx:	x is red
Ex:	x is expensive	Wx:	x is worthwhile
Fx:	x is fun	i:	Independence Hall
Gx:	x is green	k:	Kermit
Hx:	x is heavy	p:	Oscar's piano
Px:	x is probable	s:	Spondulix

Translations:

1. $Ri \to \exists x Rx$ [Q p.33]
2. $\forall x Rx \to Ri$ [Q p.33]
3. $\neg\exists x(Gx \wedge Rx)$ [Q p.33]
4. $\neg\neg\exists x(Gx \wedge Rx)$ [Q p.33]
5. This can be interpreted in two ways. The most natural interpretation is (i) *no* red thing is green; but we could also take it to mean (ii) not *every* red thing is green (i.e. some red things are not green).
   (i) $\forall x(Rx \to \neg Gx)$
   ("Pick anything at all: if it is red, then it is not green.")
   or equivalently: $\neg\exists x(Rx \wedge Gx)$
   ("There does not exist anything which is both red and green.")
   (ii) $\neg\forall x(Rx \to Gx)$ or equivalently: $\exists x(Rx \wedge \neg Gx)$ [Q p.33]
6. $\forall x(Rx \to (Hx \vee Ex))$ [Q p.33]
7. $\forall x((Rx \wedge \neg Hx) \to Ex)$ [Q p.33]
   ("Pick anything at all: if it is red and not heavy, then it is expensive.")
8. $\forall x(Rx \to Hx) \wedge \exists x(Gx \wedge \neg Hx)$ [Q p.33]
9. $\forall x(Rx \to Hx) \wedge \neg\forall x(Hx \to Rx)$ [Q p.33]
10. $\exists x(Rx \wedge Hx) \wedge \exists x(Gx \wedge Hx)$ [Q p.33]
11. $\exists x(Rx \wedge \neg Hx) \wedge \exists x(Hx \wedge \neg Rx)$ [Q p.33]

12. $(Gk \wedge Rk) \to \neg\neg\exists x(Gx \wedge Rx)$ [Q p.34]

13. $Hp \wedge \neg(Rp \vee Ep)$ [Q p.34]

14. $(Hs \wedge Es \wedge \forall x(Ex \to Rx) \wedge \forall x(Hx \to Gx)) \to (Rs \wedge Gs)$ [Q p.34]

15. $Hk \to \exists x(Gx \wedge Hx)$ [Q p.34]

16. $\forall x Fx \to \neg\exists x Wx$ [Q p.34]

17. $\exists x Fx \wedge \exists x Wx \wedge \neg\exists x(Fx \wedge Wx)$ [Q p.34]

18. $\neg\exists x Cx \to \neg\exists x Px$ [Q p.34]

19. $\exists x Px \wedge \exists x \neg Px \wedge \neg\exists x Cx$ [Q p.34]

20. $\forall x(Cx \to Px)$ [Q p.34]

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Answers 8.3.5

Glossary:

Ax:	x can stay	Rx:	x is telling the truth
Bx:	x is brown	Sx:	x is sad
Cx:	x works at this company	Tx:	x is in trouble
Fx:	x is a leaf	Ux:	x is laughing
Gx:	x is grey	Yx:	x is lying
Hx:	x is happy	g:	Gary
Lx:	x is laughing	s:	the sky
Ox:	x is in this room	t:	Stephanie
Px:	x is a person		

Translations:

1. $\forall x(Px \to Hx)$ [Q p.34]

2. $\exists x(Px \wedge Sx)$ [Q p.34]

3. $\neg\exists x(Px \wedge Hx \wedge Sx)$ [Q p.34]

4. $\exists x(Px \wedge Sx) \to \neg\forall x(Px \to Hx)$ [Q p.34]

5. $\forall x(Px \rightarrow (\neg Hx \rightarrow \neg Lx))$
   or $\forall x((Px \wedge Lx) \rightarrow Hx)$
   or $\neg\exists x(Px \wedge Lx \wedge \neg Hx)$ [Q p.34]
6. $Lg \rightarrow \exists x(Px \wedge Hx)$ [Q p.34]
7. $\forall x((Px \wedge Lx) \rightarrow Hx)$ [Q p.34]
8. $Lg \rightarrow \forall x(Px \rightarrow Lx)$ [Q p.35]
9. $\exists x(Px \wedge Sx) \wedge \neg\forall x(Px \rightarrow Sx) \wedge \neg Sg$ [Q p.35]
10. $\neg\forall x(Px \rightarrow Sx) \rightarrow \neg Hg$ [Q p.35]
11. $\forall x(Fx \rightarrow Bx) \wedge Gs$ [Q p.35]
12. $\exists x(Fx \wedge Bx) \wedge \neg\forall x(Fx \rightarrow Bx)$ [Q p.35]
13. $\forall x(Bx \rightarrow Fx)$ [Q p.35]
14. This could be saying either of two things:
    (i) that the only *leaves* that can stay are the brown ones (but maybe non-leaves can stay too):
    $\forall x((Ax \wedge Fx) \rightarrow Bx)$ or $\forall x(Fx \rightarrow (Ax \rightarrow Bx))$
    (ii) that the only *things* that can stay are brown leaves:
    $\forall x(Ax \rightarrow (Fx \wedge Bx))$ [Q p.35]
15. $\neg Hg \rightarrow \forall x(Px \rightarrow Tx)$ [Q p.35]
16. $\neg Hg \rightarrow \forall x((Px \wedge Cx) \rightarrow Tx)$ [Q p.35]
17. $Rt \rightarrow \exists x(Px \wedge Yx)$ [Q p.35]
18. $\neg\exists x(Px \wedge Yx) \rightarrow Rt$ [Q p.35]
19. $Yt \vee (\neg\exists x(Px \wedge Rx) \wedge \forall x(Px \rightarrow Tx))$ [Q p.35]
20. $Yg \rightarrow \neg\forall x((Px \wedge Ox) \rightarrow Rx)$ [Q p.35]

[Contents]

Answers 8.4.3.1

1. Main operator: $\forall$ [Q p.35]

Step	Wff constructed at this step	from steps/by clause:
1.	Fx	/ (3i)
2.	Gx	/ (3i)
3.	$(Fx \rightarrow Gx)$	1, 2 / (3ii) line 5
4.	$\forall x(Fx \rightarrow Gx)$	3 / (3ii) line 7

2. Main operator: $\forall$ [Q p.35]

Step	Wff constructed at this step	from steps/by clause:
1.	Gx	/ (3i)
2.	$\neg Gx$	1 / (3ii) line 1
3.	$\forall x \neg Gx$	2 / (3ii) line 7

3. Main operator: $\neg$ [Q p.35]

Step	Wff constructed at this step	from steps/by clause:
1.	Fx	/ (3i)
2.	Gx	/ (3i)
3.	$(Fx \wedge Gx)$	1, 2 / (3ii) line 2
4.	$\exists x(Fx \wedge Gx)$	3 / (3ii) line 8
5.	$\neg \exists x(Fx \wedge Gx)$	4 / (3ii) line 1

4. Main operator: $\wedge$ [Q p.35]

Step	Wff constructed at this step	from steps/by clause:
1.	Fa	/ (3i)
2.	Fx	/ (3i)
3.	$\neg Fx$	2/ (3ii) line 1
4.	$\exists x \neg Fx$	3/ (3ii) line 8
5.	$\neg \exists x \neg Fx$	4/ (3ii) line 1
6.	$(Fa \wedge \neg \exists x \neg Fx)$	1,5/ (3ii) line 2

5. Main operator: $\forall$ [Q p.35]

Step	Wff constructed at this step	from steps/by clause:
1.	Fx	/ (3i)
2.	Gx	/ (3i)
3.	Gy	/ (3i)
4.	$(Gx \rightarrow Gy)$	2, 3 / (3ii) line 5
5.	$\exists y(Gx \rightarrow Gy)$	4 / (3ii) line 8
6.	$Fx \wedge \exists y(Gx \rightarrow Gy)$	1, 5 / (3ii) line 2
7.	$\forall x(Fx \wedge \exists y(Gx \rightarrow Gy))$	6 / (3ii) line 7

6. Main operator: $\wedge$ [Q p.36]

Step	Wff constructed at this step	from steps/by clause:
1.	Fx	/ (3i)
2.	Gx	/ (3i)
3.	$Fx \rightarrow Gx$	1, 2 / (3ii) line 5
4.	$\forall x(Fx \rightarrow Gx)$	3 / (3ii) line 7
5.	Fa	/ (3i)
6.	$(\forall x(Fx \rightarrow Gx) \wedge Fa)$	4, 5 / (3ii) line 2

7. Main operator: $\rightarrow$ [Q p.36]

Step	Wff constructed at this step	from steps/by clause:
1.	Fa	/ (3i)
2.	Fb	/ (3i)
3.	$\neg Fa$	1 / (3ii) line 1
4.	$\neg Fb$	2 / (3ii) line 1
5.	$(\neg Fa \wedge \neg Fb)$	3, 4 / (3ii) line 2
6.	Fx	/ (3i)
7.	$\neg Fx$	6 / (3ii) line 1
8.	$\forall x \neg Fx$	7 / (3ii) line 7
9.	$((\neg Fa \wedge \neg Fb) \rightarrow \forall x \neg Fx)$	5, 8 / (3ii) line 5

8. Main operator: $\forall$ [Q p.36]

Step	Wff constructed at this step	from steps/by clause:
1.	Fx	/ (3i)
2.	Fy	/ (3i)
3.	$(Fx \wedge Fy)$	1, 2 / (3ii) line 2
4.	Gx	/ (3i)
5.	$((Fx \wedge Fy) \rightarrow Gx)$	3, 4 / (3ii) line 5
6.	$\forall y((Fx \wedge Fy) \rightarrow Gx)$	5 / (3ii) line 7
7.	$\forall x \forall y((Fx \wedge Fy) \rightarrow Gx)$	6 / (3ii) line 7

9. Main operator: $\forall$ [Q p.36]

Step	Wff constructed at this step	from steps/by clause:
1.	Fx	/ (3i)
2.	Fy	/ (3i)
3.	$\forall yFy$	2 / (3ii) line 7
4.	$(Fx \rightarrow \forall yFy)$	1, 3 / (3ii) line 5
5.	$\forall x(Fx \rightarrow \forall yFy)$	4 / (3ii) line 7

10. Main operator: $\rightarrow$ [Q p.36]

Step	Wff constructed at this step	from steps/by clause:
1.	Fx	/ (3i)
2.	$\forall xFx$	1 / (3ii) line 7
3.	Fy	/ (3i)
4.	$\forall yFy$	3 / (3ii) line 7
5.	$(\forall xFx \rightarrow \forall yFy)$	2, 4 / (3ii) line 5

[Contents]

Answers 8.4.5.1

Free variables are underlined:

1. $T\underline{x} \wedge F\underline{x}$ Open [Q p.36]
2. $T\underline{x} \wedge T\underline{y}$ Open [Q p.36]
3. $\exists xTx \wedge \exists xFx$ Closed [Q p.36]
4. $\exists xTx \wedge \forall yF\underline{x}$ Open [Q p.36]

5. $\exists x Tx \wedge F\underline{x}$ Open [Q p.36]
6. $\exists x(Tx \wedge Fx)$ Closed [Q p.36]
7. $\forall y \exists x Ty$ Closed [Q p.36]
8. $\exists x(\forall x Tx \rightarrow \exists y Fx)$ Closed [Q p.36]
9. $\exists y \forall x Tx \rightarrow \exists y F\underline{x}$ Open [Q p.36]
10. $\forall x(\exists x Tx \wedge Fx)$ Closed [Q p.36]
11. $\forall x \exists x Tx \wedge F\underline{x}$ Open [Q p.36]
12. $\exists x T\underline{y}$ Open [Q p.36]
13. $\forall x Tx \rightarrow \exists x Fx$ Closed [Q p.36]
14. $\exists x \forall y(Tx \vee Fy)$ Closed [Q p.36]
15. $\forall x Fx \wedge G\underline{x}$ Open [Q p.36]
16. $\forall x \forall y Fx \rightarrow G\underline{y}$ Open [Q p.37]
17. $\forall x \forall y(Fx \rightarrow \forall x Gy)$ Closed [Q p.37]
18. $\exists y Gb \wedge Gc$ Closed [Q p.37]
19. $\exists y Gy \wedge \forall x(Fx \rightarrow G\underline{y})$ Open [Q p.37]
20. $\forall x((Fx \rightarrow \exists x Gx) \wedge Gx)$ Closed [Q p.37]

[Contents]

Chapter 9

Semantics of Monadic Predicate Logic

Answers 9.1.1

1. (i) True (ii) True (iii) False [Q p.38]
2. (i) False (ii) True (iii) False [Q p.38]
3. (i) False (ii) True (iii) False [Q p.38]
4. (i) True (ii) True (iii) False [Q p.38]
5. (i) True (ii) True (iii) True [Q p.39]
6. (i) False (ii) False (iii) False [Q p.39]

[Contents]

Answers 9.2.1

Model 1:

(i) False
(ii) True
(iii) True
(iv) True
(v) True

Model 2:

(i) True
(ii) False
(iii) False
(iv) True
(v) True

[Questions p.39]

Model 3:

(i) True
(ii) False
(iii) False
(iv) True
(v) True

Model 4:

(i) False
(ii) True
(iii) True
(iv) True
(v) True

Model 5:

(i) False
(ii) True
(iii) True
(iv) True
(v) True

Model 6:

(i) True
(ii) False
(iii) True
(iv) True

(v) True

[Questions p.39]
[Contents]

Answers 9.3.1

1. (i) $(Fa \wedge Ga)$ [Q p.39]
   (ii) $(Fb \wedge Ga)$ [Q p.39]
2. (i) $\forall y(Fa \rightarrow Gy)$ [Q p.39]
   (ii) $\forall y(Fb \rightarrow Gy)$ [Q p.39]
3. (i) $\forall x(Fx \rightarrow Gx) \wedge Fa$ [Q p.39]
   (ii) $\forall x(Fx \rightarrow Gx) \wedge Fb$ [Q p.40]
4. (i) $\forall x(Fx \wedge Ga)$ [Q p.40]
   (ii) $\forall x(Fx \wedge Ga)$ [Q p.40]

   NB $\alpha(x)$, i.e. $\forall x(Fx \wedge Ga)$, contains no free occurrences of x, so for *any* name $\underline{a}$, $\alpha(\underline{a}/x)$ is just $\alpha(x)$. For we replace all *free* occurrences of x with $\underline{a}$; if there are no free occurrences, nothing gets replaced.
5. (i) $\exists x(Gx \rightarrow Ga)$ [Q p.40]
   (ii) $\exists x(Gx \rightarrow Gb)$ [Q p.40]
6. (i) $\exists y(\forall x(Fx \rightarrow Fy) \vee Fa)$ [Q p.40]
   (ii) $\exists y(\forall x(Fx \rightarrow Fy) \vee Fb)$ [Q p.40]

[Contents]

Answers 9.4.3

1. (i) False [Q p.40]
   (ii) True [Q p.40]
   (iii) True [Q p.40]
   (iv) False [Q p.40]
   (v) True [Q p.40]
   (vi) True [Q p.40]

2. (i) (a) False [Q p.40]
   (b) True [Q p.41]
   (ii) (a) True [Q p.41]
   (b) False [Q p.41]
   (iii) (a) True [Q p.41]
   (b) False [Q p.41]

3. (i) True [Q p.41]
   (ii) False [Q p.41]
   (iii) True [Q p.41]
   (iv) True [Q p.41]
   (v) True [Q p.41]
   (vi) False [Q p.41]

4. (i) (a) Domain: $\{1, 2, 3, \ldots\}$
   Extensions: $F : \{1, 2\}$ $\quad G : \{1, 2, 3\}$
   (b) Domain: $\{1, 2, 3, \ldots\}$
   Extensions: $F : \{1, 2\}$ $\quad G : \{1\}$ [Q p.42]
   (ii) (a) No such model. For the formula to be true on a model, it would have to be the case that all members of the (non-empty) domain were in the extension of F (so that the first conjunct were true) and also that a certain member of the domain were not in the extension of F (so that the second conjunct were true).

(b) Domain: $\{1, 2, 3, \ldots\}$
Referent of a: 2
Extension of F : $\{2, 4, 6, \ldots\}$ [Q p.42]

(iii) (a) Domain: $\{1, 2, 3, \ldots\}$
Referent of a : 1
Extension of F : $\{2, 4, 6, \ldots\}$

(b) Domain: $\{1, 2, 3, \ldots\}$
Referent of a : 1
Extension of F : $\{1, 3, 5, \ldots\}$ [Q p.42]

(iv) (a) Domain: $\{1, 2, 3, \ldots\}$
Extensions: $F : \{1, 2, 3, \ldots\}$ $G : \{2, 4, 6, \ldots\}$

(b) Domain: $\{1, 2, 3, \ldots\}$
Extensions: $F : \{1, 3, 5, \ldots\}$ $G : \{2, 4, 6, \ldots\}$ [Q p.42]

(v) (a) Domain: $\{1, 2, 3, \ldots\}$
Extension of F : $\{1, 3, 5, \ldots\}$

(b) No such model. For the formula to be false on a model, it would have to be the case that, in that model, some member of the domain were both in the extension of F and not in the extension of F [Q p.42]

(vi) (a) Domain: $\{1, 2, 3, \ldots\}$
Extensions: $F : \{1, 3, 5, \ldots\}$ $G : \{2, 4, 6, \ldots\}$

(b) Domain: $\{1, 2, 3, \ldots\}$
Extensions: $F : \{1, 3, 5, \ldots\}$ $G : \varnothing$ [Q p.42]

(vii) (a) Domain: $\{1, 2, 3, \ldots\}$
Extension of F : $\{1, 3, 5, \ldots\}$

(b) No such model. For the formula to be false on a model, it would have to be the case that all members of the (non-empty) domain were in the extension of F (to make the antecedent true) and that no members of the domain were in the extension of F (to make the consequent false). [Q p.42]

(viii) (a) No such model. For the formula to be true on a model, it would have to be the case that, in that model, a single member of the domain were both in the extension of F and not in the extension of F.

(b) Domain: $\{1, 2, 3, \ldots\}$
Extension of F : $\{1, 3, 5, \ldots\}$ [Q p.42]

(ix) (a) Domain: $\{1, 2, 3, \ldots\}$
Extension of F : $\{1, 3, 5, \ldots\}$

(b) Domain: $\{1, 2, 3, \dots\}$
Extension of F : $\varnothing$ [Q p.42]

(x) (a) Domain: $\{1, 2, 3, \dots\}$
Extension of F : $\{1, 3, 5, \dots\}$

(b) No such model. For the formula to be false on a model, it would have to be the case that, in that model, a single member of the domain were both in the extension of F and not in the extension of F. [Q p.42]

(xi) (a) Domain: $\{1, 2, 3, \dots\}$
Extensions: $F : \{1\}$ $G : \{2\}$

(b) Domain: $\{1, 2, 3, \dots\}$
Extensions: $F : \{1\}$ $G : \varnothing$ [Q p.42]

(xii) (a) Domain: $\{1, 2, 3, \dots\}$
Extensions: $F : \{1\}$ $G : \{1, 2, 3, \dots\}$

(b) Domain: $\{1, 2, 3, \dots\}$
Extensions: $F : \{1\}$ $G : \{1\}$ [Q p.42]

(xiii) (a) Domain: $\{1, 2, 3, \dots\}$
Referents: $a : 1$
Extensions: $F : \{1\}$

(b) No such model. For the formula to be false on a model, it would have to be that all members of the domain were in the extension of F, but the referent of a was not. [Q p.42]

(xiv) (a) Domain: $\{1, 2, 3, \dots\}$
Referents: $a : 1$
Extensions: $F : \{1, 2, 3, \dots\}$

(b) Domain: $\{1, 2, 3, \dots\}$
Referents: $a : 2$
Extensions: $F : \{1\}$ [Q p.42]

(xv) (a) Domain: $\{1, 2\}$
Referents: $a : 1$ $b : 2$
Extensions: $F : \{1, 2\}$

(b) Domain: $\{1, 2\}$
Referents: $a : 1$ $b : 2$
Extensions: $F : \{1\}$ [Q p.42]

(xvi) (a) Domain: $\{1,2,3,\ldots\}$
Extensions: $F: \{1,3,5,\ldots\}$ $\quad G: \{2,4,6,\ldots\}$

(b) Domain: $\{1,2,3,\ldots\}$ Extensions: $F: \{1\}$ $\quad G: \{2\}$ [Q p.42]

(xvii) (a) Domain: $\{1,2,3,\ldots\}$
Extensions: $F: \{1\}$ $\quad G: \varnothing$

(b) Domain: $\{1,2,3,\ldots\}$
Extensions: $F: \varnothing$ $\quad G: \varnothing$ [Q p.42]

(xviii) (a) No such model. For the formula to be true on a model, it would have to be that each member of the domain was both in the extension of F and also not in the extension of F.

(b) Domain: $\{1,2,3,\ldots\}$
Extensions: $F: \{1\}$ [Q p.42]

(xix) (a) Domain: $\{1,2,3,\ldots\}$
Extensions: $F: \varnothing$ $\quad G: \{1\}$

(b) Domain: $\{1,2,3,\ldots\}$
Extensions: $F: \{1,2,3,\ldots\}$ $\quad G: \varnothing$ [Q p.42]

(xx) (a) Domain: $\{1,2,3,\ldots\}$
Extensions: $F: \varnothing$ $\quad G: \{1\}$

(b) Domain: $\{1,2,3,\ldots\}$
Extensions: $F: \{1,2,3,\ldots\}$ $\quad G: \varnothing$ [Q p.42]

5. (i) True.
Suppose there are no F's. Then, whatever in the domain d (a new name) refers to, Fd is false—so $Fd \rightarrow Gd$ is *true* (because its antecedent is false). So $\forall x(Fx \rightarrow Gx)$ is *true* when there are no F's. [Q p.42]

(ii) No.
If there are no F's, $\forall x(Fx \rightarrow Gx)$ is true (previous question), but $\exists x(Fx \wedge Gx)$ is false. [Q p.42]

[Contents]

Answers 9.5.1

1. Countermodel:
   Domain: $\{1,2\}$
   Extensions: $F : \{1\} \quad G : \{2\}$ [Q p.43]

2. No countermodel. For the premise to be true, there must be at least one object in the domain which is in the extension of F and is also in the extension of G. If this is so, then any such object is *a fortiori* in the extension of F—making the left conjunct of the conclusion true—and in the extension of G—making the right conjunct of the conclusion true: hence the conclusion is true. [Q p.43]

3. Countermodel:
   Domain: $\{1,2\}$
   Extensions: $F : \{2\} \quad G : \{1\}$ [Q p.43]

4. No countermodel. For the first premise to be true, the extension of F must be a subset of the extension of G. For the second premise to be true, the extension of G must be a subset of the extension of H. It follows that the extension of F is a subset of the extension of H—and this makes the conclusion true. [Q p.43]

5. Countermodel:
   Domain: $\{1,2,3\}$
   Extensions: $F : \{1\} \quad G : \{1,2\} \quad H : \{1,2,3\}$ [Q p.43]

[Contents]

Chapter 10

Trees for Monadic Predicate Logic

Answers 10.2.2

1. (i)

$$\begin{array}{c} \neg(Fa \rightarrow \exists xFx) \checkmark \\ Fa \\ \neg\exists xFx \checkmark \\ \forall x\neg Fx \setminus a \\ \neg Fa \\ \times \end{array}$$

Logical truth. [Q p.44]

(ii)

$$\begin{array}{c} \neg(\exists xFx \rightarrow \neg\forall x\neg Fx) \checkmark \\ \exists xFx \checkmark a \\ \neg\neg\forall x\neg Fx \checkmark \\ \forall x\neg Fx \setminus a \\ Fa \\ \neg Fa \\ \times \end{array}$$

Logical truth. [Q p.44]

(iii)

$$\neg\forall x((Fx \wedge \neg Gx) \to \exists x Gx) \checkmark$$
$$\exists x \neg((Fx \wedge \neg Gx) \to \exists x Gx) \ \checkmark a$$
$$\neg((Fa \wedge \neg Ga) \to \exists x Gx) \ \checkmark$$
$$Fa \wedge \neg Ga \ \checkmark$$
$$\neg \exists x Gx \checkmark$$
$$Fa$$
$$\neg Ga$$
$$\forall x \neg Gx \ \backslash a \checkmark$$
$$\neg Ga$$
$$\uparrow$$

Not a logical truth. Countermodel:
Domain: $\{1\}$
Extensions: $F : \{1\} \quad G : \varnothing$ [Q p.44]

(iv)

$$\neg(\forall x Fx \to \exists x Fx) \ \checkmark$$
$$\forall x Fx \ \backslash a$$
$$\neg \exists x Fx \checkmark$$
$$\forall x \neg Fx \ \backslash a$$
$$Fa$$
$$\neg Fa$$
$$\times$$

Logical truth. [Q p.44]

(v)

$$\neg((Fa \wedge (Fb \wedge Fc)) \to \forall x Fx) \ \checkmark$$
$$Fa \wedge (Fb \wedge Fc) \ \checkmark$$
$$\neg \forall x Fx \ \checkmark$$
$$Fa$$
$$(Fb \wedge Fc) \ \checkmark$$
$$Fb$$
$$Fc$$
$$\exists x \neg Fx \ \checkmark d$$
$$\neg Fd \ \checkmark$$
$$\uparrow$$

Not a logical truth. Countermodel:
Domain: $\{1, 2, 3, 4\}$
Extension of $F : \{1, 2, 3\}$ [Q p.44]

(vi)

$$
\begin{array}{c}
\neg(\exists x Fx \wedge \exists x \neg Fx)\ \checkmark \\
\begin{array}{cc}
\neg\exists x Fx\ \checkmark & \neg\exists x\neg Fx\ \checkmark \\
\forall x \neg Fx \setminus a & \forall x \neg\neg Fx \setminus a \\
\neg Fa & \neg\neg Fa\ \checkmark \\
\uparrow & Fa
\end{array}
\end{array}
$$

Not a logical truth. Countermodel:
Domain: $\{1\}$
Extension of $F : \varnothing$ [Q p.44]

(vii)

$$
\begin{array}{c}
\neg\exists x(Fx \rightarrow \forall y Fy)\ \checkmark \\
\forall x \neg(Fx \rightarrow \forall y Fy) \setminus ab \\
\neg(Fa \rightarrow \forall y Fy)\ \checkmark \\
Fa \\
\neg\forall y Fy\ \checkmark \\
\exists y \neg Fy\ \checkmark\ b \\
\neg Fb \\
\neg(Fb \rightarrow \forall y Fy)\ \checkmark \\
Fb \\
\neg\forall y Fy \\
\times
\end{array}
$$

Logical truth. [Q p.44]

(viii)

$$
\begin{array}{c}
\neg(\forall x(Fx \rightarrow Gx) \rightarrow (Fa \rightarrow Ga))\ \checkmark \\
\forall x(Fx \rightarrow Gx) \setminus a \\
\neg(Fa \rightarrow Ga)\ \checkmark \\
Fa \\
\neg Ga \\
Fa \rightarrow Ga\ \checkmark \\
\begin{array}{cc}
\neg Fa & Ga \\
\times & \times
\end{array}
\end{array}
$$

Logical truth. [Q p.44]

(ix) $\neg(\neg\forall x(Fx \wedge Gx) \leftrightarrow \exists x\neg(Fx \wedge Gx))$ ✓

Left branch:

$\neg\forall x(Fx \wedge Gx)$ ✓
$\neg\exists x\neg(Fx \wedge Gx)$ ✓
$\exists x\neg(Fx \wedge Gx)$ ✓a
$\forall x\neg\neg(Fx \wedge Gx)$ $\backslash a$
$\neg(Fa \wedge Ga)$ ✓
$\neg\neg(Fa \wedge Ga)$ ✓
$Fa \wedge Ga$ ✓
Fa
Ga

$\neg Fa$ | $\neg Ga$
$\times$ | $\times$

Right branch:

$\neg\neg\forall x(Fx \wedge Gx)$ ✓
$\exists x\neg(Fx \wedge Gx)$ ✓a
$\forall x(Fx \wedge Gx)$ $\backslash a$
$\neg(Fa \wedge Ga)$ ✓
$Fa \wedge Ga$ ✓
Fa
Ga

$\neg Fa$ | $\neg Ga$
$\times$ | $\times$

Logical truth. [Q p.44]

(x) $\neg(\neg\exists x(Fx \wedge Gx) \leftrightarrow \forall x(\neg Fx \wedge \neg Gx))$ ✓

Left branch:

$\neg\exists x(Fx \wedge Gx)$ ✓
$\neg\forall x(\neg Fx \wedge \neg Gx)$ ✓
$\forall x\neg(Fx \wedge Gx)$ $\backslash a$
$\exists x\neg(\neg Fx \wedge \neg Gx)$ ✓a
$\neg(\neg Fa \wedge \neg Ga)$ ✓
$\neg(Fa \wedge Ga)$ ✓

$\neg\neg Fa$ ✓ | $\neg\neg Ga$ ✓
Fa | Ga

$\neg Fa$ | $\neg Ga$ | $\neg Fa$ | $\neg Ga$
$\times$ | $\uparrow$ | | $\times$

Right branch:

$\neg\neg\exists x(Fx \wedge Gx)$ ✓
$\forall x(\neg Fx \wedge \neg Gx)$ $\backslash a$
$\exists x(Fx \wedge Gx)$ ✓a
$Fa \wedge Ga$ ✓
Fa
Ga
$\neg Fa \wedge \neg Ga$ ✓
$\neg Fa$
$\neg Ga$
$\times$

Not a logical truth. Countermodel:
Domain: $\{1\}$
Extension of $F : \{1\}$ $\quad G : \emptyset$ [Q p.44]

2. (i)

$$\exists x Fx \wedge \exists x Gx \ \checkmark$$
$$\neg \exists x (Fx \wedge Gx) \ \checkmark$$
$$\exists x Fx \ \checkmark a$$
$$\exists x Gx \ \checkmark b$$
$$\forall x \neg (Fx \wedge Gx) \ \backslash ab$$
$$Fa$$
$$Gb$$
$$\neg (Fa \wedge Ga)$$

$$\begin{array}{cc} \neg Fa & \neg Ga \\ \times & \neg (Fb \wedge Gb) \ \checkmark \end{array}$$

$$\begin{array}{cc} \neg Fb & \neg Gb \\ \uparrow & \times \end{array}$$

Invalid. Countermodel:
Domain: $\{1, 2\}$
Extensions: $F : \{1\} \quad G : \{2\}$ [Q p.44]

(ii)

$$\exists x \forall y (Fx \rightarrow Gy) \ \checkmark a$$
$$\neg \forall y \exists x (Fx \rightarrow Gy) \ \checkmark$$
$$\exists y \neg \exists x (Fx \rightarrow Gy) \ \checkmark b$$
$$\forall y (Fa \rightarrow Gy) \ \backslash b$$
$$\neg \exists x (Fx \rightarrow Gb) \ \checkmark$$
$$\forall x \neg (Fx \rightarrow Gb) \ \backslash a$$
$$Fa \rightarrow Gb \ \checkmark$$
$$\neg (Fa \rightarrow Gb) \ \checkmark$$
$$\times$$

Valid. [Q p.44]

(iii)

$$\begin{array}{c}
Fa \to \forall x Gx \ \checkmark \\
\neg\forall x(Fa \to Gx) \ \checkmark
\end{array}$$

$$\begin{array}{cc}
\neg Fa & \forall x Gx \ \backslash b \\
\exists x \neg(Fa \to Gx) \ \checkmark b & \exists x \neg(Fa \to Gx) \ \checkmark b \\
\neg(Fa \to Gb) \ \checkmark & \neg(Fa \to Gb) \ \checkmark \\
Fa & Fa \\
\neg Gb & \neg Gb \\
\times & Gb \\
 & \times
\end{array}$$

Valid. [Q p.45]

(iv)

$$\begin{array}{c}
Fa \to \forall x Gx \ \checkmark \\
\neg\exists x(Fa \to Gx) \ \checkmark \\
\forall x \neg(Fa \to Gx) \ \backslash a \\
\neg(Fa \to Ga) \ \checkmark \\
Fa \\
\neg Ga
\end{array}$$

$$\begin{array}{cc}
\neg Fa & \forall x Gx \ \backslash a \\
\times & Ga \\
 & \times
\end{array}$$

Valid. [Q p.45]

(v)

$$\begin{array}{c}
\forall x(Fx \lor Gx) \ \backslash ab \\
\neg\forall x Fx \ \checkmark \\
\neg\forall x Gx \ \checkmark \\
\exists x \neg Fx \ \checkmark a \\
\exists x \neg Gx \ \checkmark b \\
\neg Fa \\
\neg Gb \\
Fa \lor Ga
\end{array}$$

$$\begin{array}{cc}
Fa & Ga \\
\times & Fb \lor Gb
\end{array}$$

$$\begin{array}{cc}
Fb & Gb \\
\uparrow & \times
\end{array}$$

Invalid. Countermodel:
Domain: $\{1, 2\}$
Extensions: $F : \{2\} \quad G : \{1\}$ [Q p.45]

(vi)

$$
\begin{array}{c}
\exists x(Fx \land Gx)\ \checkmark a \\
\neg(\exists xFx \land \exists xGx)\ \checkmark \\
Fa \land Ga\ \checkmark \\
Fa \\
Ga \\
\begin{array}{cc}
\neg\exists xFx\ \checkmark & \neg\exists xGx\ \checkmark \\
\forall x\neg Fx \setminus a & \forall x\neg Gx \setminus a \\
\neg Fa & \neg Ga \\
\times & \times
\end{array}
\end{array}
$$

Valid. [Q p.45]

(vii)

$$
\begin{array}{c}
\forall x(Fx \to Gx) \setminus a \\
Fa \\
\neg Ga \\
Fa \to Ga\ \checkmark \\
\begin{array}{cc}
\neg Fa & Ga \\
\times & \times
\end{array}
\end{array}
$$

Valid. [Q p.45]

(viii)

$$
\begin{array}{c}
\neg\forall x(Fx \lor Gx)\ \checkmark \\
\neg\exists x(\neg Fx \land \neg Gx)\ \checkmark \\
\exists x\neg(Fx \lor Gx)\ \checkmark a \\
\forall x\neg(\neg Fx \land \neg Gx) \setminus a \\
\neg(Fa \lor Ga)\ \checkmark \\
\neg Fa \\
\neg Ga \\
\neg(\neg Fa \land \neg Ga)\ \checkmark \\
\begin{array}{cc}
\neg\neg Fa\ \checkmark & \neg\neg Ga\ \checkmark \\
Fa & Ga \\
\times & \times
\end{array}
\end{array}
$$

Valid. [Q p.45]

(ix)

$$\begin{array}{c}
\forall x(Fx \to Gx) \ \backslash a \\
\forall x(Gx \to Hx) \ \backslash a \\
\neg\neg\exists x(\neg Fx \wedge Hx) \ \checkmark \\
\exists x(\neg Fx \wedge Hx) \ \checkmark a \\
\neg Fa \wedge Ha \ \checkmark \\
\neg Fa \\
Ha \\
Fa \to Ga \ \checkmark \\
\begin{array}{cc}
\begin{array}{c}
\neg Fa \\
Ga \to Ha \ \checkmark \\
\begin{array}{cc} \neg Ga & Ha \\ \uparrow & \end{array}
\end{array}
&
\begin{array}{c}
Ga \\
Ga \to Ha \ \checkmark \\
\begin{array}{cc} \neg Ga & Ha \\ \times & \end{array}
\end{array}
\end{array}
\end{array}$$

Invalid. Countermodel:
Domain: $\{1\}$
Extensions: $F : \emptyset \quad G : \emptyset \quad H : \{1\}$ [Q p.45]

(x)

$$\begin{array}{c}
\forall x(Fx \vee Gx) \ \backslash a \\
\neg\neg\exists x(Fx \wedge Gx) \ \checkmark \\
\exists x(Fx \wedge Gx) \ \checkmark a \\
Fa \wedge Ga \ \checkmark \\
Fa \\
Ga \\
Fa \vee Ga \ \checkmark \\
\begin{array}{cc} Fa & Ga \\ \uparrow & \end{array}
\end{array}$$

Invalid. Countermodel:
Domain: $\{1\}$
Extensions: $F : \{1\} \quad G : \{1\}$ [Q p.45]

[Contents]

Answers 10.3.8

1. Dx: x is a dog
   Mx: x is a mammal
   Ax: x is an animal

   $\forall x(Dx \to Mx)$
   $\forall x(Mx \to Ax)$
   $\therefore\ \forall x(Dx \to Ax)$

$$
\begin{array}{c}
\forall x(Dx \to Mx)\ \ \backslash a \\
\forall x(Mx \to Ax)\ \ \backslash a \\
\neg\forall x(Dx \to Ax)\ \ \checkmark \\
\exists x\neg(Dx \to Ax)\ \ \checkmark a \\
\neg(Da \to Aa)\ \ \checkmark \\
Da \\
\neg Aa \\
Da \to Ma\ \ \checkmark \\
\begin{array}{cc}
\neg Da & Ma \\
\times & Ma \to Aa\ \ \checkmark \\
 & \begin{array}{cc} \neg Ma & Aa \\ \times & \times \end{array}
\end{array}
\end{array}
$$

Valid. [Q p.45]

2. Fx: x is frozen
   Cx: x is cold

   $\forall xFx \rightarrow \forall xCx$
   $\therefore\ \forall x(Fx \rightarrow Cx)$

$$
\begin{array}{c}
\forall xFx \rightarrow \forall xCx \ \checkmark \\
\neg\forall x(Fx \rightarrow Cx) \ \checkmark \\
\exists x\neg(Fx \rightarrow Cx) \ \checkmark a \\
\begin{array}{cc}
\neg\forall xFx \ \checkmark & \forall xCx \ \backslash a \\
\neg(Fa \rightarrow Ca) \checkmark & \neg(Fa \rightarrow Ca) \ \checkmark \\
Fa & Fa \\
\neg Ca & \neg Ca \\
\exists x\neg Fx \ \checkmark b & Ca \\
\neg Fb & \times \\
\uparrow &
\end{array}
\end{array}
$$

Invalid. Countermodel:
Domain: $\{1, 2\}$
Extensions: $F : \{1\}$ $C : \varnothing$ [Q p.45]

3. Cx: x is conscious
   Gx: x is a divine being
   Sx: x has a sonic screwdriver

   $\forall x(Cx \rightarrow (\exists yGy \vee Sx))$
   $\neg\exists xSx$
   $\therefore\ \neg\forall xCx$

$$
\begin{array}{c}
\forall x(Cx \rightarrow (\exists yGy \vee Sx)) \ \backslash ab \\
\neg\exists xSx \ \checkmark \\
\neg\neg\forall xCx \ \checkmark \\
\forall xCx \ \backslash abc \\
\forall x\neg Sx \ \backslash abc \\
Ca \\
\neg Sa \\
Ca \rightarrow (\exists yGy \vee Sa) \ \checkmark
\end{array}
$$

$$
\begin{array}{cc}
\neg Ca & \exists yGy \vee Sa \ \checkmark \\
\times &
\end{array}
$$

$$
\begin{array}{cc}
\exists yGy \ \checkmark b & Sa \\
Gb & \times \\
Cb & \\
\neg Sb & \\
Cb \rightarrow (\exists yGy \vee Sb) \ \checkmark &
\end{array}
$$

$$
\begin{array}{cc}
\neg Cb & \exists yGy \vee Sb \ \checkmark \\
\times &
\end{array}
$$

$$
\begin{array}{cc}
\exists yGy \ \checkmark c & Sb \\
Gc & \times \\
Cc & \\
\neg Sc & \\
\vdots & \\
\uparrow &
\end{array}
$$

Invalid. Countermodel:
Domain: $\{1, 2, 3, \ldots\}$
Extensions: $C : \{1, 2, 3, \ldots\}$ $G : \{2, 3, 4, \ldots\}$ $S : \emptyset$ [Q p.45]

4. Cx: x is a cow
   Sx: x is a scientist
   Fx: x can fly

   $\forall x(Cx \to Sx)$
   $\neg\exists x(Sx \wedge Fx)$
   $\therefore\ \neg\exists x(Cx \wedge Fx)$

$$\forall x(Cx \to Sx)\ \ \backslash a$$
$$\neg\exists x(Sx \wedge Fx)\ \ \checkmark$$
$$\neg\neg\exists x(Cx \wedge Fx)\ \ \checkmark$$
$$\exists x(Cx \wedge Fx)\ \ \checkmark a$$
$$\forall x\neg(Sx \wedge Fx)\ \ \backslash a$$
$$Ca \wedge Fa\ \ \checkmark$$
$$Ca$$
$$Fa$$
$$Ca \to Sa\ \ \checkmark$$

$$\begin{array}{cc} \neg Ca & Sa \\ \times & \neg(Sa \wedge Fa)\ \ \checkmark \\ & \begin{array}{cc} \neg Sa & \neg Fa \\ \times & \times \end{array} \end{array}$$

Valid. [Q p.45]

5. Px: x is a person
Hx: x is here
Sx: x is smoking

$\exists x((Px \wedge Hx) \wedge \neg Sx)$
$\therefore\ \neg\forall x((Px \wedge Hx) \to Sx)$

$$\begin{array}{c}
\exists x((Px \wedge Hx) \wedge \neg Sx)\ \checkmark a \\
\neg\neg\forall x((Px \wedge Hx) \to Sx)\ \checkmark \\
\forall x((Px \wedge Hx) \to Sx)\ \backslash a \\
(Pa \wedge Ha) \wedge \neg Sa\ \checkmark \\
(Pa \wedge Ha)\ \checkmark \\
\neg Sa \\
Pa \\
Ha \\
(Pa \wedge Ha) \to Sa\ \checkmark
\end{array}$$

$$\begin{array}{cc}
\neg(Pa \wedge Ha)\ \checkmark & Sa \\
\begin{array}{cc} \neg Pa & \neg Ha \\ \times & \times \end{array} & \times
\end{array}$$

Valid. [Q p.46]

6. Cx: x is a coward
Rx: x rocks up
Sx: x will shake
c: Catwoman
s: Superman

$Rs \rightarrow \forall x(Cx \rightarrow Sx)$
$\neg Cc$
$\therefore\ \neg Sc$

$$Rs \rightarrow \forall x(Cx \rightarrow Sx)\ \checkmark$$
$$\neg Cc$$
$$\neg\neg Sc\ \checkmark$$
$$Sc$$

$$\begin{array}{cc} \neg Rs & \forall x(Cx \rightarrow Sx)\ \backslash cs \\ & Cc \rightarrow Sc\ \checkmark \\ & Cs \rightarrow Ss\ \checkmark \end{array}$$

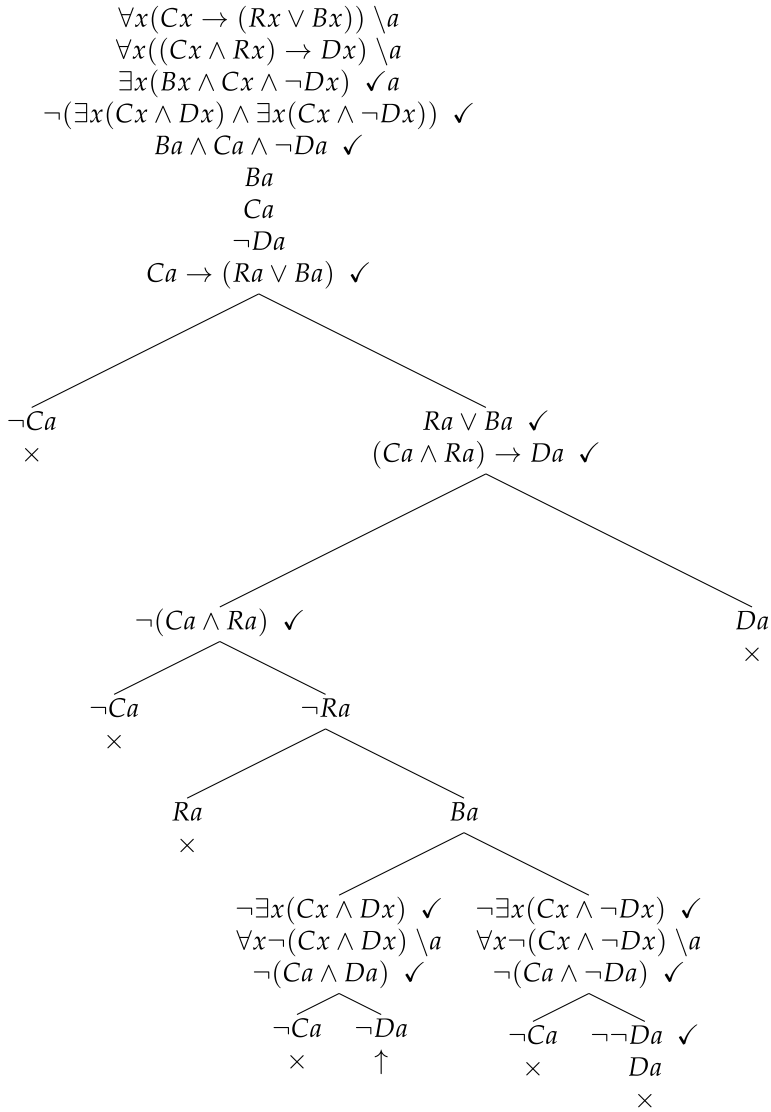
$$\begin{array}{cc} \neg Cc & Sc \end{array}$$

$$\begin{array}{cccc} \neg Cs & Ss & \neg Cs & Ss \\ \uparrow & & & \end{array}$$

Invalid. Countermodel:
Domain: $\{1, 2\}$
Referents: $c : 1 \quad s : 2$
Extensions: $C : \emptyset \quad R : \emptyset \quad S : \{1\}$ [Q p.46]

7. Bx: x is blue
   Cx: x a car
   Dx: x is defective
   Rx: x is red

   $\forall x(Cx \to (Rx \lor Bx))$
   $\forall x((Cx \land Rx) \to Dx)$
   $\exists x(Bx \land Cx \land \neg Dx)$
   $\therefore\ \exists x(Cx \land Dx) \land \exists x(Cx \land \neg Dx)$



Invalid. Countermodel:
Domain: $\{1\}$
Extensions: $B : \{1\} \quad C : \{1\} \quad D : \varnothing \quad R : \varnothing$ [Q p.46]

8. Fx: x is a fish
   Sx: x swims

$\forall x(Sx \to \exists yFy)$
$\therefore\ \exists x \neg Sx$

$$
\begin{array}{c}
\forall x(Sx \to \exists yFy) \setminus abc \\
\neg\exists x \neg Sx \\
\forall x \neg\neg Sx \setminus abc \\
\neg\neg Sa \ \checkmark \\
Sa \\
Sa \to \exists yFy \ \checkmark \\
\begin{array}{cc}
\begin{array}{c} \neg Sa \\ \times \end{array} &
\begin{array}{c}
\exists yFy \ \checkmark b \\
Fb \\
\neg\neg Sb \ \checkmark \\
Sb \\
Sb \to \exists yFy \ \checkmark \\
\begin{array}{cc}
\begin{array}{c} \neg Sb \\ \times \end{array} &
\begin{array}{c}
\exists yFy \ \checkmark c \\
Fc \\
\neg\neg Sc \ \checkmark \\
Sc \\
Sc \to \exists yFy \ \checkmark \\
\begin{array}{cc}
\begin{array}{c} \neg Sc \\ \times \end{array} &
\begin{array}{c} \exists yFy \ \checkmark d \\ Fd \\ \vdots \end{array}
\end{array}
\end{array}
\end{array}
\end{array}
\end{array}
\end{array}
$$

Invalid. Countermodel:
Domain: $\{1, 2, 3, \ldots\}$
Extensions: $F : \{2, 3, 4, \ldots\} \quad S : \{1, 2, 3, \ldots\}$ [Q p.46]

9. Bx: x was built before 1970
Kx: x runs on kerosene
Rx: x is a robot
a: Autovac 23E

$\forall x((Rx \wedge Bx) \to Kx)$
$Ba \wedge \neg Ka$
$\therefore\ \neg Ra$

$$\begin{array}{c}
\forall x((Rx \wedge Bx) \to Kx)\ \backslash a \\
Ba \wedge \neg Ka\ \checkmark \\
\neg\neg Ra\ \checkmark \\
Ra \\
Ba \\
\neg Ka \\
(Ra \wedge Ba) \to Ka\ \checkmark \\
\begin{array}{cc}
\begin{array}{c} \neg(Ra \wedge Ba)\ \checkmark \\ \begin{array}{cc} \neg Ra & \neg Ba \\ \times & \times \end{array} \end{array} & \begin{array}{c} Ka \\ \times \\ \\ \end{array}
\end{array}
\end{array}$$

Valid. [Q p.46]

10. Ax: x is an athlete
Ix: x is an intellectual
Px: x is a person
Tx: x is tall
g: Graham

$\forall x((Tx \land Px) \to (Ax \lor Ix))$
$\exists x(Px \land Ax \land Ix)$
$\forall x((Px \land Ax \land Ix) \to \neg Tx)$
Pg
$\therefore\ Ag \to (\neg Ig \lor \neg Tg)$

$$
\begin{array}{c}
\forall x((Tx \land Px) \to (Ax \lor Ix)) \\
\exists x(Px \land Ax \land Ix) \\
\forall x((Px \land Ax \land Ix) \to \neg Tx)\ \backslash g \\
Pg \\
\neg(Ag \to (\neg Ig \lor \neg Tg))\ \checkmark \\
Ag \\
\neg(\neg Ig \lor \neg Tg)\ \checkmark \\
\neg\neg Ig\ \checkmark \\
\neg\neg Tg\ \checkmark \\
Ig \\
Tg \\
(Pg \land Ag \land Ig) \to \neg Tg\ \checkmark
\end{array}
$$

$$
\begin{array}{cc}
\neg(Pg \land Ag \land Ig)\ \checkmark & \neg Tg \\
\begin{array}{ccc} \neg Pg & \neg Ag & \neg Ig \\ \times & \times & \times \end{array} & \times
\end{array}
$$

Valid.

[Q p.46]

[Contents]

Chapter 11

Models, Propositions, and Ways the World Could Be

There are no exercises for chapter 11. [Contents]

Chapter 12

General Predicate Logic

Answers 12.1.3.1

1. Yes [Q p.48]
2. Yes [Q p.48]
3. Yes [Q p.48]
4. Yes [Q p.48]
5. No [Q p.48]
6. No [Q p.48]
7. No [Q p.48]
8. Yes [Q p.48]
9. No [Q p.48]
10. Yes [Q p.48]

[Contents]

Answers 12.1.6

Glossary:

a:	Alice
b:	Bill
c:	Clare
d:	Dave
e:	Edward
f:	*Fiesta*
j:	*The Bell Jar*
m:	Mary
t:	the Eiffel tower
Tx:	x is tall
Hxy:	x heard y
Lxy:	x likes y
Rxy:	x has read y
Txy:	x is taller than y
$Pxyz$:	x prefers y to z

Translations:

1. Hba [Q p.49]
2. $\neg Hba$ [Q p.49]
3. $Hba \wedge \neg Hab$ [Q p.49]
4. $Hba \to Hab$ [Q p.49]
5. $Hba \leftrightarrow Haa$ [Q p.49]
6. $Hba \vee Hab$ [Q p.49]
7. $Tcd \wedge \neg Tce$ [Q p.49]
8. $Pmac$ [Q p.49]
9. $\neg Pmdc \wedge \neg Pmcd$ [Q p.49]
10. $Tec \wedge \neg Te$ [Q p.49]
11. $Ttc \wedge Ttd$ [Q p.49]
12. $Tdt \to Td$ [Q p.49]

13. $Ttd \land Pcdt$ [Q p.49]

14. $Tad \to Pdda$ [Q p.49]

15. $Pdec \to Tet$ [Q p.49]

16. $Pdec \to \neg Tc$ [Q p.49]

17. $Rmf \land Lmf$ [Q p.49]

18. $\neg Ldf \land \neg Rdf$ [Q p.49]

19. $\neg Ldj \to \neg Rdj$ [Q p.49]

20. $Rdjf \land \neg Rdj \land \neg Rdf$ [Q p.49]

[Contents]

Answers 12.1.9

1. Glossary:

a:	Alice
b:	Bill
Cx:	x is a chair
Bx:	x is broken
Rx:	x is a room
Bxy:	x is bigger than y
Cxy:	x contains y

Translations:

(i) $\exists x \forall y Bxy$ [Q p.50]

(ii) $\exists x \forall y Byx$ [Q p.50]

(iii) $Bab \to \exists x Bxb$ [Q p.50]

(iv) $\forall x Bxb \to Bab$ [Q p.50]

(v) $\exists x \forall y Bxy \to \exists x Bxx$ [Q p.50]

(vi) $(Bab \land Bba) \to \forall x Bxx$ [Q p.50]

(vii) $\exists x \forall y (Bay \to Bxy)$ [Q p.50]

(viii) $\forall x (Bxa \to \forall y (Bay \to Bxy))$ [Q p.50]

(ix) $\forall x (Rx \to \exists y (Cy \land Cxy))$ [Q p.50]

(x) $\exists x(Rx \wedge \exists y(Cy \wedge Cxy \wedge By)) \wedge$
$\exists x(Rx \wedge \forall y((Cy \wedge Cxy) \rightarrow By)) \wedge$
$\neg\exists x(Rx \wedge \forall y((Cy \wedge Cxy) \rightarrow \neg By))$ [Q p.50]

2. Glossary:

Bx:	x is a beagle
Cx:	x is a chihuahua
Dx:	x is a dog
Px:	x is a person
Bxy:	x is bigger than y
Hxy:	x is happier than y
Oxy:	x owns y

Translations:

(i) $\forall x(Px \rightarrow \exists y(Dy \wedge Oxy))$ [Q p.50]

(ii) $\forall x(Dx \rightarrow \exists y(Py \wedge Oyx))$ [Q p.50]

(iii) $\exists x\exists y(Bx \wedge Cy \wedge Oxy)$ [Q p.50]

(iv) $\neg\exists x(Bx \wedge Oxx)$ [Q p.50]

(v) $\neg\exists x\exists y(Bx \wedge Cy \wedge Byx)$ [Q p.50]

(vi) $\exists x\exists y(Cx \wedge By \wedge Bxy)$ [Q p.50]

(vii) $\exists x(Dx \wedge \forall y(Py \rightarrow Hxy))$ [Q p.50]

(viii) $\forall x\forall y((Px \wedge Py \wedge \exists z(Dz \wedge Oxz) \wedge \neg\exists w(Dw \wedge Oyw)) \rightarrow Hxy)$ [Q p.50]

(ix) $\forall x\forall y((Dx \wedge Dy) \rightarrow (Bxy \rightarrow Hxy))$ [Q p.50]

(x) $\exists x(Bx \wedge \forall y(Cy \rightarrow Bxy) \wedge \forall z(Pz \rightarrow Bzx))$ [Q p.50]

3. Glossary:

a:	Alice
b:	Bill
o:	Woolworths
Cx:	x is a cat
Dx:	x is a dog
Gx:	x is grey
Tx:	x is timid
Bxy:	x is bigger than y
Gxy:	x growls at y
$Wxyz$:	x wants to buy y from z

Translations:

(i) $Ta \wedge Da \wedge \exists x(Cx \wedge Bxa)$ [Q p.51]

(ii) $\forall x((Dx \wedge Bxa) \to Bxb)$ [Q p.51]

(iii) $Tb \wedge Cb \wedge \forall x(Dx \to Bxb)$ [Q p.51]

(iv) $\forall x((Tx \wedge Dx) \to \exists y(Gy \wedge Cy \wedge Gxy))$ [Q p.51]

(v) $\forall x(Dx \to \forall y((Ty \wedge Cy) \to Gxy))$ [Q p.51]

(vi) $\exists x(Tx \wedge Dx \wedge \forall y((Gy \wedge Cy) \to Gxy))$ [Q p.51]

(vii) $\neg\exists x\exists y(Tx \wedge Dx \wedge Gy \wedge Cy \wedge Gxy)$ [Q p.51]

(viii) $\exists xWaxo \wedge \neg\exists xWbxo$ [Q p.51]

(ix) $\exists x(Waxo \wedge \neg Wbxo)$ [Q p.51]

(x) $\forall x(Waxo \to Gbx)$ [Q p.51]

4. Glossary:

d:	Dave
e:	Elvis
f:	Frank
r:	the Rolling Stones
Px:	x is a person
Sx:	x is a song
Tx:	x was in the top twenty
Axy:	x is admires y
Rxy:	x recorded y
$Pxyz$:	x prefers y to z

Translations:

(i) $\forall x(Px \rightarrow Adx)$ [Q p.51]

(ii) $\neg\exists x(Px \land Axd)$ [Q p.51]

(iii) $\neg Add$ [Q p.51]

(iv) $\neg\exists x(Px \land Axx)$ [Q p.51]

(v) $\forall x((Px \land \neg Axx) \rightarrow Adx)$ [Q p.51]

(vi) $\forall x((Px \land Axx) \rightarrow Axd)$ [Q p.51]

(vii) $Afe \land Pfre$ [Q p.51]

(viii) $\forall x\forall y((Sx \land Rrx \land Sy \land Rey) \rightarrow Pfxy)$ [Q p.51]

(ix) $\exists x(Sx \land Tx \land Rrx) \land \neg\exists x(Sx \land Tx \land Rex)$ [Q p.51]

(x) $\forall x\forall y((Sx \land Tx \land Rrx \land Sy \land Rey) \rightarrow Pexy)$ [Q p.51]

[Contents]

Answers 12.2.2

1. (i) False [Q p.52]

   (ii) True [Q p.52]

   (iii) False [Q p.52]

   (iv) True (Ldb is true if we let the new name d refer to 1.) [Q p.52]

   (v) False (The extension of L contains no ordered pair whose second member is 1, which is the referent of a; so no matter what the new name d refers to, Lda is false.) [Q p.52]

(vi) False (The extension of L contains no ordered pair which has the same object in both first and second place, so there is no possible choice of referent for the new name d which makes Ldd true.) [Q p.52]

(vii) True (No matter what we pick as the referent of d, we can then pick a referent for e such that Lde is true.) [Q p.52]

(viii) False (If we pick 1 as the referent of d, then we cannot pick a referent for e such that Led is true.) [Q p.52]

(ix) False (There is no object in the domain which is both in the extension of P, and in the first place of an ordered pair in the extension of L which has 2, which is the referent of b, in second place; so there is no possible choice of referent for the new name d which makes both Pd and Ldb true.) [Q p.52]

(x) True [Q p.52]

(xi) True [Q p.52]

(xii) True [Q p.52]

(xiii) False [Q p.52]

(xiv) True [Q p.52]

(xv) False [Q p.52]

(xvi) False [Q p.52]

(xvii) False [Q p.52]

(xviii) True [Q p.52]

2. (i) False [Q p.53]

(ii) False [Q p.53]

(iii) False [Q p.53]

(iv) True [Q p.53]

(v) True [Q p.53]

(vi) False [Q p.53]

(vii) True [Q p.53]

(viii) False [Q p.53]

(ix) False [Q p.53]

(x) True [Q p.53]

3. (i) False [Q p.53]

(ii) True [Q p.53]

(iii) True [Q p.53]

(iv) True [Q p.54]

(v) False (e.g. $\langle$Alice, Bob$\rangle$ and $\langle$Bob, Alice$\rangle$ are in the extension of S but $\langle$Alice, Alice$\rangle$ isn't.) [Q p.54]

(vi) False [Q p.54]

(vii) False [Q p.54]

(viii) False [Q p.54]

(ix) True [Q p.54]

(x) True [Q p.54]

4. (i) (a) Domain: $\{1, 2, 3\}$
   Extension of $F : \{\langle 1, 1\rangle, \langle 2, 2\rangle, \langle 3, 3\rangle\}$
   (b) Domain: $\{1, 2, 3\}$
   Extension of $F : \{\langle 1, 1\rangle, \langle 2, 2\rangle\}$ [Q p.54]

   (ii) (a) Domain: $\{1, 2, 3\}$
   Extension of $F : \{\langle 1, 2\rangle, \langle 2, 1\rangle\}$
   (b) Domain: $\{1, 2, 3\}$
   Extension of $F : \{\langle 1, 2\rangle\}$ [Q p.54]

   (iii) (a) Domain: $\{1, 2, 3\}$
   Extension of $F : \{\langle 1, 1\rangle, \langle 2, 3\rangle, \langle 3, 2\rangle\}$
   (b) Domain: $\{1, 2, 3\}$
   Extension of $F : \{\langle 1, 1\rangle, \langle 2, 3\rangle\}$ [Q p.54]

   (iv) (a) Domain: $\{1, 2, 3\}$
   Extension of $F : \{\langle 1, 1\rangle, \langle 1, 2\rangle, \langle 1, 3\rangle\}$
   (b) Domain: $\{1, 2, 3\}$
   Extension of $F : \{\langle 1, 2\rangle, \langle 1, 3\rangle\}$ [Q p.54]

   (v) (a) Domain: $\{1, 2, 3\}$
   Extension of $F : \{\langle 1, 2\rangle, \langle 2, 3\rangle, \langle 3, 1\rangle\}$
   (b) Domain: $\{1, 2, 3\}$
   Extension of $F : \{\langle 1, 1\rangle, \langle 1, 2\rangle, \langle 1, 3\rangle\}$ [Q p.54]

   (vi) (a) Domain: $\{1, 2, 3\}$
   Extension of $F : \{\langle 1, 1\rangle\}$
   (b) Domain: $\{1, 2, 3\}$
   Extension of $F : \emptyset$ [Q p.54]

(vii) (a) Domain: $\{1\}$
Extension of F : $\{\langle 1, 1\rangle\}$

(b) Domain: $\{1\}$
Extension of F : $\{\emptyset\}$ [Q p.54]

(viii) (a) Domain: $\{1, 2, 3\}$
Referent of a: 1
Extension of F : $\{\langle 1, 2\rangle\}$

(b) Domain: $\{1, 2, 3\}$
Referent of a: 1
Extension of F : $\{\langle 1, 1\rangle\}$ [Q p.54]

(ix) (a) No such model. For the proposition to be true, the ordered pair consisting of the referent of a, followed by the referent of a, would have to both be in the extension of F and not in the extension of F

(b) Domain: $\{1, 2, 3\}$
Referent of a: 1
Extension of F : $\{\langle 1, 1\rangle\}$ [Q p.54]

(x) (a) No such model. For the second and third conjuncts to be true, the extension of F must contain the ordered pair of the referent of a followed by the referent of b, but must not contain the ordered pair obtained by switching the order of these two referents. This is incompatible with the first conjunct.

(b) Domain: $\{1, 2, 3\}$
Referents: a: 1 b: 2
Extension of F : $\{\langle 1, 1\rangle\}$ [Q p.54]

[Contents]

Answers 12.3.1

1. (i)

$$
\begin{array}{c}
\neg\forall x(Rxx \rightarrow \exists yRxy)\ \checkmark \\
\exists x\neg(Rxx \rightarrow \exists yRxy)\ \checkmark a \\
\neg(Raa \rightarrow \exists yRay)\ \checkmark \\
Raa \\
\neg\exists yRay\ \checkmark \\
\forall y\neg Ray\ \backslash a \\
\neg Raa \\
\times
\end{array}
$$

Logical truth. [Q p.54]

(ii)

$$
\begin{array}{c}
\neg\forall x(\exists yRxy \rightarrow \exists zRzx)\ \checkmark \\
\exists x\neg(\exists yRxy \rightarrow \exists zRzx)\ \checkmark a \\
\neg(\exists yRay \rightarrow \exists zRza)\ \checkmark \\
\exists yRay\ \checkmark b \\
\neg\exists zRza\ \checkmark \\
\forall z\neg Rza\ \backslash ab \\
Rab \\
\neg Raa \\
\neg Rba \\
\uparrow
\end{array}
$$

Not a logical truth. Countermodel:
Domain: $\{1, 2\}$
Extension of R : $\{\langle 1, 2\rangle\}$ [Q p.54]

(iii)

$$
\begin{array}{c}
\neg(\forall xRax \rightarrow \forall x\exists yRyx)\ \checkmark \\
\forall xRax\ \backslash b \\
\neg\forall x\exists yRyx\ \checkmark \\
\exists x\neg\exists yRyx\ \checkmark b \\
\neg\exists yRyb\ \checkmark \\
\forall y\neg Ryb\ \backslash a \\
Rab \\
\neg Rab \\
\times
\end{array}
$$

Logical truth. [Q p.54]

(iv)

$$\begin{array}{c}
\neg(\forall x\exists y\exists z Ryxz \rightarrow \exists x\exists y Rxay) \\
\forall x\exists y\exists z Ryxz \ \backslash a \\
\neg\exists x\exists y Rxay \ \checkmark \\
\forall x\neg\exists y Rxay \ \backslash b \\
\exists y\exists z Ryaz \ \checkmark b \\
\exists z Rbaz \ \checkmark c \\
Rbac \\
\neg\exists y Rbay \ \checkmark \\
\forall y\neg Rbay \ \backslash c \\
\neg Rbac \\
\times
\end{array}$$

Logical truth. [Q p.54]

(v)

$$\begin{array}{c}
\neg\neg\forall x\exists y Rxy \ \checkmark \\
\forall x\exists y Rxy \ \backslash abc \\
\exists y Ray \ \checkmark b \\
Rab \\
\exists y Rby \ \checkmark c \\
Rbc \\
\exists y Rcy \ \checkmark d \\
Rcd \\
\vdots \\
\uparrow
\end{array}$$

Not a logical truth. Countermodel:
Domain: $\{1, 2, 3, \ldots\}$
Extension of R : $\{\langle 1, 2\rangle, \langle 2, 3\rangle, \langle 3, 4\rangle, \ldots\}$ [Q p.55]

(vi)

$$\neg\forall x\forall y\forall z((Rxy \wedge Ryz) \to Rxz) \ \checkmark$$
$$\exists x\neg\forall y\forall z((Rxy \wedge Ryz) \to Rxz) \ \checkmark a$$
$$\neg\forall y\forall z((Ray \wedge Ryz) \to Raz) \ \checkmark$$
$$\exists y\neg\forall z((Ray \wedge Ryz) \to Raz) \ \checkmark b$$
$$\neg\forall z((Rab \wedge Rbz) \to Raz) \ \checkmark$$
$$\exists z\neg((Rab \wedge Rbz) \to Raz) \ \checkmark c$$
$$\neg((Rab \wedge Rbc) \to Rac) \ \checkmark$$
$$Rab \wedge Rbc \ \checkmark$$
$$\neg Rac$$
$$Rab$$
$$Rbc$$
$$\uparrow$$

Not a logical truth. Countermodel:
Domain: $\{1, 2, 3\}$
Extension of R : $\{\langle 1, 2\rangle, \langle 2, 3\rangle\}$ [Q p.55]

(vii)

$$\neg(\exists x\forall yRxy \to \forall x\exists yRxy) \ \checkmark$$
$$\exists x\forall yRxy \ \checkmark a$$
$$\neg\forall x\exists yRxy \ \checkmark$$
$$\exists x\neg\exists yRxy \ \checkmark b$$
$$\forall yRay \setminus ab$$
$$\neg\exists yRby \ \checkmark$$
$$\forall y\neg Rby \setminus ab$$
$$Raa$$
$$Rab$$
$$\neg Rba$$
$$\neg Rbb$$
$$\uparrow$$

Not a logical truth. Countermodel:
Domain: $\{1, 2\}$
Extension of R : $\{\langle 1, 1\rangle, \langle 1, 2\rangle\}$ [Q p.55]

(viii)

$$\begin{array}{c} \neg(\exists y \forall x Rxy \rightarrow \forall x \exists y Rxy) \ \checkmark \\ \exists y \forall x Rxy \ \checkmark a \\ \neg \forall x \exists y Rxy \ \checkmark \\ \exists x \neg \exists y Rxy \ \checkmark b \\ \forall x Rxa \setminus b \\ \neg \exists y Rby \ \checkmark \\ \forall y \neg Rby \setminus a \\ Rba \\ \neg Rba \\ \times \end{array}$$

Logical truth. [Q p.55]

(ix)

$$\begin{array}{c} \neg(\exists x \forall y Rxy \rightarrow \exists x \exists y Rxy) \ \checkmark \\ \exists x \forall y Rxy \ \checkmark a \\ \neg \exists x \exists y Rxy \ \checkmark \\ \forall x \neg \exists y Rxy \setminus a \\ \forall y Ray \setminus a \\ \neg \exists y Ray \ \checkmark \\ \forall y \neg Ray \setminus a \\ Raa \\ \neg Raa \\ \times \end{array}$$

Logical truth. [Q p.55]

(x)

$$\neg(\forall x\forall y\exists zRxyz \vee \forall x\forall y\forall z\neg Rxyz)\ \checkmark$$
$$\neg\forall x\forall y\exists zRxyz\ \checkmark$$
$$\neg\forall x\forall y\forall z\neg Rxyz\ \checkmark$$
$$\exists x\neg\forall y\exists zRxyz\ \checkmark a$$
$$\exists x\neg\forall y\forall z\neg Rxyz\ \checkmark b$$
$$\neg\forall y\exists zRayz\ \checkmark$$
$$\exists y\neg\exists zRayz\ \checkmark c$$
$$\neg\forall y\forall z\neg Rbyz\ \checkmark$$
$$\exists y\neg\forall z\neg Rbyz\ \checkmark d$$
$$\neg\exists zRacz\ \checkmark$$
$$\forall z\neg Racz\ \backslash abcde$$
$$\neg\forall z\neg Rbdz\ \checkmark$$
$$\exists z\neg\neg Rbdz\ \checkmark e$$
$$\neg\neg Rbde\ \checkmark$$
$$Rbde$$
$$\neg Raca$$
$$\neg Racb$$
$$\neg Racc$$
$$\neg Racd$$
$$\neg Race$$
$$\uparrow$$

Not a logical truth. Countermodel:
Domain: $\{1,2,3,4,5\}$
Extension of R : $\{\langle 2,4,5\rangle\}$ [Q p.55]

2. (i)

$$\forall x\forall y\forall z((Rxy \wedge Ryz) \rightarrow Rxz)\ \backslash a$$
$$Rab$$
$$Rba$$
$$\neg\exists xRxx\ \checkmark$$
$$\forall x\neg Rxx\ \backslash a$$
$$\forall y\forall z((Ray \wedge Ryz) \rightarrow Raz)\ \backslash b$$
$$\forall z((Rab \wedge Rbz) \rightarrow Raz)\ \backslash a$$
$$(Rab \wedge Rba) \rightarrow Raa\ \checkmark$$

$$\neg(Rab \wedge Rba) \qquad Raa$$
$$\neg Rab \quad \neg Rba \qquad \neg Raa$$
$$\times \quad \times \qquad \times$$

Valid. [Q p.55]

(ii)

$$\begin{array}{c}
\forall x Fxa \rightarrow \exists x Fax \ \checkmark \\
\exists x Fxa \ \checkmark b \\
\neg \exists x Fax \ \checkmark \\
\forall x \neg Fax \ \backslash abc
\end{array}$$

$$\begin{array}{cc}
\neg \forall x Fxa \ \checkmark & \exists x Fax \ \checkmark c \\
\exists x \neg Fxa \ \checkmark c & Fac \\
Fba & \neg Fac \\
\neg Fca & \times \\
\neg Faa & \\
\neg Fab & \\
\neg Fac & \\
\uparrow &
\end{array}$$

Invalid. Countermodel:
Domain: $\{1,2,3\}$
Referent of a : 1
Extension of F : $\{\langle 2,1\rangle\}$ [Q p.55]

(iii)

$$\begin{array}{c}
\exists x \exists y \exists z (Rxy \wedge Rzy) \ \checkmark a \\
\neg \exists x Rxx \ \checkmark \\
\forall x \neg Rxx \ \backslash abc \\
\exists y \exists z (Ray \wedge Rzy) \ \checkmark b \\
\exists z (Rab \wedge Rzb) \ \checkmark c \\
Rab \wedge Rcb \ \checkmark \\
Rab \\
Rcb \\
\neg Raa \\
\neg Rbb \\
\neg Rcc \\
\uparrow
\end{array}$$

Invalid. Countermodel:
Domain: $\{1,2,3\}$
Extension of F : $\{\langle 1,2\rangle, \langle 3,2\rangle\}$ [Q p.55]

(iv)

$$\begin{array}{c}
\forall x \forall y (Rxy \rightarrow Ryx) \ \backslash b \\
\exists x Rxa \ \checkmark b \\
\neg \exists x Rax \ \checkmark \\
\forall x \neg Rax \ \backslash b \\
Rba \\
\forall y (Rby \rightarrow Ryb) \ \backslash a \\
Rba \rightarrow Rab \ \checkmark \\
\begin{array}{cc}
\neg Rba & Rab \\
\times & \neg Rab \\
 & \times
\end{array}
\end{array}$$

Valid. [Q p.55]

(v)

$$\begin{array}{c}
\forall x \forall y (\neg Rxy \rightarrow Ryx) \ \backslash a \\
\neg \forall x \exists y Ryx \ \checkmark \\
\exists x \neg \exists y Ryx \ \checkmark a \\
\neg \exists y Rya \ \checkmark \\
\forall y \neg Rya \ \backslash a \\
\neg Raa \\
\forall y (\neg Ray \rightarrow Rya) \ \backslash a \\
\neg Raa \rightarrow Raa \ \checkmark \\
\begin{array}{cc}
\neg\neg Raa & Raa \\
Raa & \times \\
\times &
\end{array}
\end{array}$$

Valid. [Q p.55]

(vi)

$$\begin{array}{c}
\forall x \forall y (Rxy \rightarrow (Fx \wedge Gy)) \ \backslash a \\
\neg\neg \exists x Rxx \ \checkmark \\
\exists x Rxx \ \checkmark a \\
Raa \\
\forall y (Ray \rightarrow (Fa \wedge Gy)) \ \backslash a \\
Raa \rightarrow (Fa \wedge Ga) \ \checkmark \\
\begin{array}{cc}
\neg Raa & Fa \wedge Ga \ \checkmark \\
 & Fa \\
 & Ga \\
 & \uparrow
\end{array}
\end{array}$$

Invalid. Countermodel:
Domain: $\{1\}$
Extensions: $F : \{1\} \quad G : \{1\} \quad R : \{\langle 1, 1 \rangle\}$ [Q p.55]

(vii)

$$
\begin{array}{c}
\forall x(Fx \to (\forall y Rxy \lor \neg\exists y Rxy)) \setminus a \\
Fa \\
\neg Rab \\
\neg\neg Raa \ \checkmark \\
Raa \\
Fa \to (\forall y Ray \lor \neg\exists y Ray) \ \checkmark
\end{array}
$$

$$
\begin{array}{cc}
\begin{array}{c} \neg Fa \\ \times \end{array}
&
\begin{array}{c}
\forall y Ray \lor \neg\exists y Ray \ \checkmark \\
\begin{array}{cc}
\begin{array}{c} \forall y Ray \setminus b \\ Rab \\ \times \end{array}
&
\begin{array}{c} \neg\exists y Ray \ \checkmark \\ \forall y \neg Ray \setminus a \\ \neg Raa \\ \times \end{array}
\end{array}
\end{array}
\end{array}
$$

Valid. [Q p.55]

(viii)

$$
\begin{array}{c}
\forall x \forall y(\exists z(Rzx \land Rzy) \to Rxy) \setminus b \\
\forall x Rax \setminus bc \\
\neg\forall x\forall y Rxy \ \checkmark \\
\exists x \neg\forall y Rxy \ \checkmark b \\
\neg\forall y Rby \ \checkmark \\
\exists y \neg Rby \ \checkmark c \\
\neg Rbc \\
\forall y(\exists z(Rzb \land Rzy) \to Rby) \setminus c \\
\exists z(Rzb \land Rzc) \to Rbc \ \checkmark
\end{array}
$$

$$
\begin{array}{cc}
\begin{array}{c}
\neg\exists z(Rzb \land Rzc) \ \checkmark \\
\forall z \neg(Rzb \land Rzc) \setminus a \\
\neg(Rab \land Rac) \ \checkmark \\
Rab \\
Rac \\
\begin{array}{cc} \neg Rab & \neg Rac \\ \times & \times \end{array}
\end{array}
&
\begin{array}{c} Rbc \\ \times \end{array}
\end{array}
$$

Valid. [Q p.55]

(ix)

$$\begin{array}{c}
\forall x \exists y Rxy \setminus bacde \\
\neg \exists x Rxb \ \checkmark \\
\forall x \neg Rxb \setminus b \\
\neg Rbb \\
\exists y Rby \ \checkmark a \\
Rba \\
\exists y Ray \ \checkmark c \\
Rac \\
\exists y Rcy \ \checkmark d \\
Rcd \\
\exists y Rdy \ \checkmark e \\
Rde \\
\exists y Rey \ \checkmark f \\
Ref \\
\vdots \\
\uparrow
\end{array}$$

Invalid. Countermodel:
Domain: $\{1, 2, 3, 4, 5, \dots\}$
Referent of b: 2
Extension of R : $\{\langle 2, 1\rangle, \langle 1, 3\rangle, \langle 3, 4\rangle, \langle 4, 5\rangle, \langle 5, 6\rangle, \dots\}$ [Q p.55]

(x)

$$\begin{array}{c}
\exists x \forall y (Fy \rightarrow Rxy) \ \checkmark a \\
\exists x \forall y \neg Ryx \ \checkmark b \\
\neg \exists x \neg Fx \ \checkmark \\
\forall x \neg\neg Fx \setminus b \\
\forall y (Fy \rightarrow Ray) \setminus b \\
\forall y \neg Ryb \setminus a \\
\neg Rab \\
\neg\neg Fb \ \checkmark \\
Fb \\
Fb \rightarrow Rab \ \checkmark \\
\begin{array}{cc}
\neg Fb & Rab \\
\times & \times
\end{array}
\end{array}$$

Valid. [Q p.56]

3. (i) a: Alice
b: Bill
c: Carol
Oxy: x is older than y

Oab
Obc
$\therefore Oac$

$$\begin{array}{c} Oab \\ Obc \\ \neg Oac \\ \uparrow \end{array}$$

Invalid. Countermodel:
Domain: $\{1, 2, 3\}$
Referents: $a : 1 \quad b : 2 \quad c : 3$
Extension of $O : \{\langle 1, 2\rangle, \langle 2, 3\rangle\}$ [Q p.56]

(ii) a: Alice
b: Bill
c: Carol
Oxy: x is older than y

Oab
Obc
$\forall x \forall y \forall z((Oxy \wedge Oyz) \to Oxz))$
$\therefore Oac$

$$\begin{array}{c} Oab \\ Obc \\ \forall x \forall y \forall z((Oxy \wedge Oyz) \to Oxz)) \ \backslash a \\ \neg Oac \\ \forall y \forall z((Oay \wedge Oyz) \to Oaz)) \ \backslash b \\ \forall z((Oab \wedge Obz) \to Oaz)) \ \backslash c \\ ((Oab \wedge Obc) \to Oac)) \ \checkmark \end{array}$$

$$\begin{array}{cc} \neg(Oab \wedge Obc) \ \checkmark & Oac \\ \begin{array}{cc} \neg Oab & \neg Obc \\ \times & \times \end{array} & \times \end{array}$$

Valid. [Q p.56]

(iii) a: me
b: you
d: Dave
Bx: x is a banker
Txy: x trusts y

$\forall x(Tbx \rightarrow Tax)$
$\forall x(Bx \rightarrow Tbx)$
Bd
$\therefore\ Tad$

$$\forall x(Tbx \rightarrow Tax)\ \backslash d$$
$$\forall x(Bx \rightarrow Tbx)\ \backslash d$$
$$Bd$$
$$\neg Tad$$
$$Bd \rightarrow Tbd\ \checkmark$$

$\neg Bd$	Tbd
$\times$	$Tbd \rightarrow Tad\ \checkmark$

$\neg Tbd$	Tad
$\times$	$\times$

Valid. [Q p.56]

(iv) Px: x is a person
Lxy: x loves y

$\forall x(Px \to \exists y(Py \land Lxy))$
$\therefore\ \forall x(Px \to \exists y(Py \land Lyx))$

$$
\begin{array}{c}
\forall x(Px \to \exists y(Py \land Lxy)) \ \backslash ab \\
\neg\forall x(Px \to \exists y(Py \land Lyx)) \ \checkmark \\
\exists x\neg(Px \to \exists y(Py \land Lyx)) \ \checkmark a \\
\neg(Pa \to \exists y(Py \land Lya)) \ \checkmark \\
Pa \\
\neg\exists y(Py \land Lya) \ \checkmark \\
\forall y\neg(Py \land Lya) \ \backslash ab \\
\neg(Pa \land Laa) \ \checkmark
\end{array}
$$

$$
\begin{array}{cc}
\neg Pa & \neg Laa \\
\times & Pa \to \exists y(Py \land Lay) \ \checkmark
\end{array}
$$

$$
\begin{array}{cc}
\neg Pa & \exists y(Py \land Lay) \ \checkmark b \\
\times & Pb \land Lab \ \checkmark \\
 & Pb \\
 & Lab \\
 & \neg(Pb \land Lba) \ \checkmark
\end{array}
$$

$$
\begin{array}{cc}
\neg Pb & \neg Lba \\
\times & Pb \to \exists y(Py \land Lby) \ \checkmark
\end{array}
$$

$$
\begin{array}{cc}
\neg Pb & \exists y(Py \land Lby) \ \checkmark c \\
\times & Pc \land Lbc \ \checkmark \\
 & Pc \\
 & Lbc \\
 & \vdots \\
 & \uparrow
\end{array}
$$

Invalid. Countermodel:
Domain: $\{1, 2, 3 \ldots\}$
Extensions: $L : \{\langle 1, 2\rangle, \langle 2, 3\rangle, \langle 3, 4\rangle \ldots\}$ $P : \{1, 2, 3, \ldots\}$

[Q p.56]

(v) n: Nancy
Rx: x is a restaurateur
Axy: x can afford to feed y
Wx: x is very wealthy

Rn
$\forall x(Rx \rightarrow (Anx \leftrightarrow \neg Axx))$
$\therefore\ Wn$

$$
\begin{array}{c}
Rn \\
\forall x(Rx \rightarrow (Anx \leftrightarrow \neg Axx)) \ \backslash n \\
\neg Wn \\
Rn \rightarrow (Ann \leftrightarrow \neg Ann) \ \checkmark \\
\begin{array}{cc}
\begin{array}{c} \neg Rn \\ \times \end{array} &
\begin{array}{c}
Ann \leftrightarrow \neg Ann \ \checkmark \\
\begin{array}{cc}
\begin{array}{c} Ann \\ \neg Ann \\ \times \end{array} &
\begin{array}{c} \neg Ann \\ \neg\neg Ann \ \checkmark \\ Ann \\ \times \end{array}
\end{array}
\end{array}
\end{array}
\end{array}
$$

Valid. [Q p.56]

(vi) e: the Eiffel tower
l: Lake Burley Griffin
Cx: x is in Canberra
Px: x is in Paris
Bxy: x is more beautiful than y

$\forall x \forall y((Px \wedge Cy) \rightarrow Bxy)$
$Pe \wedge Cl$
$\therefore\ Bel$

$$
\begin{array}{c}
\forall x \forall y((Px \wedge Cy) \rightarrow Bxy)\ \backslash e \\
Pe \wedge Cl\ \checkmark \\
\neg Bel \\
Pe \\
Cl \\
\forall y((Pe \wedge Cy) \rightarrow Bey)\ \backslash l \\
((Pe \wedge Cl) \rightarrow Bel)\ \checkmark \\
\begin{array}{cc}
\neg(Pe \wedge Cl)\ \checkmark & Bel \\
\begin{array}{cc} \neg Pe & \neg Cl \\ \times & \times \end{array} & \times
\end{array}
\end{array}
$$

Valid. [Q p.56]

(vii) Jx: x is a journalist
Px: x is a politician
Txy: x talks to y

$\forall x \forall y((Px \wedge Txy) \to Py)$
$\neg \exists x(Px \wedge Jx)$
$\therefore\ \neg \exists x \exists y(Px \wedge Jy \wedge Txy)$

$$
\begin{array}{c}
\forall x \forall y((Px \wedge Txy) \to Py) \ \backslash a \\
\neg \exists x(Px \wedge Jx) \ \checkmark \\
\neg\neg \exists x \exists y(Px \wedge Jy \wedge Txy) \ \checkmark \\
\exists x \exists y(Px \wedge Jy \wedge Txy) \ \checkmark a \\
\exists y(Pa \wedge Jy \wedge Tay) \ \checkmark b \\
Pa \wedge Jb \wedge Tab \ \checkmark \\
Pa \\
Jb \\
Tab \\
\forall x \neg(Px \wedge Jx) \ \backslash ab \\
\neg(Pa \wedge Ja) \ \checkmark \\
\begin{array}{cc}
\begin{array}{c} \neg Pa \\ \times \end{array} &
\begin{array}{c}
\neg Ja \\
\neg(Pb \wedge Jb) \ \checkmark \\
\begin{array}{cc}
\begin{array}{c}
\neg Pb \\
\forall y((Pa \wedge Tay) \to Py) \ \backslash b \\
(Pa \wedge Tab) \to Pb \ \checkmark \\
\begin{array}{cc}
\begin{array}{c}
\neg(Pa \wedge Tab) \ \checkmark \\
\begin{array}{cc}
\begin{array}{c} \neg Pa \\ \times \end{array} & \begin{array}{c} \neg Tab \\ \times \end{array}
\end{array}
\end{array} &
\begin{array}{c} Pb \\ \times \end{array}
\end{array}
\end{array} &
\begin{array}{c} \neg Jb \\ \times \end{array}
\end{array}
\end{array}
\end{array}
\end{array}
$$

Valid. [Q p.56]

(viii) Sxy: x is smaller than y

$\neg\exists x\forall y Sxy$
$\therefore \neg\exists x\forall y Syx$

$$
\begin{array}{c}
\neg\exists x\forall y Sxy \ \checkmark \\
\neg\neg\exists x\forall y Syx \ \checkmark \\
\exists x\forall y Syx \ \checkmark a \\
\forall x\neg\forall y Sxy \ \checkmark abcd \\
\forall y Sya \setminus abcd \\
Saa \\
\neg\forall y Say \ \checkmark \\
\exists y\neg Say \ \checkmark b \\
\neg Sab \\
Sba \\
\neg\forall y Sby \ \checkmark \\
\exists y\neg Sby \ \checkmark c \\
\neg Sbc \\
Sca \\
\neg\forall y Scy \ \checkmark \\
\exists y\neg Scy \ \checkmark d \\
\neg Scd \\
Sda \\
\neg\forall y Sdy \ \checkmark \\
\exists y\neg Sdy \\
\vdots \\
\uparrow
\end{array}
$$

Invalid. Countermodel:
Domain: $\{1, 2, 3, 4, \ldots\}$
Extension of S : $\{\langle 1,1\rangle, \langle 2,1\rangle, \langle 3,1\rangle, \langle 4,1\rangle \ldots\}$ [Q p.56]

(ix) d: David
m: Margaret
Fx: x is French
Mx: x is a movie
Sx: x is commercially successful
Lxy: x likes y

$\forall x(Mx \rightarrow (\neg Sx \vee (Lmx \wedge Ldx)))$
$\neg\exists x(Fx \wedge Mx \wedge Lmx \wedge Ldx)$
$\therefore\ \neg\exists x(Fx \wedge Mx \wedge Sx)$

$$\begin{array}{c}
\forall x(Mx \rightarrow (\neg Sx \vee (Lmx \wedge Ldx)))\ \backslash a \\
\neg\exists x(Fx \wedge Mx \wedge Lmx \wedge Ldx)\ \checkmark \\
\neg\neg\exists x(Fx \wedge Mx \wedge Sx)\ \checkmark \\
\exists x(Fx \wedge Mx \wedge Sx)\ \checkmark a \\
\forall x\neg(Fx \wedge Mx \wedge Lmx \wedge Ldx)\ \backslash a \\
Fa \wedge Ma \wedge Sa\ \checkmark \\
Fa \\
Ma \\
Sa \\
Ma \rightarrow (\neg Sa \vee (Lma \wedge Lda))\ \checkmark
\end{array}$$

$$\begin{array}{cc}
\neg Ma & \neg Sa \vee (Lma \wedge Lda)\ \checkmark \\
\times &
\end{array}$$

$$\begin{array}{cc}
\neg Sa & Lma \wedge Lda\ \checkmark \\
\times & Lma \\
 & Lda \\
 & \neg(Fa \wedge Ma \wedge Lma \wedge Lda)\ \checkmark
\end{array}$$

$$\begin{array}{cccc}
\neg Fa & \neg Ma & \neg Lma & \neg Lda \\
\times & \times & \times & \times
\end{array}$$

Valid. [Q p.56]

(x) Cxy: x causes y

$\exists x \forall y Cxy$
$\therefore \neg \exists x \forall y Cyx$

$$\begin{array}{c}
\exists x \forall y Cxy \ \checkmark a \\
\neg\neg \exists x \forall y Cyx \ \checkmark \\
\exists x \forall y Cyx \ \checkmark b \\
\forall y Cay \setminus ab \\
\forall y Cyb \setminus ab \\
Caa \\
Cab \\
Cab \\
Cbb \\
\uparrow
\end{array}$$

Invalid. Countermodel:
Domain: $\{1, 2\}$
Extension of C : $\{\langle 1, 1\rangle, \langle 1, 2\rangle, \langle 2, 2\rangle\}$ [Q p.56]

[Contents]

Answers 12.4.1

The trees are not given in these answers.

1. Glossary:

e:	that egg
r:	Roger
Fx:	x is a food
Exy:	x will eat y

Translation:

$\forall x(Fx \rightarrow Erx)$
$\therefore Ere$

Postulate:

Fe

[Q p.57]

2. Glossary:

a:	180
b:	170
l:	Bill
n:	Ben
Gxy:	x is greater than y
Wxy:	x weighs y pounds
Vxy:	x is heavier than y

Translation:

Wla
Wnb
$\therefore Vln$

Postulates:

Gab
$\forall x \forall y \forall z \forall w((Wxy \wedge Wzw \wedge Gyw) \rightarrow Vxz)$

[Q p.57]

3. Glossary:

a:	5
b:	10
j:	John
n:	Nancy
Fxy:	x ran further than y
Gxy:	x is greater than y
Rxy:	x ran y miles

Translation:

Rja
Rnb
$\therefore Fnj$

Postulates:

Gba
$\forall x \forall y \forall z \forall w((Rxy \wedge Rzw \wedge Gyw) \rightarrow Fxz)$

[Q p.57]

4. Glossary:

b:	*Buddenbrooks*
s:	Sophie
t:	Thomas Mann
Nx:	x is a novel
Axy:	x is the author of y
Exy:	x enjoys y

   Translation:

   $\forall x((Nx \wedge Atx) \rightarrow Esx)$
   $\therefore Esb$

   Postulates:

   Nb
   Atb

[Q p.57]

5. Glossary:

b:	Borges
c:	Chris
Nx:	x is a novel
Axy:	x is the author of y
Exy:	x enjoys y

   Translation:

   $\forall x(Ecx \leftrightarrow Nx)$
   $\therefore \neg\exists x(Abx \wedge Ecx)$

   Postulate:

   $\neg\exists x(Nx \wedge Abx)$

[Q p.57]

[Contents]

Answers 12.5.4

Note that a formula may have more than one prenex equivalent (i.e. there may be other correct answers).

1. $\forall x \forall y (Px \vee Qy)$ [Q p.57]
2. $\exists x \exists y (Px \vee Qy)$ [Q p.57]
3. $\exists x \forall y (Px \rightarrow Py)$ [Q p.57]
4. $\exists x \forall y \exists z \forall w ((Pz \rightarrow Pw) \wedge (Px \rightarrow Py))$ [Q p.57]
5. $\exists x \exists y \forall z \neg (Sx \wedge (Ty \rightarrow Uxz))$ [Q p.57]

[Contents]

Chapter 13

Identity

Answers 13.2.2

Glossary

a:	Adam	Hx:	x is happy
b:	Ben	Mx:	x is a man
c:	Chris	Ox:	x is a town
d:	Diane	Px:	x is a person
e:	Edward	Tx:	x is a television show
f:	*Family Guy*	Wx:	x is a woman
g:	you	Cxy:	x is colder than y
h:	Sydney	Kxy:	x knows y
i:	Jindabyne	Lxy:	x is larger than y
j:	Jonathon	Oxy:	x owns y
k:	Melbourne	Sxy:	x is by y's side
l:	Canberra	Txy:	x is taller than y
m:	Mary	Vxy:	x watches y
n:	I/me	Wxy:	x wants y
s:	*Seinfeld*	$Bxyz$:	y is between x and z
Cx:	x is a chihuahua	$Pxyz$:	x prefers y to z
Dx:	x is a dog		

Translations:

1. $\forall x(x \neq c \rightarrow Lcx)$ [Q p.58]
2. $\forall x((x \neq c \wedge Dx) \rightarrow Bx) \wedge Cc$ [Q p.58]
3. $\exists x(Dx \wedge Sxb \wedge x \neq c) \rightarrow Hb$ [Q p.58]
4. $\forall x((Px \wedge Scx \wedge x \neq j) \rightarrow Hc)$ [Q p.58]

5. $\forall x(Dx \rightarrow Ljx)$ [Q p.58]
6. $\forall x(Wmx \rightarrow \exists y(Py \land y \neq m \land Oyx))$ [Q p.58]
7. $\exists x \exists y(Px \land x \neq m \land Wxy \land Omy)$ [Q p.58]
8. $\exists x(Omx \land \neg Wmx)$ [Q p.58]
9. $\exists x(Bx \land Omx) \rightarrow \forall y \forall z((Py \land Bz \land y \neq m) \rightarrow \neg Oyz)$ [Q p.58]
10. $\forall x \forall y((Px \land x \neq m \land Wmy) \rightarrow \neg Oxy)$ [Q p.58]
11. $\forall x(Px \rightarrow Pxsf)$ [Q p.58]
12. $\forall x((Tx \land x \neq s) \rightarrow Pasx)$ [Q p.58]
13. $\forall x((Tx \land x \neq f) \rightarrow Paxf)$ [Q p.58]
14. $Vjf \land \forall x((Tx \land x \neq f) \rightarrow \neg Vjx)$ [Q p.59]
15. $Vjf \land \forall x((Px \land x \neq j) \rightarrow \neg Vxf)$ [Q p.59]
16. $Wd \land \forall x((Wx \land x \neq d) \rightarrow Tdx)$ [Q p.59]
17. $\forall x((Mx \land Txd) \leftrightarrow x = e)$ [Q p.59]
18. $Ted \land \exists x(Wx \land x \neq d \land Tex)$ [Q p.59]
19. $\neg \exists x(Px \land Tdx \land Txe)$ [Q p.59]
20. $\exists x(Px \land x \neq e \land x \neq d)$ [Q p.59]
21. $\forall x((Px \land Kxb) \leftrightarrow x = g)$ [Q p.59]
22. $\exists x(Px \land Knx \land x \neq b)$ [Q p.59]
23. $\forall x((Kbx \land Px \land x \neq c \land x \neq n) \rightarrow Hx)$ [Q p.59]
24. $\forall x((Px \land Hx \land Knx) \leftrightarrow x = b)$ [Q p.59]
25. $Pb \land Hb \land Knb \land \forall x((Px \land Hx \land Knx \land x \neq b) \rightarrow Tbx)$ [Q p.59]
26. $Oi \land Bhik \land \forall x((Ox \land Bhxk \land x \neq i) \rightarrow Cix)$ [Q p.59]
27. $\exists x(Ox \land Bhxk \land Cxl)$ [Q p.59]
28. $\forall x((Ox \land x \neq i) \rightarrow \exists y(Oy \land Cyx))$ [Q p.59]
29. $\neg \exists x(Ox \land Bhxk \land (Lxl \lor Cxi))$ [Q p.59]

30. $Oi \wedge Bhik \wedge \forall x((Ox \wedge Bhxk \wedge x \neq i) \to Pnix)$ [Q p.59]

[Contents]

Answers 13.3.1

1. (i) True [Q p.60]
   (ii) False [Q p.60]
   (iii) True [Q p.60]
   (iv) True [Q p.60]
   (v) False [Q p.60]
   (vi) True [Q p.60]

2. (i) False [Q p.60]
   (ii) True [Q p.60]
   (iii) True [Q p.60]
   (iv) True [Q p.60]
   (v) True [Q p.60]
   (vi) False [Q p.60]

3. (i) (a) Domain: $\{1, 2, 3\}$
   Referent of a : 1
   Extension of F : $\{1\}$
   (b) Domain: $\{1, 2, 3\}$
   Referent of a : 1
   Extension of F : $\{1, 2\}$ [Q p.61]
   (ii) (a) Domain: $\{1, 2\}$
   Referents: $a : 1 \quad b : 1$
   (b) Domain: $\{1, 2\}$
   Referents: $a : 1 \quad b : 2$ [Q p.61]
   (iii) (a) Domain: $\{1, 2, 3\}$
   Extension of R : $\{\langle 1, 2\rangle, \langle 1, 3\rangle\}$
   (b) Domain: $\{1, 2, 3\}$
   Extension of R : $\{\langle 1, 2\rangle\}$ [Q p.61]

(iv) (a) Domain: $\{1,2,3\}$
Extension of $R : \{\langle 1,1\rangle, \langle 2,2\rangle\}$

(b) Domain: $\{1,2,3\}$
Extension of $R : \{\langle 1,1\rangle, \langle 1,2\rangle\}$ [Q p.61]

(v) (a) Domain: $\{1,2,3\}$
Extension of $R : \{\langle 1,2,1\rangle, \langle 1,3,1\rangle, \langle 2,1,2\rangle, \langle 2,3,2\rangle, \langle 3,1,3\rangle, \langle 3,2,3\rangle\}$

(b) Domain: $\{1,2,3\}$
Extension of $R : \{\langle 1,2,1\rangle, \langle 1,3,1\rangle\}$ [Q p.61]

(vi) (a) No such model: symmetry of identity.

(b) Domain: $\{1,2,3\}$
Referent of $a : 1$ [Q p.61]

(vii) (a) Domain: $\{1,2,3\}$
Extension of $F : \{1\}$

(b) Domain: $\{1,2,3\}$
Extension of $F : \{1,2\}$ [Q p.61]

(viii) (a) Domain: $\{1,2,3\}$
Extensions: $F : \{1,2\}$ $G : \{1\}$

(b) Domain: $\{1,2,3\}$
Extensions: $F : \{1\}$ $G : \{1,2\}$ [Q p.61]

(ix) (a) Domain: $\{1,2,3\}$
Extensions: $F : \{1,2\}$ $R : \{\langle 1,2\rangle, \langle 2,1\rangle\}$

(b) Domain: $\{1,2,3\}$
Extensions: $F : \{1,2\}$ $R : \{\langle 1,2\rangle, \langle 2,2\rangle\}$ [Q p.61]

(x) (a) Domain: $\{1,2,3\}$
Referents: $a : 1$
Extensions: $F : \{1\}$ $R : \emptyset$

(b) Domain: $\{1,2,3\}$
Referents: $a : 1$
Extensions: $F : \{1\}$ $R : \{\langle 1,1\rangle\}$ [Q p.61]

(xi) (a) Domain: $\{1,2,3\}$
Extensions: $R : \{\langle 1,2,3\rangle\}$

(b) Domain: $\{1,2,3\}$
Extensions: $R : \{\langle 1,2,1\rangle\}$ [Q p.61]

(xii) (a) Domain: $\{1,2,3\}$
Extensions: $R : \{\langle 1,2,3\rangle\}$

(b) Domain: $\{1,2,3\}$
Extensions: $R : \{\langle 1,2,1\rangle\}$ [Q p.61]

(xiii) (a) Domain: $\{1,2,3\}$
Extensions: $F : \{2,3\}$

(b) Domain: $\{1,2,3\}$
Extensions: $F : \{2\}$ [Q p.61]

(xiv) (a) Domain: $\{1,2,3\}$
Extensions: $F : \{1,2\}$ $R : \{\langle 1,2\rangle\}$

(b) Domain: $\{1,2,3\}$
Extensions: $F : \{1,2\}$ $R : \{\langle 1,1\rangle\}$ [Q p.61]

(xv) (a) Domain: $\{1,2,3\}$
Extensions: $R : \{\langle 1,2,3\rangle, \langle 1,1,1\rangle, \langle 2,2,2\rangle, \langle 3,3,3\rangle\}$

(b) Domain: $\{1,2,3\}$
Extensions: $R : \{\langle 1,2,3\rangle, \langle 1,1,1\rangle, \langle 2,2,2\rangle\}$ [Q p.61]

(xvi) (a) Domain: $\{1,2,3\}$
Extensions: $R : \{\langle 1,1\rangle\}$

(b) No such model: the only y identical to x is x itself. [Q p.61]

(xvii) (a) Domain: $\{1,2,3\}$
Referents: $a : 1$ $b : 1$
Extensions: $F : \{1\}$

(b) Domain: $\{1,2,3\}$
Referents: $a : 1$ $b : 2$
Extensions: $F : \{1,2\}$ [Q p.61]

(xviii) (a) Domain: $\{1,2,3\}$
Extensions: $F : \{1,2\}$

(b) Domain: $\{1,2,3\}$
Extensions: $F : \{1,2,3\}$ [Q p.61]

[Contents]

Answers 13.4.3

1. (i)

$$\begin{array}{c}
Rab \to \neg Rba \ \checkmark \\
Rab \\
a = b \\
\begin{array}{cc}
\neg Rab & \neg Rba \\
\times & Raa \\
 & Rba \\
 & \times
\end{array}
\end{array}$$

Unsatisfiable. [Q p.61]

(ii)

$$\begin{array}{c}
Rab \\
\neg Rbc \\
a = b \\
Raa \\
Rba \\
Rbb \\
\neg Rac \\
\uparrow
\end{array}$$

Satisfiable.
Domain: $\{1, 2\}$
Referents: $a : 1 \quad b : 1 \quad c : 2$
Extension of $R : \{\langle 1, 1\rangle\}$ [Q p.61]

(iii)

$$\forall x(Fx \to x = a) \ \backslash ab$$
$$Fa$$
$$a \neq b$$
$$Fa \to a = a \ \checkmark$$
$$\begin{array}{cc} \neg Fa & a = a \\ \times & Fb \to b = a \ \checkmark \\ & \begin{array}{cc} \neg Fb & b = a \\ \uparrow & a \neq a \\ & \times \end{array} \end{array}$$

Satisfiable.
Domain: $\{1, 2\}$
Referents: $a : 1 \quad b : 2$
Extension of $F : \{1\}$

[Q p.61]

(iv)

$$\forall x(Fx \to Gx) \ \backslash abc$$
$$\exists x Fx \ \checkmark c$$
$$\neg Ga$$
$$a = b$$
$$Fc$$
$$Fa \to Ga \ \checkmark$$
$$\begin{array}{cc} \neg Fa & Ga \\ \neg Fb & \times \\ Fb \to Gb \ \checkmark & \\ \begin{array}{cc} \neg Fb & Gb \\ Fc \to Gc \ \checkmark & \neg Gb \\ \begin{array}{cc} \neg Fc & Gc \\ \times & \neg Gb \\ & \uparrow \end{array} & \times \end{array} & \end{array}$$

Satisfiable.
Domain: $\{1, 2\}$
Referents: $a : 1 \quad b : 1 \quad c : 2$
Extensions: $F : \{2\} \quad G : \{2\}$

[Q p.61]

(v)

$$\begin{array}{c}
\forall x(x \neq a \to Rax) \ \backslash b \\
\forall x \neg Rxb \ \backslash ab \\
a \neq b \\
\neg Rab \\
\neg Rbb \\
b \neq a \to Rab \ \checkmark \\
\begin{array}{cc}
\neg(b \neq a) \ \checkmark & Rab \\
b = a & \times \\
a \neq a & \\
\times &
\end{array}
\end{array}$$

Unsatisfiable. [Q p.61]

(vi)

$$\begin{array}{c}
\exists x \forall y(Fy \to x = y) \ \checkmark c \\
Fa \\
Fb \\
\forall y(Fy \to c = y) \ \backslash abc \\
Fa \to c = a \ \checkmark \\
\begin{array}{cc}
\neg Fa & c = a \\
\times & Fb \to c = b \ \checkmark \\
 & \begin{array}{cc}
 \neg Fb & c = b \\
 \times & Fc \to c = c \\
  & \begin{array}{cc}
  \neg Fc & c = c \\
  \neg Fa & \uparrow \\
  \times &
  \end{array}
 \end{array}
\end{array}
\end{array}$$

Satisfiable.
Domain: $\{1\}$
Referents: $a : 1 \quad b : 1 \quad c : 1$
Extensions: $F : \{1\}$ [Q p.61]

(vii)

$$
\begin{array}{c}
\forall x \forall y (Rxy \to x = y) \setminus a \\
Rab \\
a \neq b \\
\forall y (Ray \to a = y) \setminus b \\
Rab \to a = b \ \checkmark \\
\begin{array}{cc}
\neg Rab & a = b \\
\times & a \neq a \\
 & \times
\end{array}
\end{array}
$$

Unsatisfiable. [Q p.62]

(viii)

$$
\begin{array}{c}
\forall x ((Fx \wedge Rxa) \to x \neq a) \setminus b \\
Fb \wedge Rba \ \checkmark \\
a = b \\
Fb \\
Rba \\
(Fb \wedge Rba) \to b \neq a \ \checkmark \\
\begin{array}{cc}
\begin{array}{c}
\neg(Fb \wedge Rba) \ \checkmark \\
\begin{array}{cc}
\neg Fb & \neg Rba \\
\times & \times
\end{array}
\end{array}
&
\begin{array}{c}
b \neq a \\
a \neq a \\
\times
\end{array}
\end{array}
\end{array}
$$

Unsatisfiable. [Q p.62]

(ix)

$\exists x \exists y \exists z Rxyz \ \checkmark a$

$\forall x(x = x \rightarrow x = a) \setminus abc$

$\exists y \exists z Rayz \ \checkmark b$

$\exists z Rabz \ \checkmark c$

$Rabc$

$a = a \rightarrow a = a \ \checkmark$

$\neg a = a \qquad a = a$

$\times \qquad b = b \rightarrow b = a \ \checkmark$

$\neg b = b \qquad b = a$

$\times \qquad c = c \rightarrow c = a \ \checkmark$

$\neg c = c \qquad c = a$

$\times \qquad Raba$

$Raaa$

$Raab$

$Raac$

$Rabb$

$Raca$

$Racb$

$Racc$

$Rbaa$

$Rbab$

$Rbac$

$Rbba$

$Rbbb$

$Rbbc$

$Rbca$

$Rbcb$

$Rbcc$

$Rcaa$

$Rcab$

$Rcac$

$Rcba$

$Rcbb$

$Rcbc$

$Rcca$

$Rccb$

$Rccc$

$\uparrow$

Satisfiable. (See next page for model.)

Domain: $\{1\}$
Referents: $a : 1 \quad b : 1 \quad c : 1$
Extensions: $R : \{\langle 1, 1, 1\rangle\}$ [Q p.62]

(x)

$$\begin{array}{c}
\forall x \neg Rxx \ \backslash a \\
\forall x \forall y x = y \ \backslash a \\
\exists x Rax \ \checkmark b \\
Rab \\
\forall y a = y \ \backslash b \\
a = b \\
Raa \\
\neg Raa \\
\times
\end{array}$$

Unsatisfiable. [Q p.62]

2. (i)

$$\begin{array}{c}
\exists x Fx \ \checkmark \mathrm{a} \\
\exists y Gy \ \checkmark b \\
\forall x \forall y x = y \ \backslash a \\
\neg \exists x (Fx \wedge Gx) \ \checkmark \\
\forall x \neg (Fx \wedge Gx) \ \backslash a \\
Fa \\
Gb \\
\forall y a = y \ \backslash b \\
a = b \\
Ga \\
\neg (Fa \wedge Ga) \ \checkmark \\
\begin{array}{cc} \neg Fa & \neg Ga \\ \times & \times \end{array}
\end{array}$$

Valid. [Q p.62]

(ii)

$$\exists x \exists y(Fx \wedge Gy \wedge \forall z(z = x \vee z = y)) \ \checkmark a$$
$$\neg\exists x(Fx \wedge Gx) \ \checkmark$$
$$\forall x \neg(Fx \wedge Gx) \ \backslash ab$$
$$\exists y(Fa \wedge Gy \wedge \forall z(z = a \vee z = y)) \ \checkmark b$$
$$Fa \wedge Gb \wedge \forall z(z = a \vee z = b) \ \checkmark$$
$$Fa$$
$$Gb$$
$$\forall z(z = a \vee z = b) \ \backslash ab$$
$$\neg(Fa \wedge Ga) \ \checkmark$$

$$\begin{array}{cc}
\neg Fa & \neg Ga \\
\times & \neg(Fb \wedge Gb) \ \checkmark
\end{array}$$

$$\begin{array}{cc}
\neg Fb & \neg Gb \\
a = a \vee a = b \ \checkmark & \times
\end{array}$$

$$\begin{array}{cc}
a = a & a = b \\
b = a \vee b = b \ \checkmark & \neg Fa \\
 & \times
\end{array}$$

$$\begin{array}{cc}
b = a & b = b \\
\neg Fa & \uparrow \\
\times &
\end{array}$$

Invalid. Countermodel:
Domain: $\{1, 2\}$
Referents: $a : 1 \quad b : 2$
Extensions: $F : \{1\} \quad G : \{2\}$ [Q p.62]

(iii)

$$
\begin{array}{c}
Rab \\
\neg\forall x\forall y\forall z(((Rxy \wedge Ryz) \wedge x = z) \to Ryy) \ \checkmark \\
\exists x\neg\forall y\forall z(((Rxy \wedge Ryz) \wedge x = z) \to Ryy) \ \checkmark c \\
\neg\forall y\forall z(((Rcy \wedge Ryz) \wedge c = z) \to Ryy) \ \checkmark \\
\exists y\neg\forall z(((Rcy \wedge Ryz) \wedge c = z) \to Ryy) \ \checkmark d \\
\neg\forall z(((Rcd \wedge Rdz) \wedge c = z) \to Rdd) \ \checkmark \\
\exists z\neg(((Rcd \wedge Rdz) \wedge c = z) \to Rdd) \ \checkmark e \\
\neg(((Rcd \wedge Rde) \wedge c = e) \to Rdd) \ \checkmark \\
(Rcd \wedge Rde) \wedge c = e) \ \checkmark \\
\neg Rdd \\
Rcd \wedge Rde \ \checkmark \\
c = e \\
Rcd \\
Rde \\
Red \\
Rdc \\
\uparrow
\end{array}
$$

Invalid. Countermodel:
Domain: $\{1, 2, 3, 4\}$
Referents: $a : 1 \quad b : 2 \quad c : 3 \quad d : 4 \quad e : 3$
Extension of R : $\{\langle 1, 2\rangle, \langle 3, 4\rangle, \langle 4, 3\rangle\}$

[Q p.62]

(iv)

$$
\begin{array}{c}
\forall x\forall y(Rxy \to Ryx) \ \backslash a \\
\exists x(Rax \wedge x \neq b) \ \checkmark c \\
\neg\exists x(Rxa \wedge x \neq b) \ \checkmark \\
\forall x\neg(Rxa \wedge x \neq b) \ \backslash c \\
Rac \wedge c \neq b \ \checkmark \\
Rac \\
c \neq b \\
\forall y(Ray \to Rya) \ \backslash c \\
Rac \to Rca \ \checkmark
\end{array}
$$

$$
\begin{array}{cc}
\neg Rac & Rca \\
\times & \neg(Rca \wedge c \neq b) \ \checkmark \\
 & \begin{array}{cc}
\neg Rca & \neg(c \neq b) \ \checkmark \\
\times & c = b \\
 & \times
\end{array}
\end{array}
$$

Valid.

[Q p.62]

(v)

$$
\begin{array}{c}
\forall x \forall y\, x = y \;\; \backslash a \\
\neg \forall x \forall y (Rxy \rightarrow Ryx) \;\; \checkmark \\
\exists x \neg \forall y (Rxy \rightarrow Ryx) \;\; \checkmark a \\
\neg \forall y (Ray \rightarrow Rya) \;\; \checkmark \\
\exists y \neg (Ray \rightarrow Rya) \;\; \checkmark b \\
\neg (Rab \rightarrow Rba) \;\; \checkmark \\
Rab \\
\neg Rba \\
\forall y\, a = y \;\; \backslash b \\
a = b \\
Raa \\
Rba \\
\times
\end{array}
$$

Valid. [Q p.62]

(vi)

$$
\begin{array}{c}
\forall x \forall y \forall z ((Rxy \wedge Rxz) \rightarrow y = z) \;\backslash a \\
Rab \wedge Rcd \;\; \checkmark \\
b \neq d \\
\neg a \neq c \;\; \checkmark \\
a = c \\
Rab \\
Rcd \\
\forall y \forall z ((Ray \wedge Raz) \rightarrow y = z) \;\backslash b \\
\forall z ((Rab \wedge Raz) \rightarrow b = z) \;\backslash d \\
(Rab \wedge Rad) \rightarrow b = d \;\; \checkmark \\
\begin{array}{cc}
\neg (Rab \wedge Rad) \;\; \checkmark & b = d \\
\begin{array}{cc}
\neg Rab & \neg Rad \\
\times & \neg Rcd \\
 & \times
\end{array} & \begin{array}{c} b \neq b \\ \times \end{array}
\end{array}
\end{array}
$$

Valid. [Q p.62]

(vii)

$$\begin{array}{c}
\exists x \exists y(Rxy \wedge x = y) \ \checkmark a \\
\neg\neg\forall x Rxx \ \checkmark \\
\forall x Rxx \setminus ab \\
\exists y(Ray \wedge a = y) \ \checkmark b \\
Rab \wedge a = b \ \checkmark \\
Rab \\
a = b \\
Raa \\
Rbb \\
Rba \\
\uparrow
\end{array}$$

Invalid. Countermodel:
Domain: $\{1\}$
Referents: $a : 1 \quad b : 1$
Extension of R : $\{\langle 1, 1\rangle\}$ [Q p.62]

(viii)

$$\begin{array}{c}
\forall x(x = a \vee x = b) \setminus cab \\
\neg\forall x x = a \ \checkmark \\
\exists x \neg x = a \ \checkmark c \\
\neg c = a \\
c = a \vee c = b \ \checkmark
\end{array}$$

$$\begin{array}{cc}
\begin{array}{c} c = a \\ \neg c = c \\ \times \end{array} &
\begin{array}{c} c = b \\ a = a \vee a = b \ \checkmark \\
\begin{array}{cc}
\begin{array}{c} a = a \\ b = a \vee b = b \ \checkmark \\
\begin{array}{cc}
\begin{array}{c} b = a \\ c = a \\ \neg c = c \\ \times \end{array} &
\begin{array}{c} b = b \\ \neg b = a \\ \uparrow \end{array}
\end{array}
\end{array} &
\begin{array}{c} a = b \\ a = c \\ \neg c = c \\ \times \end{array}
\end{array}
\end{array}
\end{array}$$

Invalid. Countermodel:
Domain: $\{1, 2\}$
Referents: $a : 1 \quad b : 2 \quad c : 2$ [Q p.62]

(ix)

$$\begin{array}{c}
\forall x Rax \setminus abc \\
\neg\forall x \forall y x = y \ \checkmark \\
\neg\exists x \exists y \exists z (Rxy \wedge Rxz \wedge y \neq z) \ \checkmark \\
\exists x \neg\forall y x = y \ \checkmark b \\
\neg\forall y b = y \ \checkmark \\
\exists y \neg b = y \ \checkmark c \\
\neg b = c \\
Raa \\
Rab \\
Rac \\
\forall x \neg\exists y \exists z (Rxy \wedge Rxz \wedge y \neq z) \setminus a \\
\neg\exists y \exists z (Ray \wedge Raz \wedge y \neq z) \ \checkmark \\
\forall y \neg\exists z (Ray \wedge Raz \wedge y \neq z) \setminus b \\
\neg\exists z (Rab \wedge Raz \wedge b \neq z) \ \checkmark \\
\forall z \neg(Rab \wedge Raz \wedge b \neq z) \setminus c \\
\neg(Rab \wedge Rac \wedge b \neq c) \ \checkmark \\
\begin{array}{ccc}
\neg Rab & \neg Rac & \neg b \neq c \ \checkmark \\
\times & \times & b = c \\
 & & \neg b = b \\
 & & \times
\end{array}
\end{array}$$

Valid. [Q p.62]

(x)

$$\begin{array}{c}
\forall x x = a \setminus bc \\
\neg\forall x x = b \ \checkmark \\
\exists x \neg x = b \ \checkmark c \\
\neg c = b \\
b = a \\
c = a \\
b = c \\
\neg b = b \\
\times
\end{array}$$

Valid. [Q p.62]

3. (i) s: Stan
Fx: x is a firefighter

$(Fs \wedge \forall x(Fx \to x = s)) \to \neg\exists x(Fx \wedge x \neq s)$

$$
\begin{array}{c}
\neg((Fs \wedge \forall x(Fx \to x = s)) \to \neg\exists x(Fx \wedge x \neq s)) \\
Fs \wedge \forall x(Fx \to x = s) \ \checkmark \\
\neg\neg\exists x(Fx \wedge x \neq s) \ \checkmark \\
\exists x(Fx \wedge x \neq s) \ \checkmark a \\
Fs \\
\forall x(Fs \to x = s) \ \backslash a \\
Fa \wedge a \neq s \ \checkmark \\
Fa \\
a \neq s \\
Fs \to a = s \ \checkmark \\
\begin{array}{cc}
\neg Fs & a = s \\
\times & \times
\end{array}
\end{array}
$$

Logical truth. [Q p.63]

(ii) c: Lewis Carroll
j: Julius Caesar
Lx: x is left-handed

$(Lj \wedge \neg Lc) \to c \neq j$

$$
\begin{array}{c}
\neg((Lj \wedge \neg Lc) \to c \neq j) \ \checkmark \\
Lj \wedge \neg Lc \ \checkmark \\
\neg c \neq j \ \checkmark \\
c = j \\
Lj \\
\neg Lc \\
\neg Lj \\
\times
\end{array}
$$

Logical truth. [Q p.63]

(iii) a: Apollo
s: the sun
Wxy: x is warming y

$\forall x(s \neq x \leftrightarrow Wsx) \to Wsa$

$$
\begin{array}{c}
\neg(\forall x(s \neq x \leftrightarrow Wsx) \to Wsa) \\
\forall x(s \neq x \leftrightarrow Wsx) \ \backslash as \\
\neg Wsa \\
s \neq a \leftrightarrow Wsa \ \checkmark \\
\begin{array}{cc}
\begin{array}{c} s \neq a \\ Wsa \\ \times \end{array} &
\begin{array}{c} \neg s \neq a \ \checkmark \\ \neg Wsa \\ s = a \\ s \neq s \leftrightarrow Wss \ \checkmark \\
\begin{array}{cc}
\begin{array}{c} s \neq s \\ Wss \\ \times \end{array} &
\begin{array}{c} \neg s \neq s \\ \neg Wss \\ s = s \\ \neg Was \\ \neg Waa \\ \uparrow \end{array}
\end{array}
\end{array}
\end{array}
\end{array}
$$

Not a logical truth. Countermodel:
Domain: $\{1\}$
Referents: $a : 1 \quad s : 1$
Extension of $W : \emptyset$ [Q p.63]

(iv) k: Kevin Bacon
m: Michael J. Fox

$k \neq k \to k = m$

$$
\begin{array}{c}
\neg(k \neq k \to k = m) \ \checkmark \\
k \neq k \\
k \neq m \\
\times
\end{array}
$$

Logical truth. ($k \neq k$ is logically false; *any* conditional with a logically false antecedent is logically true.) [Q p.63]

(v) c: Clemens
t: Twain
Ax: x is an author
Wx: x is witty

$(\forall x((Ax \wedge Wx) \rightarrow x = t) \wedge Ac) \rightarrow \neg Wc$

$$
\begin{array}{c}
\neg((\forall x((Ax \wedge Wx) \rightarrow x = t) \wedge Ac) \rightarrow \neg Wc) \ \checkmark \\
\forall x((Ax \wedge Wx) \rightarrow x = t) \wedge Ac \ \checkmark \\
\neg \neg Wc \ \checkmark \\
Wc \\
\forall x((Ax \wedge Wx) \rightarrow x = t) \ \backslash ct \\
Ac \\
(Ac \wedge Wc) \rightarrow c = t \ \checkmark
\end{array}
$$

$$
\begin{array}{cc}
\begin{array}{c}
\neg(Ac \wedge Wc) \ \checkmark \\
\begin{array}{cc}
\neg Ac & \neg Wc \\
\times & \times
\end{array}
\end{array}
&
\begin{array}{c}
c = t \\
(At \wedge Wt) \rightarrow t = t \ \checkmark \\
\begin{array}{cc}
\begin{array}{c}
\neg(At \wedge Wt) \ \checkmark \\
\begin{array}{cc}
\neg At & \neg Wt \\
\neg Ac & \neg Wc \\
\times & \times
\end{array}
\end{array}
&
\begin{array}{c}
t = t \\
Wt \\
At \\
\uparrow
\end{array}
\end{array}
\end{array}
\end{array}
$$

Not a logical truth. Countermodel:
Domain: $\{1\}$
Referents: $c : 1 \quad t : 1$
Extensions: $A : \{1\} \quad W : \{1\}$ [Q p.63]

(vi) Sx: x is a spy
Txy: x trusts y

$\forall x \forall y((Sx \wedge Sy \wedge x \neq y) \to \neg Txy)$

$$\begin{array}{c}
\neg\forall x \forall y((Sx \wedge Sy \wedge x \neq y) \to \neg Txy) \ \checkmark \\
\exists x \neg\forall y((Sx \wedge Sy \wedge x \neq y) \to \neg Txy) \ \checkmark a \\
\neg\forall y((Sa \wedge Sy \wedge a \neq y) \to \neg Tay) \ \checkmark \\
\exists y \neg((Sa \wedge Sy \wedge a \neq y) \to \neg Tay) \ \checkmark b \\
\neg((Sa \wedge Sb \wedge a \neq b) \to \neg Tab) \ \checkmark \\
Sa \wedge Sb \wedge a \neq b \ \checkmark \\
\neg\neg Tab \ \checkmark \\
Tab \\
Sa \\
Sb \\
a \neq b \\
\uparrow
\end{array}$$

Not a logical truth. Countermodel:
Domain: $\{1,2\}$
Referents: $a:1 \quad b:2$
Extensions: $S:\{1,2\} \quad T:\{\langle 1,2\rangle\}$ [Q p.63]

(vii) a: this ant

$\forall x x = a \vee \neg\exists x x = a$

$$\begin{array}{c}
\neg(\forall x x = a \vee \neg\exists x x = a) \ \checkmark \\
\neg\forall x x = a \ \checkmark \\
\neg\neg\exists x x = a \ \checkmark \\
\exists x x = a \ \checkmark \mathrm{b} \\
\exists x \neg x = a \ \checkmark c \\
b = a \\
\neg c = a \\
\neg c = b \\
\uparrow
\end{array}$$

Not a logical truth. Countermodel:
Domain: $\{1,2\}$
Referents: $a:1 \quad b:1 \quad c:2$ [Q p.63]

(viii) d: Doug
s: Santa Claus
Axy: x is afraid of y

$\forall x(x \neq s \to Adx) \to (Add \vee d = s)$

$$
\begin{array}{c}
\neg(\forall x(x \neq s \to Adx) \to (Add \vee d = s)) \checkmark \\
\forall x(x \neq s \to Adx) \setminus d \\
\neg(Add \vee d = s) \checkmark \\
\neg Add \\
\neg d = s \\
d \neq s \to Add \checkmark \\
\begin{array}{cc}
\neg d \neq s \checkmark & Add \\
d = s & \times \\
\neg d = d & \\
\times &
\end{array}
\end{array}
$$

Logical truth. [Q p.63]

(ix) m: Mark
s: Samuel
Rxy: x respects y

$(Rms \wedge \forall x(Rmx \to x = s)) \to \neg Rmm$

$$
\begin{array}{c}
\neg((Rms \wedge \forall x(Rmx \to x = s)) \to \neg Rmm) \ \checkmark \\
Rms \wedge \forall x(Rmx \to x = s) \ \checkmark \\
\neg\neg Rmm \ \checkmark \\
Rmm \\
Rms \\
\forall x(Rmx \to x = s) \ \backslash ms \\
Rmm \to m = s \ \checkmark
\end{array}
$$

$$
\begin{array}{cc}
\neg Rmm & m = s \\
\times & Rms \to s = s \ \checkmark
\end{array}
$$

$$
\begin{array}{cc}
\neg Rms & s = s \\
\times & Rss \\
 & Rsm \\
 & \uparrow
\end{array}
$$

Not a logical truth. Countermodel:
Domain: $\{1\}$
Referents: $m : 1 \quad s : 1$
Extension: $R : \{\langle 1, 1\rangle\}$ [Q p.63]

(x) m: I/me
Px: x is a physical body

$Pm \vee \exists x(x = m \wedge \neg Px)$

$$
\begin{array}{c}
\neg(Pm \vee \exists x(x = m \wedge \neg Px)) \ \checkmark \\
\neg Pm \\
\neg\exists x(x = m \wedge \neg Px) \ \checkmark \\
\forall x \neg(x = m \wedge \neg Px) \ \backslash m \\
\neg(m = m \wedge \neg Pm) \ \checkmark \\
\begin{array}{cc}
\neg m = m & \neg\neg Pm \ \checkmark \\
\times & Pm \\
 & \times
\end{array}
\end{array}
$$

Logical truth. [Q p.63]

[Contents]

Answers 13.5.1

1. (i) Gx: x is a gremlin

$$\forall x \forall y \forall z((Gx \wedge Gy \wedge Gz) \to (x = y \vee x = z \vee y = z))$$

$$\begin{array}{c}
\neg\forall x \forall y \forall z((Gx \wedge Gy \wedge Gz) \to (x = y \vee x = z \vee y = z)) \ \checkmark \\
\exists x \neg\forall y \forall z((Gx \wedge Gy \wedge Gz) \to (x = y \vee x = z \vee y = z)) \ \checkmark a \\
\neg\forall y \forall z((Ga \wedge Gy \wedge Gz) \to (a = y \vee a = z \vee y = z)) \ \checkmark \\
\exists y \neg\forall z((Ga \wedge Gy \wedge Gz) \to (a = y \vee a = z \vee y = z)) \ \checkmark b \\
\neg\forall z((Ga \wedge Gb \wedge Gz) \to (a = b \vee a = z \vee b = z)) \ \checkmark \\
\exists z \neg((Ga \wedge Gb \wedge Gz) \to (a = b \vee a = z \vee b = z)) \ \checkmark c \\
\neg((Ga \wedge Gb \wedge Gc) \to (a = b \vee a = c \vee b = c)) \ \checkmark \\
Ga \wedge Gb \wedge Gc \ \checkmark \\
\neg(a = b \vee a = c \vee b = c) \ \checkmark \\
Ga \\
Gb \\
Gc \\
\neg a = b \\
\neg a = c \\
\neg b = c \\
\uparrow
\end{array}$$

Not a logical truth. Countermodel:
Domain: $\{1, 2, 3\}$
Referents: $a : 1 \quad b : 2 \quad c : 3$
Extension of $G : \{1, 2, 3\}$ [Q p.63]

(ii) Bx: x is a Beatle

$$\exists x \exists y \exists z (Bx \wedge By \wedge Bz \wedge (x \neq y \wedge x \neq z \wedge y \neq z))$$

$$\begin{array}{c}
\neg \exists x \exists y \exists z (Bx \wedge By \wedge Bz \wedge (x \neq y \wedge x \neq z \wedge y \neq z)) \checkmark \\
\forall x \neg \exists y \exists z (Bx \wedge By \wedge Bz \wedge (x \neq y \wedge x \neq z \wedge y \neq z)) \ \backslash a \\
\neg \exists y \exists z (Ba \wedge By \wedge Bz \wedge (a \neq y \wedge a \neq z \wedge y \neq z)) \checkmark \\
\forall y \neg \exists z (Ba \wedge By \wedge Bz \wedge (a \neq y \wedge a \neq z \wedge y \neq z)) \ \backslash a \\
\neg \exists z (Ba \wedge Ba \wedge Bz \wedge (a \neq a \wedge a \neq z \wedge a \neq z)) \checkmark \\
\forall z \neg (Ba \wedge Ba \wedge Bz \wedge (a \neq a \wedge a \neq z \wedge a \neq z)) \ \backslash a \\
\neg (Ba \wedge Ba \wedge Ba \wedge (a \neq a \wedge a \neq a \wedge a \neq a)) \checkmark
\end{array}$$

$$\begin{array}{cccc}
\neg Ba & \neg Ba & \neg Ba & \neg (a \neq a \wedge a \neq a \wedge a \neq a) \checkmark \\
\uparrow & \uparrow & \uparrow & \begin{array}{ccc}
\neg (a \neq a) \checkmark & \neg (a \neq a) \checkmark & \neg (a \neq a) \checkmark \\
a = a & a = a & a = a \\
\uparrow & \uparrow & \uparrow
\end{array}
\end{array}$$

Not a logical truth. Countermodel:
Domain: $\{1\}$
Referent of $a : 1$
Extension of $B : \emptyset$

[Q p.63]

(iii) k: Kevin Bacon

$$\exists x x = k \wedge \forall x \forall y((x = k \wedge y = k) \rightarrow x = y)$$

$$\begin{array}{c}
\neg(\exists x x = k \wedge \forall x \forall y((x = k \wedge y = k) \rightarrow x = y)) \;\checkmark \\
\begin{array}{cc}
\begin{array}{c}
\neg \exists x x = k \;\checkmark \\
\forall x \neg x = k \;\backslash k \\
\neg k = k \\
\times
\end{array}
&
\begin{array}{c}
\neg \forall x \forall y((x = k \wedge y = k) \rightarrow x = y) \;\checkmark \\
\exists x \neg \forall y((x = k \wedge y = k) \rightarrow x = y) \;\checkmark a \\
\neg \forall y((a = k \wedge y = k) \rightarrow a = y) \;\checkmark \\
\exists y \neg((a = k \wedge y = k) \rightarrow a = y) \;\checkmark b \\
\neg((a = k \wedge b = k) \rightarrow a = b) \;\checkmark \\
a = k \wedge b = k \;\checkmark \\
\neg a = b \\
a = k \\
b = k \\
\neg k = b \\
\neg k = k \\
\times
\end{array}
\end{array}
\end{array}$$

Logical truth. (This may seem odd at first sight—but note what it means: on every model *of the fragment* of GPLI used to state the wff in question—the fragment which contains the name k—the wff comes out true. Of course!—that's not strange at all: every such model assigns exactly one referent to k.) [Q p.63]

(iv) Ox: x is an ocean

$\exists x \exists y(Ox \wedge Oy \wedge x \neq y) \rightarrow \exists x Ox$

$$
\begin{array}{c}
\neg(\exists x \exists y(Ox \wedge Oy \wedge x \neq y) \rightarrow \exists x Ox) \\
\exists x \exists y(Ox \wedge Oy \wedge x \neq y) \ \checkmark a \\
\neg \exists x Ox \ \checkmark \\
\forall x \neg Ox \ \backslash a \\
\exists y(Oa \wedge Oy \wedge a \neq y) \ \checkmark b \\
Oa \wedge Ob \wedge a \neq b \ \checkmark \\
Oa \\
Ob \\
a \neq b \\
\neg Oa \\
\times
\end{array}
$$

Logical truth. [Q p.63]

(v) Dx: x is a dog
Lxy: x is larger than y

$\forall x \forall y((Dx \wedge Dy \wedge x \neq y \wedge Lxy) \rightarrow \neg Lyx)$

$$
\begin{array}{c}
\neg \forall x \forall y((Dx \wedge Dy \wedge x \neq y \wedge Lxy) \rightarrow \neg Lyx) \ \checkmark \\
\exists x \neg \forall y((Dx \wedge Dy \wedge x \neq y \wedge Lxy) \rightarrow Lyx) \ \checkmark a \\
\neg \forall y((Da \wedge Dy \wedge a \neq y \wedge Lay) \rightarrow \neg Lya) \ \checkmark \\
\exists y \neg((Da \wedge Dy \wedge a \neq y \wedge Lay) \rightarrow \neg Lya) \ \checkmark b \\
\neg((Da \wedge Db \wedge a \neq b \wedge Lab) \rightarrow \neg Lba) \ \checkmark \\
Da \wedge Db \wedge a \neq b \wedge Lab \ \checkmark \\
\neg\neg Lba \ \checkmark \\
Lba \\
Da \\
Db \\
a \neq b \\
Lab \\
\uparrow
\end{array}
$$

Not a logical truth. Countermodel:
Domain: $\{1, 2\}$
Referents: $a : 1 \quad b : 2$
Extensions: $D : \{1, 2\} \quad L : \{\langle 1, 2\rangle, \langle 2, 1\rangle\}$ [Q p.64]

(vi) Ax: x is an apple

$$(\exists x Ax \wedge \forall x \forall y((Ax \wedge Ay) \to x = y)) \to \exists x Ax$$

$$\begin{array}{c}
\neg((\exists x Ax \wedge \forall x \forall y((Ax \wedge Ay) \to x = y)) \to \exists x Ax)\ \checkmark \\
\exists x Ax \wedge \forall x \forall y((Ax \wedge Ay) \to x = y)\ \checkmark \\
\neg \exists x Ax\ \checkmark \\
\forall x \neg Ax \setminus a \\
\exists x Ax\ \checkmark a \\
\forall x \forall y((Ax \wedge Ay) \to x = y) \\
Aa \\
\neg Aa \\
\times
\end{array}$$

Logical truth. [Q p.64]

(vii) Ax: x is an apple

$$\neg(\exists x \exists y(Ax \wedge Ay \wedge x \neq y) \wedge \forall x \forall y((Ax \wedge Ay) \to x = y))$$

$$\begin{array}{c}
\neg\neg(\exists x \exists y(Ax \wedge Ay \wedge x \neq y) \wedge \forall x \forall y((Ax \wedge Ay) \to x = y))\ \checkmark \\
\exists x \exists y(Ax \wedge Ay \wedge x \neq y) \wedge \forall x \forall y((Ax \wedge Ay) \to x = y)\ \checkmark \\
\exists x \exists y(Ax \wedge Ay \wedge x \neq y)\ \checkmark a \\
\forall x \forall y((Ax \wedge Ay) \to x = y) \setminus a \\
\exists y(Aa \wedge Ay \wedge a \neq y)\ \checkmark b \\
Aa \wedge Ab \wedge a \neq b\ \checkmark \\
Aa \\
Ab \\
a \neq b \\
\forall y((Aa \wedge Ay) \to a = y) \setminus b \\
(Aa \wedge Ab) \to a = b\ \checkmark
\end{array}$$

$$\begin{array}{ccc}
\multicolumn{2}{c}{\neg(Aa \wedge Ab)\ \checkmark} & a = b \\
\neg Aa & \neg Ab & a \neq a \\
\times & \times & \times
\end{array}$$

Logical truth. [Q p.64]

(viii) Sx: x is a snake

$\neg\exists xSx \vee \exists x\exists y(Sx \wedge Sy \wedge x \neq y)$

$$
\begin{array}{c}
\neg(\neg\exists xSx \vee \exists x\exists y(Sx \wedge Sy \wedge x \neq y)) \ \checkmark \\
\neg\neg\exists xSx \ \checkmark \\
\neg\exists x\exists y(Sx \wedge Sy \wedge x \neq y) \ \checkmark \\
\exists xSx \ \checkmark \\
Sa \\
\forall x\neg\exists y(Sx \wedge Sy \wedge x \neq y) \ \backslash a \\
\neg\exists y(Sa \wedge Sy \wedge a \neq y) \ \checkmark \\
\forall y\neg(Sa \wedge Sy \wedge a \neq y) \ \backslash a \\
\neg(Sa \wedge Sa \wedge a \neq a) \ \checkmark \\
\begin{array}{ccc}
\neg Sa & \neg Sa & \neg a \neq a \ \checkmark \\
\times & \times & a = a \\
 & & \uparrow
\end{array}
\end{array}
$$

Not a logical truth. Countermodel:
Domain: $\{1\}$
Referent of a : 1
Extension of S : $\{1\}$ [Q p.64]

2. (i) Rx: x is in the room

$$\exists x \exists y \exists z(Rx \wedge Ry \wedge Rz \wedge x \neq y \wedge x \neq z \wedge y \neq z)$$
$$\therefore\ \exists x \exists y(Rx \wedge Ry \wedge x \neq y)$$

$$\begin{array}{c}
\exists x \exists y \exists z(Rx \wedge Ry \wedge Rz \wedge x \neq y \wedge x \neq z \wedge y \neq z)\ \checkmark a \\
\neg \exists x \exists y(Rx \wedge Ry \wedge x \neq y)\ \checkmark \\
\exists y \exists z(Ra \wedge Ry \wedge Rz \wedge a \neq y \wedge a \neq z \wedge y \neq z)\ \checkmark b \\
\exists z(Ra \wedge Rb \wedge Rz \wedge a \neq b \wedge a \neq z \wedge b \neq z)\ \checkmark c \\
Ra \wedge Rb \wedge Rc \wedge a \neq b \wedge a \neq c \wedge b \neq c\ \checkmark \\
Ra \\
Rb \\
Rc \\
a \neq b \\
a \neq c \\
b \neq c \\
\forall x \neg \exists y(Rx \wedge Ry \wedge x \neq y)\ \backslash a \\
\neg \exists y(Ra \wedge Ry \wedge a \neq y)\ \checkmark \\
\forall y \neg(Ra \wedge Ry \wedge a \neq y)\ \backslash b \\
\neg(Ra \wedge Rb \wedge a \neq b)\ \checkmark \\
\begin{array}{ccc}
\neg Ra & \neg Rb & \neg a \neq b \\
\times & \times & \times
\end{array}
\end{array}$$

Valid. [Q p.64]

(ii) c: Canada
Bx: x is a bear
Nxy: x is in y

$\exists x \exists y(Bx \wedge By \wedge Nxc \wedge Nyc \wedge x \neq y)$
$\therefore\ \forall x \forall y \forall z((Bx \wedge By \wedge Bz \wedge Nxc \wedge Nyc \wedge Nzc) \to (x = y \vee x = z \vee y = z))$

$$
\begin{array}{c}
\exists x \exists y(Bx \wedge By \wedge Nxc \wedge Nyc \wedge x \neq y)\ \checkmark a \\
\neg \forall x \forall y \forall z((Bx \wedge By \wedge Bz \wedge Nxc \wedge Nyc \wedge Nzc) \to (x = y \vee x = z \vee y = z))\ \checkmark \\
\exists y(Ba \wedge By \wedge Nac \wedge Nyc \wedge a \neq y)\ \checkmark b \\
Ba \wedge Bb \wedge Nac \wedge Nbc \wedge a \neq b\ \checkmark \\
Ba \\
Bb \\
Nac \\
Nbc \\
a \neq b \\
\exists x \neg \forall y \forall z((Bx \wedge By \wedge Bz \wedge Nxc \wedge Nyc \wedge Nzc) \to (x = y \vee x = z \vee y = z))\ \checkmark d \\
\neg \forall y \forall z((Bd \wedge By \wedge Bz \wedge Ndc \wedge Nyc \wedge Nzc) \to (d = y \vee d = z \vee y = z))\ \checkmark \\
\exists y \neg \forall z((Bd \wedge By \wedge Bz \wedge Ndc \wedge Nyc \wedge Nzc) \to (d = y \vee d = z \vee y = z))\ \checkmark e \\
\neg \forall z((Bd \wedge Be \wedge Bz \wedge Ndc \wedge Nec \wedge Nzc) \to (d = e \vee d = z \vee e = z))\ \checkmark \\
\exists z \neg((Bd \wedge Be \wedge Bz \wedge Ndc \wedge Nec \wedge Nzc) \to (d = e \vee d = z \vee e = z))\ \checkmark f \\
\neg((Bd \wedge Be \wedge Bf \wedge Ndc \wedge Nec \wedge Nfc) \to (d = e \vee d = f \vee e = f))\ \checkmark \\
Bd \wedge Be \wedge Bf \wedge Ndc \wedge Nec \wedge Nfc \\
\neg(d = e \vee d = f \vee e = f) \\
Bd \\
Be \\
Bf \\
Ndc \\
Nec \\
Nfc \\
\neg d = e \\
\neg d = f \\
\neg e = f \\
\uparrow
\end{array}
$$

Invalid. Countermodel:
Domain: $\{1, 2, 3, 4, 5, 6\}$
Referents: $a : 1 \quad b : 2 \quad c : 3 \quad d : 4 \quad e : 5 \quad f : 6$
Extensions: $B : \{1, 2, 4, 5, 6\}$
$N : \{\langle 1, 3\rangle, \langle 2, 3\rangle, \langle 4, 3\rangle, \langle 5, 3\rangle, \langle 6, 3\rangle\}$ [Q p.64]

(iii) Bx: x is a barber
Hxy: x cuts y's hair

$\forall x \forall y((Bx \wedge By) \to x = y)$
$\therefore \forall x(Bx \to Hxx) \vee \forall x \forall y((Bx \wedge By) \to \neg Hxy)$

$$
\begin{array}{c}
\forall x \forall y((Bx \wedge By) \to x = y) \ \backslash a \\
\neg(\forall x(Bx \to Hxx) \vee \forall x \forall y((Bx \wedge By) \to \neg Hxy)) \ \checkmark \\
\neg \forall x(Bx \to Hxx) \ \checkmark \\
\neg \forall x \forall y((Bx \wedge By) \to \neg Hxy) \ \checkmark \\
\exists x \neg(Bx \to Hxx) \ \checkmark a \\
\exists x \neg \forall y((Bx \wedge By) \to \neg Hxy) \ \checkmark b \\
\neg(Ba \to Haa) \ \checkmark \\
Ba \\
\neg Haa \\
\neg \forall y((Bb \wedge By) \to \neg Hby) \ \checkmark \\
\exists y \neg((Bb \wedge By) \to \neg Hby) \ \checkmark c \\
\neg((Bb \wedge Bc) \to \neg Hbc) \ \checkmark \\
Bb \wedge Bc \ \checkmark \\
\neg\neg Hbc \ \checkmark \\
Bb \\
Bc \\
Hbc \\
\forall y((Ba \wedge By) \to a = y) \ \backslash bc \\
(Ba \wedge Bb) \to a = b \ \checkmark
\end{array}
$$

$$
\begin{array}{cc}
\begin{array}{c}
\neg(Ba \wedge Bb) \ \checkmark \\
\begin{array}{cc} \neg Ba & \neg Bb \\ \times & \times \end{array}
\end{array}
&
\begin{array}{c}
a = b \\
Hac \\
(Ba \wedge Bc) \to a = c \ \checkmark \\
\begin{array}{cc}
\begin{array}{c} \neg(Ba \wedge Bc) \ \checkmark \\ \begin{array}{cc} \neg Ba & \neg Bc \\ \times & \times \end{array} \end{array}
&
\begin{array}{c} a = c \\ Haa \\ \times \end{array}
\end{array}
\end{array}
\end{array}
$$

Valid. [Q p.64]

(iv) Hxy: x is heavier than y

$$\forall x\forall y\forall z(x = y \vee x = z \vee y = z)$$
$$\forall x\forall y(Hxy \vee Hyx)$$
$$\therefore \forall x(\forall y(x \neq y \to Hxy) \vee \forall y(x \neq y \to Hyx))$$

$$\begin{array}{c}
\forall x\forall y\forall z(x = y \vee x = z \vee y = z) \ \backslash a \\
\forall x\forall y(Hxy \vee Hyx) \ \backslash a \\
\neg\forall x(\forall y(x \neq y \to Hxy) \vee \forall y(x \neq y \to Hyx)) \checkmark \\
\exists x\neg(\forall y(x \neq y \to Hxy) \vee \forall y(x \neq y \to Hyx)) \checkmark a \\
\neg(\forall y(a \neq y \to Hay) \vee \forall y(a \neq y \to Hya)) \checkmark \\
\neg\forall y(a \neq y \to Hay) \checkmark \\
\neg\forall y(a \neq y \to Hya) \checkmark \\
\exists y\neg(a \neq y \to Hay) \checkmark b \\
\neg(a \neq b \to Hab) \checkmark \\
a \neq b \\
\neg Hab \\
\exists y\neg(a \neq y \to Hya) \checkmark c \\
\neg(a \neq c \to Hca) \checkmark \\
a \neq c \\
\neg Hca \\
\forall y\forall z(a = y \vee a = z \vee y = z) \ \backslash b \\
\forall z(a = b \vee a = z \vee b = z) \ \backslash c \\
a = b \ \vee a = c \ \vee b = c \ \checkmark \\
\begin{array}{ccc}
a = b & a = c & b = c \\
\times & \times & \neg Hac \\
 & & \forall y(Hay \vee Hya) \ \backslash c \\
 & & (Hac \vee Hca) \checkmark \\
 & & \begin{array}{cc} Hac & Hca \\ \times & \times \end{array}
\end{array}
\end{array}$$

Valid.

[Q p.64]

(v) Ax: x is an athlete
Fx: x is a footballer
Gx: x is a golfer

$\exists x(Fx \land Ax)$
$\exists x(Gx \land Ax)$
$\therefore \exists x \exists y(Ax \land Ay \land x \neq y)$

$\exists x(Fx \land Ax)$ ✓a
$\exists x(Gx \land Ax)$ ✓b
$\neg\exists x \exists y(Ax \land Ay \land x \neq y)$ ✓
$\forall x \neg\exists y(Ax \land Ay \land x \neq y)$ $\backslash ab$
$Fa \land Aa$ ✓
Fa
Aa
$Gb \land Ab$ ✓
Gb
Ab
$\neg\exists y(Aa \land Ay \land a \neq y)$ ✓
$\forall y \neg(Aa \land Ay \land a \neq y)$ $\backslash ab$
$\neg(Aa \land Aa \land a \neq a)$ ✓

- $\neg Aa$ ×
- $\neg Aa$ ×
- $\neg a \neq a$ ✓
  $a = a$
  $\neg(Aa \land Ab \land a \neq b)$ ✓
  - $\neg Aa$ ×
  - $\neg Ab$ ×
  - $\neg a \neq b$ ✓
    $a = b$
    $\neg\exists y(Ab \land Ay \land b \neq y)$ ✓
    $\forall y \neg(Ab \land Ay \land b \neq y)$ $\backslash ab$
    $\neg(Ab \land Aa \land b \neq a)$ ✓
    - $\neg Ab$ ×
    - $\neg Aa$ ×
    - $\neg b \neq a$ ✓
      $b = a$
      $\neg(Ab \land Ab \land b \neq b)$ ✓
      - $\neg Ab$ ×
      - $\neg Ab$ ×
      - $\neg b \neq b$ ✓
        $b = b$
        Fb
        Ga
        ↑

Invalid. Countermodel:
Domain: $\{1\}$
Referents: $a : 1$ $b : 1$
Extensions: $F : \{1\}$ $G : \{1\}$ $A : \{1\}$ [Q p.64]

(vi) Pxy: x is a part of y

$\forall x Pxx$
$\therefore \forall x \exists y \exists z(Pyx \wedge Pzx \wedge y \neq z)$

$$
\begin{array}{c}
\forall x Pxx \ \backslash a \\
\neg\forall x \exists y \exists z(Pyx \wedge Pzx \wedge y \neq z) \ \checkmark \\
\exists x \neg\exists y \exists z(Pyx \wedge Pzx \wedge y \neq z) \ \checkmark a \\
\neg\exists y \exists z(Pya \wedge Pza \wedge y \neq z) \ \checkmark \\
\forall y \neg\exists z(Pya \wedge Pza \wedge y \neq z) \ \backslash a \\
\neg\exists z(Paa \wedge Pza \wedge a \neq z) \ \checkmark \\
\forall z \neg(Paa \wedge Pza \wedge a \neq z) \ \backslash a \\
\neg(Paa \wedge Paa \wedge a \neq a) \ \checkmark \\
\begin{array}{ccc}
\neg Paa & \neg Paa & \neg a \neq a \ \checkmark \\
Paa & Paa & a = a \\
\times & \times & Paa \\
 & & \uparrow
\end{array}
\end{array}
$$

Invalid. Countermodel:
Domain: $\{1\}$
Referent of a : 1
Extension of P : $\{\langle 1,1 \rangle\}$ [Q p.64]

(vii) e: the Eiffel tower

$\exists x \exists y(x = e \wedge y = e \wedge x \neq y)$
$\therefore \neg \exists x x = e$

$$\begin{array}{c}
\exists x \exists y(x = e \wedge y = e \wedge x \neq y) \;\; \checkmark a \\
\neg\neg \exists x x = e \\
\exists y(a = e \wedge y = e \wedge a \neq y) \;\; \checkmark b \\
a = e \wedge b = e \wedge a \neq b \;\; \checkmark \\
a = e \\
b = e \\
a \neq b \\
e \neq b \\
e \neq e \\
\times
\end{array}$$

Valid. (The premise is logically false, hence any conclusion at all follows logically from it.) [Q p.64]

(viii) c: the Chief of Police
j: Jemima
m: I/me
Axy: x is afraid of y

$Amj \wedge Amc$
$\therefore j = c \vee \exists x \exists y(Amx \wedge Amy \wedge x \neq y)$

$$
\begin{array}{c}
Amj \wedge Amc \ \checkmark \\
\neg(j = c \vee \exists x \exists y(Amx \wedge Amy \wedge x \neq y)) \ \checkmark \\
Amj \\
Amc \\
j \neq c \\
\neg \exists x \exists y(Amx \wedge Amy \wedge x \neq y) \ \checkmark \\
\forall x \neg \exists y(Amx \wedge Amy \wedge x \neq y) \ \backslash j \\
\neg \exists y(Amj \wedge Amy \wedge j \neq y) \ \checkmark \\
\forall y \neg(Amj \wedge Amy \wedge j \neq y) \ \backslash c \\
\neg(Amj \wedge Amc \wedge j \neq c) \ \checkmark \\
\begin{array}{ccc}
\neg Amj & \neg Amc & \neg j \neq c \\
\times & \times & \times
\end{array}
\end{array}
$$

Valid. [Q p.64]

[Contents]

Answers 13.6.1.1

a:	Vance
c:	Joseph Conrad
i:	*The Inheritors*
l:	*Lord Jim*
s:	*The Shadow Line*
Axy:	x authored y
Fxy:	x is father of y
Rxy:	x reads y
Txy:	x is taller than y

1. $\exists x(\forall y(Ays \leftrightarrow y = x) \wedge c = x)$ [Q p.65]

2. $\exists x(\forall y(Ays \leftrightarrow y = x) \wedge Axl)$ [Q p.65]

3. $\exists x(\forall y(Ays \leftrightarrow y = x) \wedge \exists z(\forall y(Ayl \leftrightarrow y = z) \wedge x = z))$ [Q p.65]

4. $\exists x(\forall y(Ayl \leftrightarrow y = x) \wedge \forall y(Axy \rightarrow Ray))$ [Q p.65]

5. $Aci \wedge \neg\exists x(\forall y(Ayi \leftrightarrow y = x) \wedge c = x)$

   Another possible translation:

   $Aci \wedge \exists x(\forall y(Ayi \leftrightarrow y = x) \wedge c \neq x)$

   The first translation is preferable if we assume that the person making the claim being translated knows that *The Inheritors* was authored by two persons, one of whom was Joseph Conrad. [Q p.65]

6. $\exists x(\forall y(Ays \leftrightarrow y = x) \wedge \forall y(Ayl \rightarrow Txy))$ [Q p.65]

7. $\exists x(\forall y(Ays \leftrightarrow y = x) \wedge \exists yTyx)$ [Q p.65]

8. $\exists x(\forall y(Ays \leftrightarrow y = x) \wedge Txc) \wedge \exists x(\forall y(Ayl \leftrightarrow y = x) \wedge Tcx)$ [Q p.65]

9. $\exists x(\forall y(Ays \leftrightarrow y = x) \wedge \exists z(\forall y(Fyx \leftrightarrow y = z) \wedge Tzc))$ [Q p.65]

10. $\exists x(\forall y(Ays \leftrightarrow y = x) \wedge \exists z(\forall y(Fyx \leftrightarrow y = z) \wedge Tzx))$ [Q p.65]

[Contents]

Answers 13.6.2.1

Glossary:

a:	Vance
c:	Joseph Conrad
i:	*The Inheritors*
l:	*Lord Jim*
s:	*The Shadow Line*
Axy:	x authored y
Fxy:	x is father of y
Rxy:	x reads y
Txy:	x is taller than y

1. $c = \imath xAxs$ [Q p.65]
2. $A\imath xAxsl$ [Q p.65]
3. $\imath xAxs = \imath xAxl$ [Q p.65]
4. $\forall y(A\imath xAxly \rightarrow Ray))$ [Q p.65]
5. $Aci \wedge c \neq \imath xAxi$

   Note that this corresponds to the second translation given in Answers 13.6.1.1 Question 5 (see p.258). [Q p.65]
6. $\forall y(Ayl \rightarrow T\imath xAxsy)$ [Q p.65]
7. $\exists yTy\imath xAxs$ [Q p.65]
8. $T\imath xAxsc \wedge Tc\imath xAxl$ [Q p.65]
9. $T\imath yFy\imath xAxsc$ [Q p.65]
10. $T\imath yFy\imath xAxs\imath xAxs$ [Q p.65]

[Contents]

Answers 13.6.3.1

Glossary:

a:	Vance
c:	Joseph Conrad
i:	*The Inheritors*
l:	*Lord Jim*
s:	*The Shadow Line*
f:	the father of the author of *The Shadow Line*
i_2:	the author of *The Inheritors*
l_2:	the author of *Lord Jim*
s_2:	the author of *The Shadow Line*
Axy:	x authored y
Fxy:	x is father of y
Rxy:	x reads y
Txy:	x is taller than y

1. translation: $c = s_2$

   uniqueness postulate for s_2 (the author of *The Shadow Line*):
   $\forall x(Axs \leftrightarrow x = s_2)$ [Q p.66]

2. translation: As_2l

   uniqueness postulate for s_2: as above [Q p.66]

3. translation: $s_2 = l_2$

   uniqueness postulate for s_2: as above

   uniqueness postulate for l_2 (the author of *Lord Jim*):
   $\forall x(Axl \leftrightarrow x = l_2)$ [Q p.66]

4. translation: $\forall x(Al_2x \to Rax)$

   uniqueness postulate for l_2: as above [Q p.66]

5. translation: $Aci \land c \neq i_2$

   uniqueness postulate for i_2 (the author of *The Inheritors*):
   $\forall x(Axi \leftrightarrow x = i_2)$

   Note that this corresponds to the second translation given in Answers 13.6.1.1 Question 5 (see p.258). [Q p.66]

6. translation: $\forall x(Axl \rightarrow Ts_2x)$

   uniqueness postulate for s_2: as above [Q p.66]

7. translation: $\exists xTxs_2$

   uniqueness postulate for s_2: as above [Q p.66]

8. translation: $Ts_2c \wedge Tcl_2$

   uniqueness postulates for s_2 and l_2: as above [Q p.66]

9. translation: Tfc

   uniqueness postulate for f (the father of the author of *The Shadow Line*): $\forall x(Fxs_2 \leftrightarrow x = f)$

   uniqueness postulate for s_2 (which features in the uniqueness postulate for f): as above [Q p.66]

10. translation: Tfs_2

    uniqueness postulates for f and s_2: as above [Q p.66]

[Contents]

Answers 13.7.4

1. Glossary:

a_n:	n
$s(x,y)$:	$x+y$
$p(x,y)$:	$x \times y$
$q(x)$:	x squared
Ex:	x is even
Ox:	x is odd
Lxy:	$x < y$

   (i) $s(a_2,a_2) = a_4$ [Q p.66]

   (ii) $p(a_2,a_2) = a_4$ [Q p.66]

   (iii) $s(a_2,a_2) = p(a_2,a_2)$ [Q p.66]

   (iv) $q(a_2) = p(a_2,a_2)$ [Q p.66]

   (v) $\forall x \forall y q(s(x,y)) = p(s(x,y),s(x,y))$ [Q p.66]

(vi) $\forall x \forall y q(s(x,y)) = s(s(q(x), p(a_2, p(x,y))), q(y))$
Note that we have represented $2xy$ using the *two-place* function symbol p. This means that we have to represent it as $p(a_2, p(x,y))$—i.e. $2 \times (x \times y)$—or as $p(p(a_2,x),y)$—i.e. $(2 \times x) \times y$. Both choices are equally good. Similar comments apply to representing $x^2 + 2xy + y^2$ using the *two-place* function symbol s. [Q p.66]

(vii) $\forall x((Ex \vee Ox) \rightarrow Ep(a_2, x))$ [Q p.66]

(viii) $\forall x((Ox \rightarrow Op(a_3, x)) \wedge (Ex \rightarrow Ep(a_3, x)))$ [Q p.66]

(ix) $\forall x Lp(a_5, x)p(a_6, x)$ [Q p.66]

(x) $\forall x \forall y(Lxy \rightarrow Lp(a_3, x)p(a_4, y))$ [Q p.66]

2. (i) True [Q p.67]

(ii) False [Q p.67]

(iii) True [Q p.67]

(iv) True [Q p.67]

(v) True [Q p.67]

(vi) True [Q p.67]

(vii) False [Q p.67]

(viii) True [Q p.68]

(ix) True [Q p.68]

(x) False [Q p.68]

3. (i) False [Q p.68]

(ii) False [Q p.68]

(iii) True [Q p.68]

(iv) False [Q p.68]

(v) True [Q p.68]

(vi) True [Q p.68]

(vii) True [Q p.68]

(viii) True [Q p.68]

(ix) False [Q p.68]

(x) False [Q p.68]

4. (i) (a) Domain: $\{1,2\}$
Referents: a: 1 b: 2
Value of f: $\{\langle 1,1\rangle, \langle 2,1\rangle\}$
(b) Domain: $\{1,2\}$
Referents: a: 1 b: 2
Value of f: $\{\langle 1,1\rangle, \langle 2,2\rangle\}$ [Q p.69]

(ii) (a) Domain: $\{1,2\}$
Referents: a: 1 b: 2
Value of f: $\{\langle 1,1\rangle, \langle 2,2\rangle\}$
(b) Domain: $\{1,2\}$
Referents: a: 1 b: 2
Value of f: $\{\langle 1,1\rangle, \langle 2,1\rangle\}$ [Q p.69]

(iii) (a) No model: reflexivity of identity.
(b) Domain: $\{1,2\}$
Referents: a: 1 b: 2
Value of f: $\{\langle 1,1\rangle, \langle 2,1\rangle\}$ [Q p.69]

(iv) (a) Domain: $\{1,2\}$
Value of f: $\{\langle 1,1\rangle, \langle 2,1\rangle\}$
(b) No model: functions are totally defined. [Q p.69]

(v) (a) Domain: $\{1\}$
Value of f: $\{\langle 1,1\rangle\}$
(b) Domain: $\{1,2\}$
Value of f: $\{\langle 1,1\rangle, \langle 2,1\rangle\}$ [Q p.69]

(vi) (a) Domain: $\{1,2\}$
Value of s: $\{\langle 1,1,1\rangle, \langle 1,2,2\rangle, \langle 2,1,2\rangle, \langle 2,2,1\rangle\}$
(b) Domain: $\{1,2\}$
Value of s: $\{\langle 1,1,1\rangle, \langle 1,2,1\rangle, \langle 2,1,2\rangle, \langle 2,2,1\rangle\}$ [Q p.69]

(vii) (a) Domain: $\{1,2\}$
Values: f: $\{\langle 1,1\rangle, \langle 2,2\rangle\}$
s: $\{\langle 1,1,1\rangle, \langle 1,2,2\rangle, \langle 2,1,2\rangle, \langle 2,2,1\rangle\}$
(b) Domain: $\{1,2\}$
Values: f: $\{\langle 1,2\rangle, \langle 2,2\rangle\}$
s: $\{\langle 1,1,1\rangle, \langle 1,2,2\rangle, \langle 2,1,2\rangle, \langle 2,2,1\rangle\}$ [Q p.69]

(viii) (a) Domain: $\{1,2\}$
Values: f: $\{\langle 1,2\rangle, \langle 2,1\rangle\}$
s: $\{\langle 1,1,1\rangle, \langle 1,2,2\rangle, \langle 2,1,2\rangle, \langle 2,2,1\rangle\}$

(b) Domain: $\{1,2\}$
Values: f: $\{\langle 1,2\rangle, \langle 2,1\rangle\}$
s: $\{\langle 1,1,1\rangle, \langle 1,2,2\rangle, \langle 2,1,1\rangle, \langle 2,2,2\rangle\}$ [Q p.69]

(ix) (a) Domain: $\{1,2\}$
Values: f: $\{\langle 1,2\rangle, \langle 2,1\rangle\}$
s: $\{\langle 1,1,1\rangle, \langle 1,2,1\rangle, \langle 2,1,2\rangle, \langle 2,2,2\rangle\}$

(b) Domain: $\{1,2\}$
Values: f: $\{\langle 1,2\rangle, \langle 2,1\rangle\}$
s: $\{\langle 1,1,2\rangle, \langle 1,2,2\rangle, \langle 2,1,1\rangle, \langle 2,2,1\rangle\}$ [Q p.69]

(x) (a) Domain: $\{1,2\}$
Value of s: $\{\langle 1,1,1\rangle, \langle 1,2,1\rangle, \langle 2,1,2\rangle, \langle 2,2,1\rangle\}$

(b) No model: the proposition says that for *every* x and y in the domain, there is *exactly one* object z in the domain such that $s(x,y) = z$; it is built into the semantics of GPLIF that on every model, s (a two-place function symbol) is assigned a value of a sort that makes this true. [Q p.69]

[Contents]

Chapter 14

Metatheory

Answers 14.1.1.1

1. 0 [Q p.70]
2. 3 [Q p.70]
3. 1 [Q p.70]
4. 3 [Q p.70]
5. 3 [Q p.70]
6. 3 [Q p.70]
7. 5 [Q p.70]
8. 5 [Q p.70]
9. 10 [Q p.70]
10. 15 [Q p.70]

[Contents]

Answers 14.1.2.1

1. Let p be a path featuring $(\alpha \wedge \beta)$. Suppose there is a model $\mathfrak{M}$ on which every proposition on p is true. Let p' be the path obtained from p by adding α and β. By clause 3 of §9.4.2, we know that α must be true on $\mathfrak{M}$, since $(\alpha \wedge \beta)$ is. By clause 3 of §9.4.2, we know that β must

be true on $\mathfrak{M}$, since $(\alpha \wedge \beta)$ is. Since α and β are the only propositions that were added to p to get p′, we know that every proposition on p′ is true on $\mathfrak{M}$. So there is a model on which all propositions on p′ are true. Therefore the tree rule for unnegated conjunction is truth-preserving. [Q p.71]

2. Let p be a path featuring $\neg(\alpha \wedge \beta)$. Suppose there is a model $\mathfrak{M}$ on which every proposition on p is true. Let q be the path obtained from p by adding $\neg\alpha$, and let r be the path obtained from p by adding $\neg\beta$. Since $\neg(\alpha \wedge \beta)$ is true on $\mathfrak{M}$, we know that $(\alpha \wedge \beta)$ is false on $\mathfrak{M}$, by clause 2 of §9.4.2. Thus, either α is false on $\mathfrak{M}$ or β is false on $\mathfrak{M}$, by clause 3 of §9.4.2. Thus, either $\neg\alpha$ is true on $\mathfrak{M}$ or $\neg\beta$ is true on $\mathfrak{M}$, by clause 2 of §9.4.2. Thus, either all propositions on q are true on $\mathfrak{M}$, or all propositions on r are true on $\mathfrak{M}$. So either there is a model on which every proposition on q is true, or there is a model on which every proposition on r is true. Therefore the tree rule for negated conjunction is truth-preserving. [Q p.71]

3. Let p be a path featuring $(\alpha \rightarrow \beta)$. Suppose there is a model $\mathfrak{M}$ on which every proposition on p is true. Let q be the path obtained from p by adding $\neg\alpha$, and let r be the path obtained from p by adding β. Since $(\alpha \rightarrow \beta)$ is true on $\mathfrak{M}$, we know that either α is false on $\mathfrak{M}$ or β is true on $\mathfrak{M}$, by clause 6 of §9.4.2. Thus, either $\neg\alpha$ is true on $\mathfrak{M}$ or β is true on $\mathfrak{M}$, by clause 2 of §9.4.2. Thus, either all propositions on q are true on $\mathfrak{M}$, or all propositions on r are true on $\mathfrak{M}$. So either there is a model on which every proposition on q is true, or there is a model on which every proposition on r is true. Therefore the tree rule for negated conjunction is truth-preserving. [Q p.71]

4. Let p be a path featuring $\neg(\alpha \rightarrow \beta)$. Suppose there is a model $\mathfrak{M}$ on which every proposition on p is true. Let p′ be the path obtained from p by adding α and $\neg\beta$. Since $\neg(\alpha \rightarrow \beta)$ is true on $\mathfrak{M}$, we know that $\alpha \rightarrow \beta$ is false on $\mathfrak{M}$, by clause 2 of §9.4.2. So α is true on $\mathfrak{M}$ and β is false on $\mathfrak{M}$, by clause 6 of §9.4.2. So α is true on $\mathfrak{M}$ and $\neg\beta$ is true on $\mathfrak{M}$, by clause 2 of §9.4.2. So all propositions on p′ are true on $\mathfrak{M}$. So there is a model on which all propositions on p′ are true. Therefore the tree rule for negated conditional is truth-preserving. [Q p.71]

5. Let p be a path featuring $(\alpha \leftrightarrow \beta)$. Suppose there is a model $\mathfrak{M}$ on which every proposition on p is true. Let q be the path obtained from p by adding α and β, and let r be the path obtained from p by adding $\neg\alpha$ and $\neg\beta$. Either α and β are both true on $\mathfrak{M}$, or α and β are both

false on $\mathfrak{M}$, by clause 7 of §9.4.2. Thus either α and β are both true on $\mathfrak{M}$, or $\neg\alpha$ and $\neg\beta$ are both true on $\mathfrak{M}$, by clause 2 of §9.4.2. .So either there is a model on which every proposition on q is true, or there is a model on which every proposition on r is true. Therefore the tree rule for unnegated biconditional is truth-preserving. [Q p.71]

6. Let p be a path featuring $\neg(\alpha \leftrightarrow \beta)$. Suppose there is a model $\mathfrak{M}$ on which every proposition on p is true. Let q be the path obtained from p by adding α and $\neg\beta$, and let r be the path obtained from p by adding $\neg\alpha$ and β. We know that $(\alpha \leftrightarrow \beta)$ is false on $\mathfrak{M}$, by clause 2 of §9.4.2. So either α is true on $\mathfrak{M}$ and β is false on $\mathfrak{M}$, or α is false on $\mathfrak{M}$ and β is true on $\mathfrak{M}$, by clause 7 of §9.4.2. So either α and $\neg\beta$ are true on $\mathfrak{M}$, or $\neg\alpha$ and β are true on $\mathfrak{M}$, by clause 2 of §9.4.2. So either every proposition on q is true on $\mathfrak{M}$, or every proposition on r is true on $\mathfrak{M}$. So either there is a model on which every proposition on q is true, or there is a model on which every proposition on r is true. Therefore the tree rule for negated biconditional is truth-preserving.

   [Q p.71]

7. Let p be a path featuring $\neg\neg\alpha$. Suppose there is a model $\mathfrak{M}$ on which every proposition on p is true. Let p′ be the path obtained from p by adding α. We know that $\neg\alpha$ is false on $\mathfrak{M}$, by clause 2 of §9.4.2. And so we know that α is true on $\mathfrak{M}$, again by clause 2 of §9.4.2. So there is a model on which all propositions on p′ are true. Therefore the tree rule for negated conditional is truth-preserving. [Q p.71]

[Contents]

Answers 14.1.3.1

1. α is of the form $(\beta \wedge \delta)$; so the formula $\neg\alpha$ which we are considering is of the form $\neg(\beta \wedge \delta)$. Then $\neg\beta$ or $\neg\delta$ also occurs on p. The complexities of these wffs are less than the complexity of $\neg(\beta \wedge \delta)$, so by the induction hypothesis, whichever of them is on p is true on $\mathfrak{M}$. So $\neg(\beta \wedge \delta)$ is also true on $\mathfrak{M}$. [Q p.71]

2. α is of the form $(\beta \rightarrow \delta)$; so the formula $\neg\alpha$ which we are considering is of the form $\neg(\beta \rightarrow \delta)$. Then β and $\neg\delta$ also occur on p. The complexities of these wffs are less than the complexity of $\neg(\beta \rightarrow \delta)$, so by the induction hypothesis, β and $\neg\delta$ are true on $\mathfrak{M}$. So $\neg(\beta \rightarrow \delta)$ is also true on $\mathfrak{M}$. [Q p.71]

3. α is of the form $(\beta \leftrightarrow \delta)$; so the formula $\neg\alpha$ which we are considering is of the form $\neg(\beta \leftrightarrow \delta)$. Then either β and $\neg\delta$, or $\neg\beta$ and δ, also occur on p. The complexities of all these wffs are less than the complexity of $\neg(\beta \leftrightarrow \delta)$, so by the induction hypothesis, whichever pair of them is on p, both formulas in the pair are true on $\mathfrak{M}$. Either way, $\neg(\beta \leftrightarrow \delta)$ is also true on $\mathfrak{M}$. [Q p.71]

4. α is of the form $\exists \underline{x}\beta$; so the formula $\neg\alpha$ which we are considering is of the form $\neg\exists\underline{x}\beta$. Then $\forall\underline{x}\neg\beta$ also occurs on p. By the clause earlier in step (III) covering the case of wffs on p whose main operator is the universal quantifier, $\forall\underline{x}\neg\beta$ is true on $\mathfrak{M}$. So by the reasoning in §10.1.1 (which establishes that $\neg\exists\underline{x}\beta$ and $\forall\underline{x}\neg\beta$ are true and false in exactly the same models), $\neg\exists\underline{x}\beta$ is also true on $\mathfrak{M}$. [Q p.71]

5. γ is of the form $(\alpha \leftrightarrow \beta)$. Then either α and β, or $\neg\alpha$ and $\neg\beta$, also occur on p. (i) Suppose it is the former pair. The complexities of α and β are less than the complexity of $(\alpha \leftrightarrow \beta)$, so by the induction hypothesis α and β are true on $\mathfrak{M}$. So by the rule governing the truth of biconditionals in models, $(\alpha \leftrightarrow \beta)$ is also true on $\mathfrak{M}$. (ii) Suppose it is the latter pair. The complexities of $\neg\alpha$ and $\neg\beta$ are not necessarily less than the complexity of $(\alpha \leftrightarrow \beta)$: if β is an atomic wff (i.e. it has complexity 0), then $\neg\alpha$ and $(\alpha \leftrightarrow \beta)$ have the same complexity; if α is an atomic wff, then $\neg\beta$ and $(\alpha \leftrightarrow \beta)$ have the same complexity. If neither α nor β is atomic, then the complexities of $\neg\alpha$ and $\neg\beta$ are less than the complexity of $(\alpha \leftrightarrow \beta)$; so by the induction hypothesis $\neg\alpha$ and $\neg\beta$ are true on $\mathfrak{M}$; and so by the rule governing the truth of biconditionals in models, $(\alpha \leftrightarrow \beta)$ is also true on $\mathfrak{M}$. If one or both of α or β is atomic, we need to reason in a way similar to that used in the clause earlier in step (III) covering the case of wffs on p whose main operator is the conditional. [Q p.71]

[Contents]

Chapter 15

Other Methods of Proof

Answers 15.1.5

1. (i)

1.	$\neg P \to Q$	A
2.	$\neg P$	A
3.	Q	1, 2 (MP)

[Q p.72]

(ii)

1.	P	A
2.	$P \to (\neg Q \to P)$	(A1)
3.	$\neg Q \to P$	1, 2 (MP)

[Q p.72]

(iii)

1.	$\neg Q$	A
2.	$\neg Q \to (\neg P \to \neg Q)$	(A1)
3.	$(\neg P \to \neg Q) \to ((\neg P \to Q) \to P)$	(A3)
4.	$(\neg P \to \neg Q)$	1, 2 (MP)
5.	$(\neg P \to Q) \to P$	3, 4 (MP)

[Q p.72]

(iv)

1.	$P \to (P \to P)$	(A1)
2.	$P \to ((P \to P) \to P)$	(A1)
3.	$(P \to ((P \to P) \to P)) \to ((P \to (P \to P)) \to (P \to P))$	(A2)
4.	$(P \to (P \to P)) \to (P \to P)$	2, 3 (MP)
5.	$P \to P$	1, 4 (MP)

[Q p.72]

(v)

1.	$\neg(P \to \neg Q)$	A
2.	$(\neg Q \to \neg(P \to \neg Q)) \to ((\neg Q \to (P \to \neg Q)) \to Q)$	(A3)
3.	$\neg(P \to \neg Q) \to (\neg Q \to \neg(P \to \neg Q))$	(A1)
4.	$\neg Q \to \neg(P \to \neg Q)$	1, 3 (MP)
5.	$(\neg Q \to (P \to \neg Q)) \to Q$	2, 4 (MP)
6.	$\neg Q \to (P \to \neg Q)$	(A1)
7.	Q	5, 6 (MP)

[Q p.72]

(vi)

1.	P	A
2.	$\neg P$	A
3.	$(\neg Q \to \neg P) \to ((\neg Q \to P) \to Q)$	(A3)
4.	$\neg P \to (\neg Q \to \neg P)$	(A1)
5.	$\neg Q \to \neg P$	2, 4 (MP)
6.	$(\neg Q \to P) \to Q$	3, 5 (MP)
7.	$P \to (\neg Q \to P)$	(A1)
8.	$\neg Q \to P$	1, 7 (MP)
9.	Q	6, 8 (MP)

[Q p.72]

(vii) $P \wedge Q := \neg(P \to \neg Q)$

1.	$\neg(P \to \neg Q)$	A
2.	$\neg(P \to \neg Q) \to ((P \to \neg Q) \to \neg(P \to \neg Q))$	(A1)
3.	$(P \to \neg Q) \to \neg(P \to \neg Q)$	1, 2 (MP)

[Q p.72]

2. (i)

1.	$\neg(P \to \neg Q) \vdash Q$	$*$
2.	$\vdash \neg(P \to \neg Q) \to Q$	1, DT

$*$ Lines 1–7 of proof in Answer 1v. [Q p.72]

(ii) $(P \vee Q) := (\neg P \to Q)$

1.	$P, \neg P \vdash Q$	$*$
2.	$P \vdash (\neg P \to Q)$	1, DT
3.	$\vdash P \to (\neg P \to Q)$	2, DT

$*$ Lines 1–9 of proof in Answer 1vi. [Q p.72]

(iii)	1.	$((P \to Q) \to (P \to R))$	A
	2.	P	A
	3.	Q	A
	4.	$(Q \to (P \to Q))$	(A1)
	5.	$(P \to Q)$	3, 4 (MP)
	6.	$(P \to R)$	1, 5 (MP)
	7.	R	2, 6 (MP)
	8.	$((P \to Q) \to (P \to R)), P, Q \vdash R$	1–7
	9.	$((P \to Q) \to (P \to R)), P \vdash (Q \to R)$	8, DT
	10.	$((P \to Q) \to (P \to R)) \vdash (P \to (Q \to R))$	9, DT
	11.	$\vdash ((P \to Q) \to (P \to R)) \to (P \to (Q \to R))$	10, DT

[Q p.72]

(iv)	1.	$(P \to Q)$	A
	2.	$\neg Q$	A
	3.	$(\neg Q \to (\neg\neg P \to \neg Q))$	(A1)
	4.	$((\neg\neg P \to \neg Q) \to ((\neg\neg P \to Q) \to \neg P))$	(A3)
	5.	$(\neg\neg P \to \neg Q)$	2, 3 (MP)
	6.	$((\neg\neg P \to Q) \to \neg P)$	4, 5 (MP)
	7.	$(\neg\neg P \to Q)$	$*$
	8.	$\neg P$	6, 7 (MP)
	9.	$(P \to Q), \neg Q \vdash \neg P$	1–8
	10.	$(P \to Q) \vdash (\neg Q \to \neg P)$	9, DT
	11.	$\vdash (P \to Q) \to (\neg Q \to \neg P)$	10, DT

$*$ 1, line 7 of proof in Fig. 15.8 (p.396). [Q p.72]

(v)	1.	$P \to Q$	A
	2.	$P \to \neg Q$	A
	3.	$(\neg\neg P \to \neg Q) \to ((\neg\neg P \to Q) \to \neg P)$	(A3)
	4.	$\neg\neg P \to Q$	$*$
	5.	$\neg\neg P \to \neg Q$	†
	6.	$(\neg\neg P \to Q) \to \neg P$	3, 5 (MP)
	7.	$\neg P$	4, 6 (MP)

$*$ 1, line 7 of proof in Fig. 15.8 (p.396).

† 1, line 7 of a proof which is just like that in Fig. 15.8 (p.396) except that it has $\neg Q$ in place of Q throughout. [Q p.72]

(vi)

1.	$P \to Q$	A
2.	$\neg Q \to P$	A
3.	$(P \to Q) \to (\neg Q \to \neg P)$	$*$
4.	$\neg Q \to \neg P$	1, 3 (MP)
5.	$(\neg Q \to \neg P) \to ((\neg Q \to P) \to Q)$	(A3)
6.	$(\neg Q \to P) \to Q$	4, 5 (MP)
7.	Q	2, 6 (MP)

$*$ Line 11 of proof in Answer 2iv. [Q p.72]

(vii)

1.	$P \to (Q \to R)$	A
2.	Q	A
3.	P	A
4.	$Q \to R$	1, 3 (MP)
5.	R	2, 4 (MP)
6.	$P \to (Q \to R), Q, P \vdash R$	1–5
7.	$P \to (Q \to R), Q \vdash P \to R$	6, DT
8.	$P \to (Q \to R) \vdash Q \to (P \to R)$	7, DT
9.	$\vdash (P \to (Q \to R)) \to (Q \to (P \to R))$	8, DT

[Q p.72]

3. (i)

1.	P	A
2.	$(\neg P \to P) \to ((\neg P \to \neg P) \to \neg\neg P)$	(A9′)
3.	$P \to (\neg P \to P)$	(A1)
4.	$\neg P \to P$	1, 3 (MP)
5.	$(\neg P \to \neg P) \to \neg\neg P$	2, 4 (MP)
6.	$\neg P \to (\neg P \to \neg P)$	(A1)
7.	$(\neg P \to (\neg P \to \neg P)) \to ((\neg P \to ((\neg P \to \neg P) \to \neg P)) \to (\neg P \to \neg P))$	(A2′)
8.	$(\neg P \to ((\neg P \to \neg P) \to \neg P)) \to (\neg P \to \neg P)$	6, 7 (MP)
9.	$\neg P \to ((\neg P \to \neg P) \to \neg P)$	(A1)
10.	$\neg P \to \neg P$	8, 9 (MP)
11.	$\neg\neg P$	5, 10 (MP)
12.	$P \vdash \neg\neg P$	1–11
13.	$\vdash P \to \neg\neg P$	12, DT

[Q p.72]

(ii)	1.	$P \to \neg P$	A
	2.	$(P \to \neg P) \to ((P \to \neg\neg P) \to \neg P)$	(A9$'$)
	3.	$(P \to \neg\neg P) \to \neg P$	1, 2 (MP)
	4.	$P \to \neg\neg P$	$*$
	5.	$\neg P$	3, 4 (MP)

$*$ Line 13 of proof in Answer 3i

[Q p.73]

(iii)	1.	$P \to Q$	A
	2.	$\neg Q$	A
	3.	$(P \to Q) \to ((P \to \neg Q) \to \neg P)$	(A9$'$)
	4.	$(P \to \neg Q) \to \neg P$	1, 3 (MP)
	5.	$\neg Q \to (P \to \neg Q)$	(A1)
	6.	$P \to \neg Q$	2, 5 (MP)
	7.	$\neg P$	4, 6 (MP)
	8.	$P \to Q, \neg Q \vdash \neg P$	1–7
	9.	$P \to Q \vdash \neg Q \to \neg P$	8, DT

[Q p.73]

(iv)	1.	$\neg Q$	A
	2.	Q	A
	3.	$Q \to (\neg P \to Q)$	(A1)
	4.	$\neg Q \to (\neg P \to \neg Q)$	(A1)
	5.	$\neg P \to Q$	2, 3 (MP)
	6.	$\neg P \to \neg Q$	1, 4 (MP)
	7.	$(\neg P \to Q) \to ((\neg P \to \neg Q) \to \neg\neg P)$	(A9$'$)
	8.	$(\neg P \to \neg Q) \to \neg\neg P$	5, 7 (MP)
	9.	$\neg\neg P$	6, 8 (MP)
	10.	$\neg\neg P \to P$	(A10$'$)
	11.	P	9, 10 (MP)
	12.	$\neg Q, Q \vdash P$	1–11
	13.	$\neg Q \vdash Q \to P$	12, DT
	14.	$\vdash \neg Q \to (Q \to P)$	13, DT

[Q p.73]

(v)	1.	$P \wedge Q$	A
	2.	$(P \wedge Q) \to Q$	(A5$'$)
	3.	Q	1, 2 (MP)
	4.	$Q \to (P \to Q)$	(A1)
	5.	$P \to Q$	3, 4 (MP)

[Q p.73]

(vi)

1.	$\neg Q$	A
2.	$(P \to P) \to ((Q \to P) \to ((P \vee Q) \to P))$	(A8′)
3.	$P \to P$	$*$
4.	$(Q \to P) \to ((P \vee Q) \to P)$	2, 3 (MP)
5.	$\neg Q \to (Q \to P)$	†
6.	$Q \to P$	1, 5 (MP)
7.	$(P \vee Q) \to P$	4, 6 (MP)

$*$ Mimic lines 6–10 of the proof in Answer 3i, with P in place of $\neg P$ throughout. Alternatively, line 3 of the following proof:

1.	P	A
2.	$P \vdash P$	1 (i.e. 1–1)
3.	$\vdash P \to P$	2, DT

† line 14 of proof in Answer 3iv. [Q p.73]

(vii)

1.	$\neg P \wedge \neg Q$	A
2.	$(\neg P \wedge \neg Q) \to \neg P$	(A4′)
3.	$(\neg P \wedge \neg Q) \to \neg Q$	(A5′)
4.	$\neg P$	1, 2 (MP)
5.	$\neg Q$	1, 3 (MP)
6.	$\neg P \to ((P \vee Q) \to \neg P)$	(A1)
7.	$(P \vee Q) \to \neg P$	4, 6 (MP)
8.	$(P \vee Q) \to P$	$*$
9.	$((P \vee Q) \to P) \to (((P \vee Q) \to \neg P) \to \neg(P \vee Q))$	(A9′)
10.	$((P \vee Q) \to \neg P) \to \neg(P \vee Q)$	8, 9 (MP)
11.	$\neg(P \vee Q)$	7, 10 (MP)

$*$ 5, line 7 of proof in Answer 3vi. [Q p.73]

(viii)	1.	$\neg(P \vee Q)$	A
	2.	$P \to (P \vee Q)$	(A6′)
	3.	$\neg(P \vee Q) \to (P \to \neg(P \vee Q))$	(A1)
	4.	$P \to \neg(P \vee Q)$	1, 3 (MP)
	5.	$(P \to (P \vee Q)) \to ((P \to \neg(P \vee Q)) \to \neg P)$	(A9′)
	6.	$(P \to \neg(P \vee Q)) \to \neg P$	2, 5 (MP)
	7.	$\neg P$	4, 6 (MP)
	8.	$Q \to (P \vee Q)$	(A7′)
	9.	$\neg(P \vee Q) \to (Q \to \neg(P \vee Q))$	(A1)
	10.	$Q \to \neg(P \vee Q)$	1, 9 (MP)
	11.	$(Q \to (P \vee Q)) \to ((Q \to \neg(P \vee Q)) \to \neg Q)$	(A9′)
	12.	$(Q \to \neg(P \vee Q)) \to \neg Q$	8, 11 (MP)
	13.	$\neg Q$	10, 12 (MP)
	14.	$\neg P \to (\neg Q \to (\neg P \wedge \neg Q))$	(A3′)
	15.	$\neg Q \to (\neg P \wedge \neg Q)$	7, 14 (MP)
	16.	$\neg P \wedge \neg Q$	13, 15 (MP)

[Q p.73]

(ix)	1.	$(P \wedge \neg P) \to P$	(A4′)
	2.	$(P \wedge \neg P) \to \neg P$	(A5′)
	3.	$((P \wedge \neg P) \to P) \to (((P \wedge \neg P) \to \neg P) \to \neg(P \wedge \neg P))$	(A9′)
	4.	$((P \wedge \neg P) \to \neg P) \to \neg(P \wedge \neg P)$	1, 3 (MP)
	5.	$\neg(P \wedge \neg P)$	2, 4 (MP)

[Q p.73]

(x)

1.	$P \wedge \neg P$	A
2.	$(P \wedge \neg P) \to P$	(A4′)
3.	$(P \wedge \neg P) \to \neg P$	(A5′)
4.	P	1, 2 (MP)
5.	$\neg P$	1, 3 (MP)
6.	$P \to (\neg Q \to P)$	(A1)
7.	$\neg P \to (\neg Q \to \neg P)$	(A1)
8.	$\neg Q \to P$	4, 6 (MP)
9.	$\neg Q \to \neg P$	5, 7 (MP)
10.	$(\neg Q \to P) \to ((\neg Q \to \neg P) \to \neg\neg Q)$	(A9′)
11.	$(\neg Q \to \neg P) \to \neg\neg Q$	8, 10 (MP)
12.	$\neg\neg Q$	9, 11 (MP)
13.	$\neg\neg Q \to Q$	(A10′)
14.	Q	12, 13 (MP)
15.	$P \wedge \neg P \vdash Q$	1–14
16.	$\vdash (P \wedge \neg P) \to Q$	15, DT

[Q p.73]

(xi) $P \leftrightarrow P := \neg((P \to P) \to \neg(P \to P))$

1.	$P \to P$	$*$
2.	$(P \to P) \to (((P \to P) \to \neg(P \to P)) \to (P \to P))$	(A1)
3.	$((P \to P) \to \neg(P \to P)) \to (P \to P)$	1, 2 (MP)
4.	$(((P \to P) \to \neg(P \to P)) \to (P \to P)) \to$ $((((P \to P) \to \neg(P \to P)) \to \neg(P \to P)) \to \neg((P \to P) \to \neg(P \to P)))$	(A9)
5.	$(((P \to P) \to \neg(P \to P)) \to \neg(P \to P)) \to \neg((P \to P) \to \neg(P \to P))$	3, 4 (MP)
6.	$((P \to P) \to \neg(P \to P)) \to \neg(P \to P)$	†
7.	$\neg((P \to P) \to \neg(P \to P))$	5, 6 (MP)

$*$ See line 3 of the proof in Answer 3vi.

† The proof in Answer 3ii plus DT gives $\vdash (P \to \neg P) \to \neg P$. Mimic this proof, with $(P \to P)$ in place of P throughout, and the resulting proof plus DT gives:
$\vdash ((P \to P) \to \neg(P \to P)) \to \neg(P \to P)$. [Q p.73]

(xii)	1.	P	A
	2.	$\neg P$	A
	3.	$P \to (\neg Q \to P)$	(A1)
	4.	$\neg Q \to P$	1, 3 (MP)
	5.	$(\neg Q \to P) \to ((\neg Q \to \neg P) \to \neg\neg Q)$	(A9′)
	6.	$(\neg Q \to \neg P) \to \neg\neg Q$	4, 5 (MP)
	7.	$\neg P \to (\neg Q \to \neg P)$	(A1)
	8.	$\neg Q \to \neg P$	2, 7 (MP)
	9.	$\neg\neg Q$	6, 8 (MP)
	10.	$\neg\neg Q \to Q$	(A10′)
	11.	Q	9, 10 (MP)
	12.	$P, \neg P \vdash Q$	1–11
	13.	$P \vdash \neg P \to Q$	12, DT
	14.	$\vdash P \to (\neg P \to Q)$	13, DT

[Q p.73]

4. (i)

1.	$\forall x(Fx \to Gx)$	A
2.	Fa	A
3.	$\forall x(Fx \to Gx) \to (Fa \to Ga)$	(A4)
4.	$Fa \to Ga$	1, 3 (MP)
5.	Ga	2, 4 (MP)

[Q p.73]

(ii) $\forall x(Gx \vee Fx) := \forall x(\neg Gx \to Fx)$

1.	$\forall x Fx$	A
2.	$\forall x Fx \to Fx$	(A4)
3.	Fx	1, 2 (MP)
4.	$Fx \to (\neg Gx \to Fx)$	(A1)
5.	$(\neg Gx \to Fx)$	3, 4 (MP)
6.	$\forall x(\neg Gx \to Fx)$	4 (Gen)

[Q p.73]

(iii)	1.	$\forall x \forall y(Rxy \to Ryx)$	A
	2.	Rab	A
	3.	$\forall x \forall y(Rxy \to Ryx) \to \forall y(Ray \to Rya)$	(A4)
	4.	$\forall y(Ray \to Rya)$	1, 3 (MP)
	5.	$\forall y(Ray \to Rya) \to (Rab \to Rba)$	(A4)
	6.	$Rab \to Rba$	4, 5 (MP)
	7.	Rba	2, 6 (MP)

[Q p.73]

(iv) $\exists x Fx \ := \ \neg\forall x \neg Fx$

1.	$\neg\forall x \neg Fx \to \neg Ga$	A
2.	Ga	A
3.	$Ga \to (\neg\forall x \neg Fx \to Ga)$	(A1)
4.	$\neg\forall x \neg Fx \to Ga$	2, 3 (MP)
5.	$(\neg\forall x \neg Fx \to \neg Ga) \to ((\neg\forall x \neg Fx \to Ga) \to \forall x \neg Fx)$	(A3)
6.	$(\neg\forall x \neg Fx \to Ga) \to \forall x \neg Fx$	1, 5 (MP)
7.	$\forall x \neg Fx$	4, 6 (MP)
8.	$\neg\forall x \neg Fx \to \neg Ga, Ga \vdash \forall x \neg Fx$	1–7
9.	$\neg\forall x \neg Fx \to \neg Ga \vdash Ga \to \forall x \neg Fx$	8, DT

[Q p.73]

(v) Auxiliary proof:

1.	$\neg\neg\forall x\neg Fx$	A
2.	$\neg\neg\forall x\neg Fx \to (\neg\forall x\neg Fx \to \neg\neg\forall x\neg Fx)$	(A1)
3.	$(\neg\forall x\neg Fx \to \neg\neg\forall x\neg Fx) \to ((\neg\forall x\neg Fx \to \neg\forall x\neg Fx) \to \forall x\neg Fx)$	(A3)
4.	$\neg\forall x\neg Fx \to \neg\neg\forall x\neg Fx$	1, 2 (MP)
5.	$(\neg\forall x\neg Fx \to \neg\forall x\neg Fx) \to \forall x\neg Fx$	4, 3 (MP)
6.	$\neg\forall x\neg Fx \to \neg\forall x\neg Fx$	†
7.	$\forall x\neg Fx$	5, 6 (MP)
8.	$\forall x\neg Fx \to \neg Fa$	(A4)
9.	$\neg Fa$	7, 8 (MP)
10.	$\neg\neg\forall x\neg Fx \vdash \neg Fa$	1–9
11.	$\vdash \neg\neg\forall x\neg Fx \to \neg Fa$	10, DT

† Look at the proof that $\vdash_{A_2} P \to P$ in the commentary on line 3 of the proof in Answer 3vi (marked $*$). Mimic this proof, with any wff α in place of P throughout, and the resulting proof establishes $\vdash_{A_1^{\forall=}} \alpha \to \alpha$. (Note that the proof does not use (Gen), so the restriction on DT in $A_1^{\forall=}$ is automatically satisfied.)

Main proof:

$\exists xFx \ := \ \neg\forall x\neg Fx$

1.	Fa	A
2.	$(\neg\neg\forall x\neg Fx \to \neg Fa) \to$ $((\neg\neg\forall x\neg Fx \to Fa) \to \neg\forall x\neg Fx)$	(A3)
3.	$\neg\neg\forall x\neg Fx \to \neg Fa$	$*$
4.	$(\neg\neg\forall x\neg Fx \to Fa) \to \neg\forall x\neg Fx$	2, 3 (MP)
5.	$Fa \to (\neg\neg\forall x\neg Fx \to Fa)$	(A1)
6.	$\neg\neg\forall x\neg Fx \to Fa$	1, 5 (MP)
7.	$\neg\forall x\neg Fx$	4, 6 (MP)
8.	$Fa \vdash \neg\forall x\neg Fx$	1–7
9.	$\vdash Fa \to \neg\forall x\neg Fx$	8, DT

$*$ Line 11 of Auxiliary (above). [Q p.73]

(vi)	1.	Fa	A
	2.	$a = b$	A
	3.	$x = y \to (Fx \to Fy)$	(A7)
	4.	$\forall y(x = y \to (Fx \to Fy))$	3 (Gen)
	5.	$\forall x \forall y(x = y \to (Fx \to Fy))$	4 (Gen)
	6.	$\forall x \forall y(x = y \to (Fx \to Fy)) \to$	
		$\forall y(a = y \to (Fa \to Fy))$	(A4)
	7.	$\forall y(a = y \to (Fa \to Fy))$	5, 6 (MP)
	8.	$\forall y(a = y \to (Fa \to Fy)) \to$	
		$(a = b \to (Fa \to Fb))$	(A4)
	9.	$a = b \to (Fa \to Fb)$	7, 8 (MP)
	10.	$Fa \to Fb$	2, 9 (MP)
	11.	Fb	1, 10 (MP)

[Q p.73]

(vii)	1.	$\forall x \forall y x = y$	A
	2.	$\forall x \forall y x = y \to \forall y a = y$	(A4)
	3.	$\forall y a = y$	1, 2 (MP)
	4.	$\forall y a = y \to a = b$	(A4)
	5.	$a = b$	3, 4 (MP)

[Q p.73]

(viii)	1.	$a = b$	A
	2.	$a = c$	A
	3.	$(x = y \to (x = b \to y = b))$	(A7)
	4.	$\forall x(x = y \to (x = b \to y = b))$	3 (Gen)
	5.	$\forall x(x = y \to (x = b \to y = b)) \to$	
		$(a = y \to (a = b \to y = b))$	(A4)
	6.	$(a = y \to (a = b \to y = b))$	4, 5 (MP)
	7.	$\forall y(a = y \to (a = b \to y = b))$	6 (Gen)
	8.	$\forall y(a = y \to (a = b \to y = b)) \to$	
		$(a = c \to (a = b \to c = b))$	(A4)
	9.	$(a = c \to (a = b \to c = b))$	7, 8 (MP)
	10.	$(a = b \to c = b)$	2, 9 (MP)
	11.	$c = b$	1, 10 (MP)

[Q p.73]

(ix)

1.	$a = b$	A
2.	$(x = y \rightarrow (x = a \rightarrow y = a))$	(A7)
3.	$\forall x(x = y \rightarrow (x = a \rightarrow y = a))$	2 (Gen)
4.	$\forall x(x = y \rightarrow (x = a \rightarrow y = a)) \rightarrow$ $(a = y \rightarrow (a = a \rightarrow y = a))$	(A4)
5.	$(a = y \rightarrow (a = a \rightarrow y = a))$	3, 4 (MP)
6.	$\forall y(a = y \rightarrow (a = a \rightarrow y = a))$	5 (Gen)
7.	$\forall y(a = y \rightarrow (a = a \rightarrow y = a)) \rightarrow$ $(a = b \rightarrow (a = a \rightarrow b = a))$	(A4)
8.	$a = b \rightarrow (a = a \rightarrow b = a)$	6, 7 (MP)
9.	$a = a \rightarrow b = a$	1, 8 (MP)
10.	$\forall x x = x$	(A6)
11.	$\forall x x = x \rightarrow a = a$	(A4)
12.	$a = a$	10, 11 (MP)
13.	$b = a$	9, 12 (MP)
14.	$a = b \vdash b = a$	1–13
15.	$\vdash a = b \rightarrow b = a$	14, DT

[Q p.73]

(x) Auxiliary proof A:

1.	$\neg\neg a = b$	A
2.	$\neg\neg a = b \rightarrow (\neg a = b \rightarrow \neg\neg a = b)$	(A1)
3.	$(\neg a = b \rightarrow \neg\neg a = b) \rightarrow ((\neg a = b \rightarrow \neg a = b) \rightarrow a = b)$	(A3)
4.	$\neg a = b \rightarrow \neg\neg a = b$	1, 2 (MP)
5.	$(\neg a = b \rightarrow \neg a = b) \rightarrow a = b$	4, 3 (MP)
6.	$\neg a = b \rightarrow \neg a = b$	$*$
7.	$a = b$	6, 5 (MP)
8.	$\neg\neg a = b \vdash a = b$	1–7
9.	$\vdash \neg\neg a = b \rightarrow a = b$	8, DT

$*$ See comment on line 6 of proof in Answer 4v (marked †).

Auxiliary proof B:

1.	$\neg Fb$	A
2.	$\neg\neg a = b$	A
3.	$\neg\neg a = b \to a = b$	$*$
4.	$a = b$	2, 3 (MP)
5.	$a = b \to b = a$	†
6.	$b = a$	4, 5 (MP)
7.	$b = a \to (\neg Fb \to \neg Fa)$	‡
8.	$\neg Fb \to \neg Fa$	6, 7 (MP)
9.	$\neg Fa$	1, 8 (MP)
10.	$\neg Fb, \neg\neg a = b \vdash \neg Fa$	1–9
11.	$\neg Fb \vdash \neg\neg a = b \to \neg Fa$	10, DT

$*$ Line 9 of Auxiliary proof A (above)

† Line 15 of proof in Answer 4ix.

‡ To get this line, use the same strategy as in lines 3–9 of the proof in Answer 4vi.

Main proof:

1.	Fa	A
2.	$\neg Fb$	A
3.	$(\neg\neg a = b \to \neg Fa) \to$ $((\neg\neg a = b \to Fa) \to \neg a = b)$	(A3)
4.	$\neg\neg a = b \to \neg Fa$	$*$
5.	$(\neg\neg a = b \to Fa) \to \neg a = b$	3, 4 (MP)
6.	$Fa \to (\neg\neg a = b \to Fa)$	(A1)
7.	$\neg\neg a = b \to Fa$	1, 6 (MP)
8.	$\neg a = b$	5, 7 (MP)

$*$ 2, line 11 of Auxiliary proof B (above).

[Q p.73]

(xi)

1.	$\neg b = a$	A
2.	$\forall x(\neg Fx \rightarrow x = a)$	A
3.	$\forall x(\neg Fx \rightarrow x = a) \rightarrow (\neg Fb \rightarrow b = a)$	(A4)
4.	$\neg Fb \rightarrow b = a$	2, 3 (MP)
5.	$\neg b = a \rightarrow (\neg Fb \rightarrow \neg b = a)$	(A1)
6.	$\neg Fb \rightarrow \neg b = a$	1, 5 (MP)
7.	$(\neg Fb \rightarrow \neg b = a) \rightarrow ((\neg Fb \rightarrow b = a) \rightarrow Fb)$	(A3)
8.	$(\neg Fb \rightarrow b = a) \rightarrow Fb$	6, 7 (MP)
9.	Fb	4, 8 (MP)

[Q p.73]

(xii)

1.	$\forall x Fx$	A
2.	$\forall x Fx \rightarrow Fy$	(A4)
3.	Fy	1, 2 (MP)
4.	$\forall y Fy$	3 (Gen)
5.	$\forall x Fx \vdash \forall y Fy$	1–4
6.	$\vdash \forall x Fx \rightarrow \forall y Fy$	5, DT

[Q p.73]

5. Sketch of answer: Adding new *axioms* does not affect the proof of DT in §15.1.1.1 (provided we retain (MP) and the existing axioms which feature in the proof). Adding new *rules* does affect the proof: each rule requires separate treatment in the induction step. For the rule (MP), the treatment of it in the induction step employs axiom (A2) (see pp.394–5). For the rule (Gen), the treatment of it in the induction step must employ axiom (A5). But (A5) includes a restriction (i.e. "α contains no free $\underline{x}$"). This means that only uses of (Gen) *with a corresponding restriction* (i.e. the rule is not applied "using a variable which is free in β") can be handled. [Q p.73]

[Contents]

Answers 15.2.3

1. (i)

1	$\mid \neg P \to P$	
2	$\mid \mid \neg P$	
3	$\mid \mid P$	$1, 2\ (\to E)$
4	$\mid \mid \neg P$	2 (RI)
5	$\mid P$	$2\text{–}4\ (\neg E)$
6	$(\neg P \to P) \to P$	$1\text{–}5\ (\to I)$

[Q p.74]

(ii)

1	$\mid (A \to C)$	
2	$\mid \mid (B \to C)$	
3	$\mid \mid \mid (A \vee B)$	
4	$\mid \mid \mid \mid A$	
5	$\mid \mid \mid \mid C$	$1, 4\ (\to E)$
6	$\mid \mid \mid \mid B$	
7	$\mid \mid \mid \mid C$	$2, 6\ (\to E)$
8	$\mid \mid \mid C$	$3, 4\text{–}5, 6\text{–}7\ (\vee E)$

[Q p.74]

(iii)

1	$\mid \neg\neg P$	
2	$\mid \mid \neg P$	
3	$\mid \mid \neg P$	2 (RI)
4	$\mid \mid \neg\neg P$	1 (RI)
5	$\mid P$	$2\text{–}4\ (\neg E)$
6	$\neg\neg P \to P$	$1\text{–}5\ (\to I)$

[Q p.74]

(iv)

$$
\begin{array}{rll}
1 & \mid\ \neg(A \vee B) & \\
2 & \mid\ \mid\ A & \\
3 & \mid\ \mid\ A \vee B & 2\ (\vee I) \\
4 & \mid\ \mid\ \neg(A \vee B) & 1\ (\mathrm{RI}) \\
5 & \mid\ \neg A & 2\text{–}4\ (\neg I) \\
6 & \mid\ \mid\ B & \\
7 & \mid\ \mid\ A \vee B & 6\ (\vee I) \\
8 & \mid\ \mid\ \neg(A \vee B) & 1\ (\mathrm{RI}) \\
9 & \mid\ \neg B & 6\text{–}8\ (\neg I) \\
10 & \mid\ \neg A \wedge \neg B & 5, 9\ (\wedge I)
\end{array}
$$

[Q p.74]

(v)

$$
\begin{array}{rll}
1 & \mid\ A & \\
2 & \mid\ \mid\ \neg A & \\
3 & \mid\ \mid\ \mid\ \neg B & \\
4 & \mid\ \mid\ \mid\ A & 1\ (\mathrm{RI}) \\
5 & \mid\ \mid\ \mid\ \neg A & 2\ (\mathrm{RI}) \\
6 & \mid\ \mid\ B & 3\text{–}5\ (\neg E)
\end{array}
$$

[Q p.74]

(vi)

$$
\begin{array}{rll}
1 & \mid\ A \to B & \\
2 & \mid\ \mid\ B \to C & \\
3 & \mid\ \mid\ \mid\ A & \\
4 & \mid\ \mid\ \mid\ B & 1, 3\ (\to E) \\
5 & \mid\ \mid\ \mid\ C & 2, 4\ (\to E) \\
6 & \mid\ \mid\ A \to C & 3\text{–}5\ (\to I)
\end{array}
$$

[Q p.74]

(vii)

1	$P \to Q$	
2	$\vert\ \neg Q$	
3	$\vert\ \vert\ P$	
4	$\vert\ \vert\ Q$	1, 3 $(\to E)$
5	$\vert\ \vert\ \neg Q$	2 (RI)
6	$\vert\ \neg P$	3–5 $(\neg I)$
7	$\neg Q \to \neg P$	2–6 $(\to I)$

[Q p.74]

(viii)

1	$A \vee B$	
2	$\vert\ \neg A$	
3	$\vert\ A \vee B$	1 (RI)
4	$\vert\ \vert\ A$	
5	$\vert\ \vert\ \vert\ \neg B$	
6	$\vert\ \vert\ \vert\ A$	4 (RI)
7	$\vert\ \vert\ \vert\ \neg A$	2 (RI)
8	$\vert\ \vert\ B$	5–7 $(\neg E)$
9	$\vert\ \vert\ B$	
10	$\vert\ \vert\ B$	8 (RI)
11	$\vert\ B$	3, 4–8, 9–10 $(\vee E)$

[Q p.74]

(ix)

1	$P \to R$	
2	$Q \to R$	
3	$P \vee Q$	
4	P	
5	R	$1, 4\ (\to E)$
6	Q	
7	R	$2, 6\ (\to E)$
8	R	$3, 4\text{–}5, 6\text{–}7\ (\vee E)$

[Q p.74]

(x)

1	$P \to Q$	
2	$P \wedge \neg Q$	
3	P	$2\ (\wedge E)$
4	Q	$1, 3\ (\to E)$
5	$\neg Q$	$2\ (\wedge E)$
6	$\neg(P \wedge \neg Q)$	$2\text{–}5\ (\neg I)$

[Q p.74]

2. (i) $\vdash_{N_2} A \vee \neg A$

1	$\neg(A \vee \neg A)$	
2	$\quad A$	
3	$\quad A \vee \neg A$	2 $(\vee I)$
4	$\quad (A \vee \neg A) \wedge \neg(A \vee \neg A)$	1, 3 $(\wedge I)$
5	$\neg A$	2–4 $(\neg I')$
6	$\quad \neg A$	
7	$\quad A \vee \neg A$	6 $(\vee I)$
8	$\quad (A \vee \neg A) \wedge \neg(A \vee \neg A)$	1, 7 $(\wedge I)$
9	$\neg\neg A$	6–8 $(\neg I')$
10	A	9 $(\neg\neg E)$
11	$A \wedge \neg A$	5, 10 $(\wedge I)$
12	$\neg\neg(A \vee \neg A)$	1–11 $(\neg I')$
13	$A \vee \neg A$	12 $(\neg\neg E)$

$\vdash_{N_3} A \vee (A \rightarrow \bot)$

1	$A \vee (A \rightarrow \bot)$	(TND)

$\vdash_{N_4} A \vee (A \rightarrow \bot)$

1	$\quad A$	
2	$\quad A \vee (A \rightarrow \bot)$	1 $(\vee I)$
3	$\quad A \rightarrow \bot$	
4	$\quad A \vee (A \rightarrow \bot)$	3 $(\vee I)$
5	$A \vee (A \rightarrow \bot)$	1–2, 3–4 (NCD)

$\vdash_{N_5} \; A \vee (A \rightarrow \bot)$

1	$((A \vee (A \rightarrow \bot)) \rightarrow \bot$	
2	$\quad A \rightarrow \bot$	
3	$\quad A \vee (A \rightarrow \bot)$	2 $(\vee I)$
4	$\quad \bot$	1, 3 $(\rightarrow E)$
5	A	2–4 (RAA)
6	$A \vee (A \rightarrow \bot)$	5 $(\vee I)$
7	$\bot$	1, 6 $(\rightarrow E)$
8	$A \vee (A \rightarrow \bot)$	1–7 (RAA)

[Q p.74]

(ii) $(A \wedge \neg A) \vdash_{N_2} \; B$

1	$(A \wedge \neg A)$	
2	B	1 $(\neg E')$

$A \wedge (A \rightarrow \bot) \vdash_{N_3/N_4/N_5} \; B$

1	$A \wedge (A \rightarrow \bot)$	
2	A	1 $(\wedge E)$
3	$A \rightarrow \bot$	1 $(\wedge E)$
4	$\bot$	2, 3 $(\rightarrow E)$
5	B	4 $(\bot E)$

[Q p.74]

(iii) $\vdash_{N_2} (\neg\neg A \rightarrow A)$

1	$\neg\neg A$	
2	A	1 $(\neg\neg E)$
3	$\neg\neg A \rightarrow A$	1–2 $(\rightarrow I)$

$\vdash_{N_3} ((A \to \bot) \to \bot) \to A$

1	$\vert\ (A \to \bot) \to \bot$	
2	$\vert\ A \vee (A \to \bot)$	(TND)
3	$\vert\ \vert\ A$	
4	$\vert\ \vert\ A$	3 (RI)
5	$\vert\ \vert\ A \to \bot$	
6	$\vert\ \vert\ \bot$	1, 5 ($\to E$)
7	$\vert\ \vert\ A$	6 ($\bot E$)
8	$\vert\ A$	2, 3–4, 5–7 ($\vee E$)
9	$((A \to \bot) \to \bot) \to A$	1–8 ($\to$I)

$\vdash_{N_4} ((A \to \bot) \to \bot) \to A$

1	$\vert\ (A \to \bot) \to \bot$	
2	$\vert\ \vert\ A$	
3	$\vert\ \vert\ A$	2 (RI)
4	$\vert\ \vert\ A \to \bot$	
5	$\vert\ \vert\ \bot$	1, 4 ($\to$E)
6	$\vert\ \vert\ A$	5 ($\bot E$)
7	$\vert\ A$	2–3, 4–6 (NCD)
8	$((A \to \bot) \to \bot) \to A$	1–7 ($\to I$)

$\vdash_{N_5} ((A \to \bot) \to \bot) \to A$

1	$\vert\ (A \to \bot) \to \bot$	
2	$\vert\ \vert\ (A \to \bot)$	
3	$\vert\ \vert\ \bot$	1, 2 ($\to E$)
4	$\vert\ A$	2–3 (RAA)
5	$((A \to \bot) \to \bot) \to A$	1–4 ($\to I$)

[Q p.74]

(iv) $\vdash_{N_2} \neg(A \wedge \neg A)$

1	$\quad\vert\ A \wedge \neg A$	
2	$\quad\vert\ A \wedge \neg A$	1 (RI)
3	$\neg(A \wedge \neg A)$	1–2 ($\neg I'$)

$\vdash_{N_3/N_4/N_5} (A \wedge (A \rightarrow \bot)) \rightarrow \bot$

1	$\quad\vert\ A \wedge (A \rightarrow \bot)$	
2	$\quad\vert\ A$	1 ($\wedge E$)
3	$\quad\vert\ A \rightarrow \bot$	1 ($\wedge E$)
4	$\quad\vert\ \bot$	2, 3 ($\rightarrow$E)
5	$(A \wedge (A \rightarrow \bot)) \rightarrow \bot$	1–4 ($\rightarrow I$)

[Q p.74]

3. (i)

1	$\quad\vert\ Fa$	
2	$\quad\vert\ Fa$	1 (RI)
3	$Fa \rightarrow Fa$	1–2 ($\rightarrow I$)
4	$\forall x(Fx \rightarrow Fx)$	3 ($\forall I$)

[Q p.74]

(ii)

1	$\vert\ \exists x(Fx \wedge Gx)$	
2	$\vert\quad\vert\ Fa \wedge Ga$	
3	$\vert\quad\vert\ Fa$	2 ($\wedge E$)
4	$\vert\quad\vert\ \exists x Fx$	3 ($\exists I$)
5	$\vert\quad\vert\ Ga$	2 ($\wedge E$)
6	$\vert\quad\vert\ \exists x Gx$	5 ($\exists I$)
7	$\vert\quad\vert\ \exists x Fx \wedge \exists x Gx$	4, 6 ($\wedge I$)
8	$\vert\ \exists x Fx \wedge \exists x Gx$	1, 2–7 ($\exists E$)

[Q p.74]

(iii)

1	$\forall x(Fx \to Gx)$	
2	$\vert\ \neg\exists xGx$	
3	$\vert\ \vert\ \exists xFx$	
4	$\vert\ \vert\ \vert\ Fa$	
5	$\vert\ \vert\ \vert\ Fa \to Ga$	$1\ (\forall E)$
6	$\vert\ \vert\ \vert\ Ga$	$4, 5\ (\to E)$
7	$\vert\ \vert\ \vert\ \exists xGx$	$6\ (\exists I)$
8	$\vert\ \vert\ \exists xGx$	$3, 4\text{–}7\ (\exists E)$
9	$\vert\ \vert\ \neg\exists xGx$	2 (RI)
10	$\vert\ \neg\exists xFx$	$3\text{–}9\ (\neg I)$

[Q p.74]

(iv)

1	$\forall x(Fx \to x = a)$	
2	$\vert\ Fb$	
3	$\vert\ Fb \to b = a$	$1\ (\forall E)$
4	$\vert\ b = a$	$2, 3\ (\to E)$
5	$\vert\ a = a$	$(= I)$
6	$\vert\ a = b$	$4, 5\ (= E)$
7	$Fb \to a = b$	$2\text{–}6\ (\to I)$

[Q p.74]

(v)

1	$\forall x \forall y x = y$	
2	$\quad Raa$	
3	$\quad\quad \neg Rbc$	
4	$\quad\quad \forall y b = y$	$1\ (\forall E)$
5	$\quad\quad b = a$	$4\ (\forall E)$
6	$\quad\quad \forall y(a = y)$	$1\ (\forall E)$
7	$\quad\quad a = c$	$6\ (\forall E)$
8	$\quad\quad \neg Rac$	$3, 5\ (= E)$
9	$\quad\quad Raa$	2 (RI)
10	$\quad\quad \neg Raa$	$7, 8\ (= E)$
11	$\quad Rbc$	$3\text{–}10\ (\neg E)$
12	$\quad \forall y Rby$	$11\ (\forall I)$
13	$\quad \forall x \forall y Rxy$	$12\ (\forall I)$

[Q p.74]

(vi)

1	$\quad \forall x Rxx$	
2	$\quad Raa$	1 (∀E)
3	$\quad \exists y Ray$	2 (∃I)
4	$\quad \forall x \exists y Rxy$	3 (∀I)
5	$\forall x Rxx \to \forall x \exists y Rxy$	1–4 (→I)

[Q p.74]

(vii)

1	$\exists x Fx$	
2	$\quad Fa$	
3	$\quad\quad \forall x \neg Fx$	
4	$\quad\quad Fa$	2 (RI)
5	$\quad\quad \neg Fa$	3 ($\forall E$)
6	$\quad \neg\forall x \neg Fx$	3–5 ($\neg I$)
7	$\neg\forall x \neg Fx$	1, 2–6 ($\exists E$)
8	$\exists x Fx \rightarrow \neg\forall x \neg Fx$	1–7 ($\rightarrow I$)

[Q p.74]

(viii)

1	$\neg\exists x Fx$	
2	$\quad Fa$	
3	$\quad \exists x Fx$	2 ($\exists I$)
4	$\quad \neg\exists x Fx$	1 (RI)
5	$\neg Fa$	2–4 ($\neg I$)
6	$\forall x \neg Fx$	5 ($\forall I$)

[Q p.74]

(ix)

1	$\forall x x = a$	
2	$\quad \neg b = c$	
3	$\quad b = a$	1 ($\forall E$)
4	$\quad c = a$	1 ($\forall E$)
5	$\quad \neg a = c$	2, 3 ($= E$)
6	$\quad a = a$	($= I$)
7	$\quad \neg a = a$	4, 5 ($= E$)
8	$b = c$	2–7 ($\neg E$)

[Q p.74]

(x)

1	$Fa \wedge \neg Fb$	
2	$\quad a = b$	
3	$\quad Fa \wedge \neg Fa$	1, 2 (= E)
4	$\quad Fa$	3 ($\wedge E$)
5	$\quad \neg Fa$	3 ($\wedge E$)
6	$\neg a = b$	2–5 ($\neg I$)
7	$(Fa \wedge \neg Fb) \rightarrow \neg a = b$	1–6 ($\rightarrow I$)
8	$\forall y((Fa \wedge \neg Fy) \rightarrow \neg a = y)$	7 ($\forall I$)
9	$\forall x \forall y((Fx \wedge \neg Fy) \rightarrow \neg x = y)$	8 ($\forall I$)

[Q p.74]

4. (i)

- Rules of N_1 reformulated in list style:

$\rightarrow$ introduction:

$\{m\}$	$m.$	α	A
$\Delta \cup \{m\}$	$n.$	β	
Δ	$k.$	$\alpha \rightarrow \beta$	$m, n\ (\rightarrow I)$

$\rightarrow$ elimination:

Γ	$m.$	α	
Δ	$n.$	$\alpha \rightarrow \beta$	
$\Gamma \cup \Delta$	$k.$	β	$m, n\ (\rightarrow E)$

$\wedge$ introduction:

Γ	$m.$	α	
Δ	$n.$	β	
$\Gamma \cup \Delta$	$k.$	$\alpha \wedge \beta$	$m, n\ (\wedge I)$

$\wedge$ elimination:

Γ	$m.$	$\alpha \wedge \beta$	
Γ	$k.$	α (or β)	$m\ (\wedge E)$

$\neg$ introduction:

$\{m\}$	$m.$	α	A
Δ or $\Delta \cup \{m\}$	$n.$	β	
Γ or $\Gamma \cup \{m\}$	$o.$	$\neg\beta$	
$\Delta \cup \Gamma$	$k.$	$\neg\alpha$	$m, n, o\ (\neg I)$

$\neg$ **elimination:**

$\{m\}$	$m.$	$\neg\alpha$	A
Δ or $\Delta \cup \{m\}$	$n.$	β	
Γ or $\Gamma \cup \{m\}$	$o.$	$\neg\beta$	
$\Delta \cup \Gamma$	$k.$	α	$m, n, o\ (\neg E)$

$\vee$ **introduction:**

Γ	$m.$	α	
Γ	$k.$	$\alpha \vee \beta$ (or $\beta \vee \alpha$)	$m\ (\vee I)$

$\vee$ **elimination:**

Γ	$m.$	$\alpha \vee \beta$	
$\{n\}$	$n.$	α	A
$\Delta \cup \{n\}$	$o.$	γ	
$\{p\}$	$p.$	β	A
$\Lambda \cup \{p\}$	$q.$	γ	
$\Gamma \cup \Delta \cup \Lambda$	$k.$	γ	$m, n, o, p, q\ (\vee E)$

- Answers to Question 1, reformulated in list style:

1.(i)	$\{1\}$	1.	$\neg P \to P$	A
	$\{2\}$	2.	$\neg P$	A
	$\{1,2\}$	3.	P	$1, 2\ (\to E)$
	$\{1\}$	4.	P	$2, 3\ (\neg E)$
	$\varnothing$	5.	$(\neg P \to P) \to P$	$1, 4\ (\to I)$

[Q p.74]

1.(ii)	$\{1\}$	1.	$A \to C$	A
	$\{2\}$	2.	$B \to C$	A
	$\{3\}$	3.	$A \vee B$	A
	$\{4\}$	4.	A	A
	$\{1,4\}$	5.	C	$1, 4\ (\to E)$
	$\{6\}$	6.	B	A
	$\{2,6\}$	7.	C	$2, 6\ (\to E)$
	$\{1,2,3\}$	8.	C	$3, 4, 5, 6, 7\ (\vee E)$

[Q p.74]

1.(iii)	$\{1\}$	1.	$\neg\neg P$	A
	$\{2\}$	2.	$\neg P$	A
	$\{1\}$	3.	P	$1, 2\ (\neg E)$
	$\varnothing$	4.	$\neg\neg P \to P$	$1, 3\ (\to I)$

[Q p.74]

1.(iv)

$\{1\}$	1.	$\neg(A \vee B)$	A
$\{2\}$	2.	A	A
$\{2\}$	3.	$A \vee B$	$2\ (\vee I)$
$\{1\}$	4.	$\neg A$	$1, 2, 3\ (\neg I)$
$\{6\}$	5.	B	A
$\{6\}$	6.	$A \vee B$	$5\ (\vee I)$
$\{1\}$	7.	$\neg B$	$1, 5, 6\ (\neg I)$
$\{1\}$	8.	$\neg A \wedge \neg B$	$4, 7\ (\wedge I)$

[Q p.74]

1.(v)

$\{1\}$	1.	A	A
$\{2\}$	2.	$\neg A$	A
$\{3\}$	3.	$\neg B$	A
$\{1,2\}$	4.	B	$1, 2, 3\ (\neg E)$

[Q p.74]

1.(vi)

$\{1\}$	1.	$A \rightarrow B$	A
$\{2\}$	2.	$B \rightarrow C$	A
$\{3\}$	3.	A	A
$\{1,3\}$	4.	B	$1, 3\ (\rightarrow E)$
$\{1,2,3\}$	5.	C	$2, 4\ (\rightarrow E)$
$\{1,2\}$	6.	$A \rightarrow C$	$3, 5\ (\rightarrow I)$

[Q p.74]

1(vii)

$\{1\}$	1.	$P \rightarrow Q$	A
$\{2\}$	2.	$\neg Q$	A
$\{3\}$	3.	P	A
$\{1,3\}$	4.	Q	$1, 3\ (\rightarrow E)$
$\{1,2\}$	5.	$\neg P$	$2, 3, 4\ (\neg I)$
$\{1\}$	6.	$\neg Q \rightarrow \neg P$	$2, 5\ (\rightarrow I)$

[Q p.74]

1.(viii)

$\{1\}$	1.	$A \vee B$	A
$\{2\}$	2.	$\neg A$	A
$\{3\}$	3.	A	A
$\{4\}$	4.	$\neg B$	A
$\{2,3\}$	5.	B	$2, 3, 4\ (\neg E)$
$\{6\}$	6.	B	A
$\{1,2\}$	7.	B	$1, 3, 5, 6\ (\vee I)$

[Q p.74]

1.(ix)

$\{1\}$	1.	$P \to R$	A
$\{2\}$	2.	$Q \to R$	A
$\{3\}$	3.	$P \vee Q$	A
$\{4\}$	4.	P	A
$\{1,4\}$	5.	R	$1, 4\ (\to E)$
$\{6\}$	6.	Q	A
$\{2,6\}$	7.	R	$2, 6\ (\to E)$
$\{1,2,3\}$	8.	R	$3, 4, 5, 6, 7\ (\vee E)$

[Q p.74]

1.(x)

$\{1\}$	1.	$P \to Q$	A
$\{2\}$	2.	$P \wedge \neg Q$	A
$\{2\}$	3.	P	$2\ (\wedge E)$
$\{1,2\}$	4.	Q	$1, 3\ (\to E)$
$\{2\}$	5.	$\neg Q$	$2\ (\wedge E)$
$\{1\}$	6.	$\neg(P \wedge \neg Q)$	$2, 4, 5\ (\neg I)$

[Q p.74]

(ii)

- Rules of N_1 reformulated in stack style:

  $\rightarrow$ **introduction:**

$$\dfrac{\begin{array}{c}[\alpha]_n\\ \vdots\\ \beta\end{array}}{\alpha \rightarrow \beta}\ (\rightarrow I)_n$$

  $\rightarrow$ **elimination:**

$$\dfrac{\alpha \qquad \alpha \rightarrow \beta}{\beta}\ (\rightarrow E)$$

  $\wedge$ **introduction:**

$$\dfrac{\alpha \qquad \beta}{\alpha \wedge \beta}\ (\wedge I)$$

  $\wedge$ **elimination:**

$$\dfrac{\alpha \wedge \beta}{\alpha \ \text{(or } \beta)}\ (\wedge E)$$

  $\neg$ **introduction:**

$$\dfrac{\begin{array}{c}[\alpha]_n\\ \vdots\\ \beta\\ \neg\beta\end{array}}{\neg\alpha}\ (\neg I)_n$$

  $\neg$ **elimination:**

$$\dfrac{\begin{array}{c}[\neg\alpha]_n\\ \vdots\\ \beta\\ \neg\beta\end{array}}{\alpha}\ (\neg E)_n$$

  $\vee$ **introduction:**

$$\dfrac{\alpha}{\alpha \vee \beta \ \text{(or } \beta \vee \alpha)}\ (\vee I)$$

$\vee$ **elimination:**

$$\dfrac{\alpha \vee \beta \qquad \begin{matrix}[\alpha]_n \\ \vdots \\ \gamma\end{matrix} \qquad \begin{matrix}[\beta]_n \\ \vdots \\ \gamma\end{matrix}}{\gamma}\ (\vee E)_n$$

repetition (R):

$$\dfrac{\alpha \qquad \beta}{\alpha}\ (\mathrm{R})$$

(This is the analogue of repetition inward (RI). We need this rule to facilitate certain applications of $(\neg I)$ and $(\neg E)$; see how it is used in the answers below. Not every stack-style natural deduction system requires this rule. For example, it is not used in the system in Dirk van Dalen, *Logic and Structure* [Springer, Berlin, fourth edition, 2004], which is a stack form of something similar to system N_5 in the text pp.412–3.)

- Answers to Question 1, reformulated in stack style:

1.(i)

$$\dfrac{\dfrac{\dfrac{[\neg P \to P]_2 \qquad [\neg P]_1}{P}\ (\to E)}{P}\ (\neg E)_1}{(\neg P \to P) \to P}\ (\to I)_2$$

[Q p.74]

1.(ii)

$$\dfrac{A \vee B \qquad \dfrac{A \to C \qquad [A]_1}{C}\ (\to E) \qquad \dfrac{B \to C \qquad [B]_1}{C}\ (\to E)}{C}\ (\vee E)_1$$

[Q p.74]

1.(iii)

$$\dfrac{\dfrac{\dfrac{[\neg\neg P]_2 \qquad [\neg P]_1}{\neg\neg P}\ (\mathrm{R})}{P}\ (\neg E)_1}{\neg\neg P \to P}\ (\to I)_2$$

[Q p.74]

1.(iv)

$$\dfrac{\dfrac{\dfrac{\neg(A \vee B) \qquad \dfrac{[A]_1}{A \vee B}\,(\vee I)}{\neg(A \vee B)}\,\text{(R)}}{\neg A}\,(\neg I)_1 \qquad \dfrac{\dfrac{\neg(A \vee B) \qquad \dfrac{[B]_2}{A \vee B}\,(\vee I)}{\neg(A \vee B)}\,\text{(R)}}{\neg B}\,(\neg I)_2}{\neg A \wedge \neg B}\,(\wedge I)$$

[Q p.74]

1.(v)

$$\dfrac{\dfrac{\dfrac{A \qquad [\neg B]_1}{A}\,\text{(R)} \qquad \neg A}{\neg A}\,\text{(R)}}{B}\,(\neg E)_1$$

[Q p.74]

1.(vi)

$$\dfrac{\dfrac{\dfrac{A \to B \qquad [A]_1}{B}\,(\to E) \qquad B \to C}{C}\,(\to E)}{A \to C}\,(\to I)_1$$

[Q p.74]

1.(vii)

$$\dfrac{\dfrac{\dfrac{\dfrac{P \to Q \qquad [P]_1}{Q}\,(\to E) \qquad [\neg Q]_2}{\neg Q}\,\text{(R)}}{\neg P}\,(\neg I)_1}{\neg Q \to \neg P}\,(\to I)_2$$

[Q p.74]

1.(viii)

$$\dfrac{A \vee B \qquad \dfrac{\dfrac{\dfrac{[A]_2 \qquad [\neg B]_1}{A}\,\text{(R)} \qquad \neg A}{\neg A}\,\text{(R)}}{B}\,(\neg E)_1 \qquad [B]_2}{B}\,(\vee E)_2$$

[Q p.74]

1.(ix)

$$\dfrac{P \vee Q \qquad \dfrac{P \to R \qquad [P]_1}{R}\,(\to E) \qquad \dfrac{Q \to R \qquad [Q]_1}{R}\,(\to E)}{R}\,(\vee E)_1$$

[Q p.74]

1.(x)

$$
\begin{prooftree}
\AxiomC{$P \to Q$}
\AxiomC{$[P \wedge \neg Q]_1$}
\RightLabel{$(\wedge E)$}
\UnaryInfC{P}
\RightLabel{$(\to E)$}
\BinaryInfC{Q}
\AxiomC{$[P \wedge \neg Q]_2$}
\RightLabel{$(\wedge E)$}
\UnaryInfC{$\neg Q$}
\RightLabel{(R)}
\BinaryInfC{$\neg Q$}
\RightLabel{$(\neg I)_1$}
\UnaryInfC{$\neg(P \wedge \neg Q)$}
\RightLabel{$(\neg I)_2$}
\UnaryInfC{$\neg(P \wedge \neg Q)$}
\end{prooftree}
$$

[Q p.74]

5. Introduction:

$$\begin{array}{|l|} \hline \alpha \\ \hline \vdots \\ \beta \\ \hline \end{array}$$

$$\begin{array}{|l|} \hline \beta \\ \hline \vdots \\ \alpha \\ \hline \end{array}$$

► $\alpha \leftrightarrow \beta$

Elimination:

$\alpha \leftrightarrow \beta$
α (or β)
► β (or α, if β above)

[Q p.75]

[Contents]

Answers 15.3.3

1. (i) (a) $\{\alpha\} \Rightarrow \emptyset$ holds logically [Q p.75]
   (b) $\{\alpha\} \Rightarrow \emptyset$ does not hold logically [Q p.75]

   (ii) (a) $\{\alpha, \beta\} \Rightarrow \emptyset$ does not hold logically [Q p.75]
   (b) $\{\alpha\} \Rightarrow \{\beta\}$ and $\{\beta\} \Rightarrow \{\alpha\}$ both hold logically [Q p.75]

2. No answers supplied. [Q p.75]

3. No answers supplied. [Q p.75]

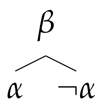
4.

$$\frac{\{\alpha, \beta\} \cup \Gamma \Rightarrow \Delta \qquad \Gamma \Rightarrow \Delta \cup \{\alpha, \beta\}}{\{\alpha \leftrightarrow \beta\} \cup \Gamma \Rightarrow \Delta} \,(\leftrightarrow\Rightarrow)$$

$$\frac{\{\alpha\} \cup \Gamma \Rightarrow \Delta \cup \{\beta\} \qquad \{\beta\} \cup \Gamma \Rightarrow \Delta \cup \{\alpha\}}{\Gamma \Rightarrow \Delta \cup \{\alpha \leftrightarrow \beta\}} \,(\Rightarrow\leftrightarrow)$$

[Q p.75]

5.



[Q p.75]

[Contents]

Chapter 16

Set Theory

There are no exercises for chapter 16. [Contents]